Monetary Policy Response to Oil Price Shocks

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Abstract

How should monetary authorities react to an oil price shock? This paper shows that, in a non-competitive economy, policies that perfectly stabilize prices entail large welfare costs, hence explaining the reluctance of policymakers to enforce them. The policy trade-off is non-trivial because oil (energy) is an input to both production and consumption.

As welfare-maximizing policies are hard to implement and communicate, I derive a simple interest rate rule that depends only on observables but mimics the optimal plan in all dimensions. The optimal rule is hard on core inflation but accommodates oil price changes.

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1 Introduction

In the last ten years a new macroeconomic paradigm has emerged centered around the New Keynesian (henceforth NK) model, which is at the core of the more involved and detailed dynamic stochastic general equilibrium (DSGE) models used for policy analysis at many central banks. Despite its apparent simplicity, the NK model is built on solid theoretical foundations and has therefore been used to draw normative conclusions on the appropriate response of monetary policy to economic shocks.

The general prescription arising from the canonical NK model (Goodfriend and King 2001) is that optimal monetary policy should aim at replicating the real allocation under flexible prices and wages, or natural output, which features constant average markups and no inflation. In the case of an oil price shock, policymakers should then simply stabilize prices, even if this leads to large drops in output and employment. Since the latter are considered efficient, monetary policy should focus on minimizing inflation volatility. There is a divine coincidence, i.e., an absence of trade-off between stabilizing inflation and stabilizing the welfare relevant output gap.\footnote{The expression "divine coincidence" was coined by Blanchard and Gali (2007a).}

The contrast between theory and practice is striking, however. When confronted with rising commodity prices, policymakers in inflation-targeting central banks do indeed perceive a trade-off. They typically favor a long-run approach to price stability by avoiding second-round effects — when wage inflation affects inflation expectations and ultimately leads to upward spiraling inflation — but by letting first-round effects on prices play out. So why the difference? Do policymakers systematically conduct
irrational, suboptimal policies? Or should we reconsider some of the assumptions em-
bedded in the NK model?

Clearly, this article is not the first to examine the implications of different mone-
tary policy reactions to oil price shocks. In a series of empirical papers, Bernanke et al. (1997, 2004) simulate counterfactual monetary policy experiments and find that monetary policy plays an important role in explaining the transmission of oil price shocks to the economy. Unfortunately, it appears that the evidence is not conclusive and that it is not robust across different policy regimes (Hamilton and Herrera 2004, Herrera and Pesavento 2009, Kilian and Lewis 2010).

A natural and alternative way to highlight the role of policy and its impact on agents’ expectations is to conduct the same exercise in microfounded, general equi-
librium models (see Leduc and Sill 2004 and Carlstrom and Fuerst 2006, for early examples). Although the results largely depend on the models’ specifications, one general insight from this line of work is that monetary policy potentially plays an im-
portant role in explaining the transmission of an oil shock to the economy.\textsuperscript{2} From a normative point of view, their analysis also suggests that tough medicine - a policy consisting of perfectly stabilizing prices - is the best policy.\textsuperscript{3} One limitation of their analysis, however, is that it is based on a comparison of the stabilization properties of simple monetary policy rules.

\textsuperscript{2}Abstracting from inflation dynamics, Carlstrom and Fuerst (2006) find that the real economy reacts similarly to an oil price shock under either a Taylor rule or a ‘Wickselian’ policy. They interpret this finding as showing that monetary policy does not play an important role in transmitting oil price shocks.

\textsuperscript{3}Dhawan and Jeske (2007) introduce energy use at the household level and obtain that stabilizing core inflation instead of headline inflation is preferable.
More recently, a rapidly growing literature has started to look into the design of optimal monetary policy responses to oil price shocks in calibrated or estimated NK models. Not surprisingly, the findings largely depend on the rigidities and production structures assumed. Yet, despite the differences, most studies come to the conclusion that there is indeed a trade-off between stabilizing inflation and the welfare relevant output gap.\(^4\)

One potential explanation that has attracted much attention is related to labor market rigidities. In contrast to earlier approaches where microfounded trade-offs were typically modeled as arising from exogenous markup shocks (Clarida et al. 1999), Blanchard and Galí (2007a; henceforth BG07) and Bodenstein et al. (2008b; henceforth BEG08) have shown that the presence of real wage rigidities leads to an endogenous policy trade-off: stabilizing prices introduces inefficient output variations. As shown in Erceg et al. (2000) and BEG08, however, assuming both sticky nominal wages and sticky prices does not in itself introduce a fundamental policy trade-off. Under reasonable calibration there will be no trade-off between stabilizing a composite index of price and wage inflation and the welfare relevant output gap.\(^5\)

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\(^4\) Plante (2009) finds that optimal monetary policy should stabilize a weighted average of core and nominal wage inflation. Winkler (2009) considers anticipated and unanticipated (deterministic) oil shocks and also finds that optimal policy cannot stabilize at the same time prices, wages and the welfare relevant output gap; indeed, following an oil price shock, optimal policy requires a larger output drop than under a traditional Taylor rule. This result can be contrasted with Kormilitsina (2009), who finds that optimal policy dampens output fluctuations compared to a Taylor rule. Her model is richer than Winkler’s and estimated on US data. Opening up the NK model, De Fiore et al. (2006) build a large three-country DSGE model - featuring two oil-importing countries and one oil exporting country - that they estimate on US and EU data. In contrast with the other papers mentioned above, the authors consider a whole array of shocks and search for the simple welfare maximizing rule. Their main finding in this context is that the optimal rule reacts strongly to inflation but accommodates output gap fluctuations, suggesting again a policy trade-off.

\(^5\) Different assumptions on nominal rigidities only give rise to different definitions of the optimal composite index to target (Woodford 2003 or Galí 2008).
Here, I focus on an alternative explanation that does not hinge on real wage rigidities but relies on the interplay between monopolistic competition and the lack of easy substitutes for energy to introduce an endogenous cost-push component in the NK Phillips curve. This study’s first contribution is to show that increases in oil prices lead to a *quantitatively meaningful* monetary policy trade-off once it is acknowledged that

(i) no fiscal transfer is available to policymakers to offset the steady-state distortion due to monopolistic competition, (ii) oil cannot easily be substituted by other factors in the short run and (iii) oil is an input both to production and to consumption (via its impact on the price of e.g., gasoline or heating oil).\(^6\)

In a nutshell, oil price hikes temporarily lead to higher oil cost shares. The larger the market distortion due to monopolistic competition, the larger the effect of a given increase in oil price on firms’ real marginal cost and the bigger is the drop in output and real wages required to stabilize prices. Perfectly stabilizing prices in a non-competitive economy introduces inefficient output variations and an endogenous monetary policy trade-off. The canonical NK model - because it typically assumes an efficient economy in the steady-state or Cobb-Douglas production (and hence constant cost shares) - dismisses out of hand the mere possibility of a trade-off.

While conditions (i) and (ii) are necessary to introduce a microfounded monetary

\(^6\)Montoro (2007) also considers an environment with flexible real wages but where oil enters the model as a non-produced input in the production function only. He finds that when oil is a gross complement to labor, an optimal monetary policy trade-off arises as oil shocks affect output and labor differently, generating a wedge between the effects on the utility of consumption and the disutility of labor. Drawing on the work of Barsky and Kilian (2004) and Kilian (2008), Nakov and Pescatori (2009) expand the canonical NK model to include an optimization-based model of the oil industry featuring both monopolistic and competitive oil producers. They show that the deviation of the best targeting rule from strict inflation targeting is substantial due to inefficient endogenous price markup variations in the oil industry.
policy trade-off, they are not sufficient to explain the policymakers’ concern for the real consequences of oil price shocks. Hence, this paper stresses that perfectly stabilizing inflation becomes really costly only when the impact of higher oil prices on households’ consumption is also taken into account. Changes in oil prices act as a distortionary tax on labor income and amplify the monetary policy trade-off. The lower the elasticity of substitution between energy and other consumption goods, the larger is the tax effect and the more detrimental are the consequences on employment and output of a given increase in oil prices. Importantly, these findings do not hinge on particular functional forms for production. All that is needed is a distorted economy where oil cost shares are allowed to vary with the price of oil.

Because of the policy trade-off, central banks can improve on both the perfect price stability solution and the recommendation obtained from a simple Taylor rule. And the welfare gains are large. One problem with welfare-based optimal policies, however, is that they rely on unobservables - like the efficient level of output - or various shadow prices, making them difficult to communicate and to implement. The second contribution of this paper is thus to derive a simple interest rate rule that mimics the optimal plan along all relevant dimensions but relies only on observables — namely core inflation and the growth rates of output and oil prices. It turns out that optimal policy is hard on core inflation but cushions the economy against the real consequences of an oil price shock by reacting strongly to output growth and negatively to oil price changes. In other words, the optimal response to a persistent increase in oil price

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7 See Orphanides and Williams (2003) for a thorough discussion of implementable monetary policy rules.
resembles the typical response of inflation targeting central banks: While long-term price stability is ensured by a credible commitment to stabilize inflation and inflation expectations, short-term real interest rates drop right after the shock to help dampen real output fluctuations.

The rest of the paper is structured as follows. Section 2 starts by building a two-sector NK model where oil enters as a gross complement to both production and consumption, thus featuring both core and headline inflation as in BEG08. Section 3 shows that the oil price shock leads to a monetary policy trade-off that is increasing in the degree of monopolistic competition and is inversely related to production and consumption elasticities of substitution. In Section 4, a linear-quadratic solution to the optimal policy problem in a timeless perspective is derived to show the sensitivity of the optimal weight on inflation in the policymaker’s loss function to structural changes in the economy. Section 5 derives a simple, implementable interest rate rule that replicates the optimal plan. Section 6 revisits the 1979 oil shock and computes the welfare losses associated with standard alternative suboptimal policy rules. Finally, Section 7 highlights the fact that this paper’s findings are robust to a production framework that features time-varying elasticities of substitution in the spirit of putty-clay models of energy use (Gilchrist and Williams 2005).

2 The model

Following BEG08, I assume a two-layer NK closed-economy setting composed of a core consumption good, which takes labor and oil as inputs, and a consumption basket
consisting of the core consumption good and oil.\textsuperscript{8} In order to keep the notations as simple as possible, there is only one source of nominal rigidity in this economy: core goods prices are sticky and firms set prices according to a Calvo scheme.\textsuperscript{9}

In contrast to BEG08, however, I relax the assumption of a unitary elasticity of substitution between oil and other goods and factors. I also explicitly consider a distorted economy: There is no fiscal transfer to offset the monopolistic competition distortion. The model being quite standard (see BEG08) I only present the main building blocks here. A full description is relegated to Appendix I available on the journal’s website.

2.1 Households

Households maximize utility out of consumption and leisure. Their consumption basket is defined as a CES aggregator $C_t$ of the core consumption goods basket $C_{Y,t}$ and the household’s demand for oil $O_{C,t}$\textsuperscript{10}

$$C_t = \left( 1 - \omega_{oc} \right) C_{Y,t}^{\frac{\chi}{\chi - 1}} + \omega_{oc} O_{C,t}^{\frac{\chi}{\chi - 1}},$$

where $\omega_{oc}$ is the oil quasi-share parameter and $\chi$ is the elasticity of substitution between oil and non-oil consumption goods.

Allowing for real wage rigidity (which may reflect some unmodeled imperfection in the labor market as in BG07), the labor supply condition relates the marginal rate of

\textsuperscript{8}The closed economy assumption allows one to ignore income distribution and international risk-sharing related issues.

\textsuperscript{9}Introducing nominal wage stickiness like in BEG08 would not change the thrust of the argument.

\textsuperscript{10}The consumption basket can be regarded as produced by perfectly competitive consumption distributors whose production function mirrors the preferences of households over consumption of oil and non-oil goods.
substitution between consumption and leisure to the geometric mean of real wages in periods $t$ and $t-1$.

$$\left(C_t^\sigma \nu H_t^\phi \right)^{(1-\eta)} = \frac{W_t}{P_t} \left(\frac{W_{t-1}}{P_{t-1}}\right)^{-\eta}. \quad (2)$$

In the benchmark calibration, i.e., unless stated otherwise, real wages are perfectly flexible, i.e., $\eta = 0$.

### 2.2 Firms

Aggregating over all firms producing core consumption goods, we get the total demand for intermediate goods $Y_t(i)$ as a function of the demand for core consumption goods $C_{Y,t}$

$$Y_t(i) = \left(\frac{P_{Y,t}(i)}{P_{Y,t}}\right)^{-\varepsilon} C_{Y,t}, \quad (3)$$

Each intermediate goods firm produces a good $Y_t(i)$ according to a constant returns-to-scale technology represented by the CES production function

$$Y_t(i) = \left(1 - \omega_{og} \right) H_t(i)^{\frac{\delta-1}{\delta}} + \omega_{og} O_{Y,t}(i)^{\frac{\delta-1}{\delta}}, \quad (4)$$

where $O_{Y,t}(i)$ and $H_t(i)$ are the quantities of oil and labor required to produce $Y_t(i)$ given the quasi-share parameters, $\omega_{og}$, and the elasticity of substitution between labor and oil, $\delta$.

The real marginal cost in terms of core consumption goods units is given by:

$$MC_t(i) \equiv MC_t = \left(1 - \omega_{og}\right)^{\delta} \left(\frac{W_t}{P_{Y,t}}\right)^{1-\delta} + \omega_{og} \left(\frac{P_{O,t}}{P_{Y,t}}\right)^{1-\delta} \left(\frac{1}{\delta}\right). \quad (5)$$
2.3 Government

To close the model, I assume that oil is extracted with no cost by the government, which sells it to the households and the firms and transfers the proceeds in a lump sum fashion to the households. I abstract from any other role for the government and assume that it runs a balanced budget in each and every period so that its budget constraint is simply given by:

$$T_t = P_{O,t}O_t,$$

for $O_t$ the total amount of oil supplied.

2.4 Calibration

For the sake of comparability, the model calibration closely follows BEG08. The quarterly discount factor $\beta$ is set at 0.993, which is consistent with an annualized real interest rate of 3 percent. The consumption utility function is chosen to be logarithmic ($\sigma = 1$) and the Frish elasticity of labor supply is set to unity ($\phi = 1$).

In the baseline calibration, I set the consumption, $\chi$, and production, $\delta$, oil elasticities of substitution to 0.3.\textsuperscript{11} Following BEG08, $\omega_{oc}$ is set such that the energy component of consumption (gasoline and fuel plus gas and electricity) equals 6 percent, which is in line with US NIPA data, and $\omega_{og}$ is chosen such that the energy share in production is 2 percent. Prices are assumed to have a duration of four quarters, so

\textsuperscript{11}Our calibration corresponds quite closely to the median estimate reported by Kilian and Murphy (2010). In order to assess the robustness of our findings, Appendix VII computes the welfare costs of suboptimal policies to alternative values for the consumption and production elasticities. The main insight from this analysis is that welfare costs are highly non-linear in the elasticities of substitution $\delta$ and $\chi$, reflecting the non-linear behavior of cost-push shocks documented in section 3, Figures 1 and 2. The lower the elasticities of substitution, the larger the cost-push shock and the higher the welfare costs from suboptimal monetary policy.
that $\theta = 0.75$. The core goods elasticity of substitution parameters $\varepsilon$ is set to 6, which implies a 20 percent markup of (core) prices over marginal costs. Finally, the logarithm of the real price of oil in terms of the consumption goods bundle $p_{o_t} = \log(P_{o,t})$ is supposed to follow an $AR(1)$ process ($\rho_o = 0.95$).

3 Divine coincidence?

Because of monopolistic competition in intermediate goods markets, the economy’s steady state is distorted. Production and employment are suboptimally low. Fully acknowledging this feature of the economy instead of subsidizing it away for convenience (as is usually done), entails important consequences for optimal policy when oil is difficult to substitute in the short run.

This section shows that the divine coincidence breaks down in a distorted economy when Cobb-Douglas production, a hallmark of the canonical NK model, is replaced by CES — or any production function that implies that oil cost shares vary with changes in oil prices.\textsuperscript{12} Cobb-Douglas production functions greatly simplify the analysis and permit nice closed-form solutions, but because they assume a unitary elasticity of substitution between factors they feature constant cost shares over the cycle regardless of the size of the monopolistic competition distortion. Following an oil price shock, natural (distorted) output drops just as much as efficient output and perfectly stabilizing prices is then the optimal policy to follow.

Yet, the case for a unitary elasticity of substitution between oil and other factors

\textsuperscript{12}See Section 7 for an illustration with a production function featuring time-varying price elasticity of oil demand: very low elasticity in the short run and unitary elasticity in the long run.
is not particularly compelling, especially at business cycle frequency. If oil is instead considered a gross complement for other factors (at least in the short run), the response of output to an oil price shock will depend on the size of the monopolistic competition distortion. The larger the distortion, the larger is the dynamic impact of a given oil price shock on the oil cost share — and therefore on output — in the flexible prices and wages equilibrium. Natural (distorted) output will drop more than efficient output. Strictly stabilizing inflation in the face of an oil price shock is thus no longer the optimal policy to follow; the divine coincidence breaks down.\textsuperscript{13}

Perfectly stabilizing prices becomes particularly costly when the impact of higher oil prices on households’ consumption is also taken into account. As stated in the introduction, increases in oil prices act as a tax on labor income; the lower the elasticity of substitution, the larger is the tax effect which amplifies the trade-off faced by monetary authorities.

Although an accurate welfare analysis requires a second order approximation of the household’s utility function and the model’s supply side (see Section 4 and Appendix III), some intuition on the mechanisms at stake can be gained by analyzing the properties of the log-linearized (see Appendix II for details) model economy at the flexible price and wage equilibrium.

\textsuperscript{13}In line with the general theory of the second best (Lipsey and Lancaster 1956), monetary authorities can aim at a higher level of welfare by trading some of the costs of inefficient output fluctuations against the distortion resulting from more inflation.
3.1 Flexible price and wage equilibrium (FPWE)

Solving the system for $mc_t = 0$ and assuming $\sigma = 1$ and $\eta = 0$ for simplicity, we get:\(^{14}\)

$$h_t = - \left[ \frac{\widetilde{\omega}_{oy} (1 - \delta)}{\Lambda} + \Theta \right] po_t, \quad (6)$$

$$y_t = - \left[ \frac{\widetilde{\omega}_{oy} (1 + \delta \phi)}{\Lambda} + \Theta \right] po_t, \quad (7)$$

and

$$w_t = - \frac{\widetilde{\omega}_{oy} (1 - \widetilde{\omega}_{oc}) + \widetilde{\omega}_{oc}}{(1 - \widetilde{\omega}_{oc}) (1 - \widetilde{\omega}_{oy})} po_t \quad (8)$$

where $\Theta = \widetilde{\omega}_{oc} (1 - \chi sy^{-1} (1 - \widetilde{\omega}_{oc})) / (1 - \widetilde{\omega}_{oc}) (\phi + 1)$, $\Lambda = (1 - \widetilde{\omega}_{oy}) (1 - \widetilde{\omega}_{oc}) (\phi + 1)$ and $0 < MC(\equiv (\varepsilon - 1) \varepsilon^{-1}) \leq 1$ reflecting the degree of monopolistic competition in the economy. Note also that $\widetilde{\omega}_{oy} \equiv \omega_{oy} P_{o}^{1-\delta} (MC \cdot P_y)^{\delta-1}$ is the share of oil in the real marginal cost, $\widetilde{\omega}_{oc} \equiv \omega_{oc} P_{o}^{1-\chi}$ is the share of oil in the CPI, $sy \equiv (1 - \omega_{oc}) (Y/C) \frac{1}{\chi-1}$ is the share of the core good in the consumption goods basket and, at the FPWE, marginal cost is:

$$0 = mc_t = (1 - \widetilde{\omega}_{oy}) (w_t - py_t) + \widetilde{\omega}_{oy} (po_t - py_t)$$

If oil is considered a gross complement to labor in production ($\delta < 1$), the oil price elasticity of real marginal costs, $\widetilde{\omega}_{oy}$, is increasing in the degree of monopolistic competition distortion as measured by $MC^{-1}$. The less competitive the economy, the larger is $\widetilde{\omega}_{oy}$ and the more sensitive are real marginal costs to increases in oil prices.

As perfect price stability means constant real marginal costs, the more distorted the

\(^{14}\)Note that lowercase letters denote the percent deviation of each variable with respect to its steady state (e.g., $c_t \equiv \log \left( \frac{C_t}{C^*} \right)$).
economy’s steady-state, the bigger is the real wage drop required to compensate for higher oil prices. In equilibrium, labor and output must then fall correspondingly.

Increases in oil prices also act as a tax on labor income when \( \chi < 1 \); the lower the elasticity of substitution, the larger the tax effect which compounds with the effect of oil price increases on marginal costs.\(^{15}\) More specifically, as changes in oil prices affect headline more than core prices, increases in oil prices have a differentiated effect on real oil prices faced by consumers, \( p_{o_t} \), and (higher) real oil prices faced by firms \( p_{o_t} - p_{y_t} \).\(^{16}\) This discrepancy is exacerbated by the fact that changes in oil prices drive a wedge between the real wage relevant for households (the consumption real wage, \( w_t \)) and the real wage faced by firms (the production real wage, \( w_t - p_{y_t} \)). The lower the elasticity of substitution between energy and other consumption goods, \( \chi \), the larger is the effect of a given increase in oil prices on \( p_{y_t} \) and the larger is the required drop in real wages \( w_t \) (and in labor and output) to stabilize real marginal costs.

Indeed, equations (6), (7) and (8) show that the response of employment, output and the real wage are increasing in \( \delta \) and \( \chi \).\(^{17}\)

\(^{15}\)Note that the tax effect tends to zero when the elasticity of substitution \( \chi \to \infty \), as in this case \( \overline{\omega}_{oc} \to 0 \) and the solution of the model collapses to the one where oil is an input to production only.

\(^{16}\)Because immediately after an increase in oil prices, the ratio of core to headline prices deteriorates \( (p_{y_t} < 0) \).

\(^{17}\)Equations (6) and (7) show that when \( \delta = \chi = 1 \), i.e., when the production functions for intermediate and final goods are Cobb-Douglas, substitution and income effects in the labor market compensate each other and employment remains constant after an oil price shock \( (\Theta = h_t = 0) \) despite a drop in output.
3.2 Endogenous cost-push shock

The cyclical wedge between the natural and efficient levels of output after an oil price shock — the endogenous cost-push shock — can be analyzed by comparing the log-linearized flex-price output responses in the distorted \((y_t^N, \text{natural})\) and undistorted \((y_t^*, \text{efficient})\) economies.\(^{18}\)

Starting from equation (7), the cyclical distortion can be written as:

\[
y_t^N - y_t^* = -(1 + \delta \phi) \left( \bar{\omega}_{og}^N / \Lambda^N - \bar{\omega}_{og}^* / \Lambda^* \right) p \omega_t, \tag{9}
\]

where I assume \(MC = 1\) in \(\bar{\omega}_{og}^*\) and \(\Lambda^*\), and \(MC < 1\) in \(\bar{\omega}_{og}^N\) and \(\Lambda^N\).

This cyclical distortion can be mapped into a cost-push shock that enters the New Keynesian Phillips curve (NKPC henceforth):

\[
\pi_{y,t} = \beta \Pi_t \pi_{y,t+1} + k_y x_t + \mu_t, \tag{10}
\]

where \(x_t = y_t - y_t^*\) is the percent deviation of output with respect to the welfare relevant output \(y_t^*\) and \(\mu_t = k_y (y_t^* - y_t^N)\) is the cost-push shock. Note that \(y_t^N = - (kB/k_y) p \omega_t\) for \(B\) a decreasing function of \(\delta\) and \(\chi\), \(k = ((1 - \theta) / \theta) (1 - \theta \beta)\) and \(k_y = k (1 - \eta) (\sigma + \phi) [(1 - \bar{\omega}_{og}) / (1 + \bar{\omega}_{og} (1 - \eta) \phi \delta)]\) (see Appendix III and IV for detailed derivations).

First, note that the wedge is constant \((y_t^N - y_t^* = 0)\) and the divine coincidence holds when a fiscal transfer is available to offset the steady state monopolistic distortion. In this case, \(\bar{\omega}_{og}^N = \bar{\omega}_{og}^*\) and \(\Lambda^N = \Lambda^*\). There is no cost-push shock and no policy

\(^{18}\)The social planner’s efficient allocation is the same as the one in the decentralized economy when prices and wages are flexible and there is no steady-state monopolistic distortion \((MC \equiv \frac{\delta}{\chi} \to 1)\). The natural allocation, on the other hand, corresponds to the flex-price and wage equilibrium in a distorted economy \((MC < 1)\).
trade-off. Second, when production functions are Cobb-Douglas \((\delta = \chi = 1)\), \(\tilde{\omega}_{oy}^* = \tilde{\omega}_{oy}^N = \omega_{oy}\) and \(\tilde{\omega}_{ac} = \omega_{ac}\) so that \(\tilde{\omega}_{oy}^N / \Lambda^N - \tilde{\omega}_{oy}^* / \Lambda^* = 0\); there is again no-trade-off and \(y_t^N - y_t^* = 0\). Third, allowing for gross complementarity \((\delta, \chi < 1)\) in a world without fiscal transfer, \(y_t^N\) will drop more than \(y_t^*\) after an oil shock as \(\tilde{\omega}_{oy}^N > \tilde{\omega}_{oy}^*\) when \(MC^N < MC^* \equiv 1\).

Clearly, the larger the steady-state distortion (i.e., the lower \(MC^N\)) and the lower \(\delta\) and \(\chi\), the larger is the cyclical wedge between \(y_t^N\) and \(y_t^*\); the larger is the cost-push shock. Moreover, the elasticity of substitution between energy and other consumption goods, \(\chi\), plays an important role in amplifying the effect of oil prices on the gap between \(y_t^N\) and \(y_t^*\): The lower \(\chi\), the larger is \(\tilde{\omega}_{ac}\), the lower is \(\Lambda\) and the larger is the difference \(\tilde{\omega}_{oy}^N / \Lambda^N - \tilde{\omega}_{oy}^* / \Lambda^*\).

Figure 1 shows the instantaneous response of the gap between natural (YN) and efficient (Y*) output to a (one period) 1-percent increase in the real price of oil as a function of \(\delta\), the production elasticity of substitution, and for different values of the consumption elasticity of substitution, \(\chi\).\(^{19}\) The gap is exponentially decreasing in both the elasticities \(\delta\) and \(\chi\). Looking at the northeastern extreme of the figure, where both elasticities are equal to one (the Cobb-Douglas case), we see that the reaction of natural and efficient outputs are the same (the gap is zero), so that stabilizing inflation or output at its natural level is welfare maximizing.\(^{20}\) Lowering the production elasticity

\(^{19}\)Note that the amplitude of the gap also depends on the Frish-elasticity of labor supply as measured by \(\phi^{-1}\). The smaller \(\phi\) (the larger the elasticity), the larger are the labor demand and output drops needed to stabilize the real marginal cost, and the larger is the cyclical gap between efficient and natural output.

\(^{20}\)Solving equations (A3.13) and (A4.3) in Appendices III and IV for \(\sigma = \phi = \delta = \chi = 1\), we get that \(y_t^N = y_t^* = -\frac{\alpha}{(1-\alpha)(1-\omega_{ac})}\partial \tau\).
only (along the curve CHI=1) gives rise to a monetary policy trade-off. Yet, the wedge becomes really large only when both the consumption and production elasticities are small (like on the curve labeled CHI=0.3).

< Figure 1 >

Figure 2 performs a similar exercise, but varies the degree of net steady-state markups \( MC^{-1} - 1 \) for different values of the elasticities \( \delta \) and \( \chi \). Again, the wedge between efficient and natural output swells for large distortions and low elasticities.

< Figure 2 >

4 Optimal monetary policy

What weight should the central bank attribute to inflation over output gap stabilization? Rotemberg and Woodford (1997) and Benigno and Woodford (2005) have shown that the central bank’s loss function - defined as the weighted sum of inflation and the welfare relevant output gap - could be derived from (a second order approximation of) the households utility function, thereby setting a natural criterion to answer this question (see Appendix III for details). Section 4.1 shows that the parameters governing the nominal and real rigidities in the model interact with the elasticities of substitution (that are assumed smaller than one) and have important consequences on the choice of policy. For reasonable parameter values, however, the weight on inflation stabilization remains larger than the one on the output gap, a result also obtained by Woodford (2003b) in a more constrained environment.

Given optimal weights in the central bank loss function, what is the optimal policy
response to an oil price shock? Section 4.2 contrasts the dynamic transmission of oil price shocks under strict inflation targeting and under the optimal precommitment policy in a timeless perspective.

4.1 Lambda

Given that the central bank’s objective is to minimize the loss function

$$\Upsilon E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \lambda u_t^2 + \pi_{y,t}^2 \right\}$$

under the economy constraint, the welfare implications of alternative policies crucially depend on the value of $\lambda$.\(^{21}\) Figure 3 describes the variation of $\lambda$, the relative weight assigned to output gap stabilization as a function of the elasticity of substitution $\delta$ and the degree of price stickiness $\theta$. Stickier prices (larger $\theta$) result in larger price dispersion and therefore larger inflation costs. In this case, monetary authorities will be less inclined to stabilize output and, for given elasticities of substitution, $\lambda$ decreases when $\theta$ becomes larger.

But $\lambda$ also depends crucially on the elasticities of substitution. The lower the elasticities, the flatter is the New Keynesian Phillips curve (NKPC), the larger is the sacrifice ratio, and the more concerned will be the central bank with the distortionary cost of inflation (i.e., the smaller will be $\lambda$).\(^{22}\) Assuming perfectly flexible real wages ($\eta = 0$), our baseline calibration ($\theta = 0.75$, $\delta = 0.3$) leads to $\lambda = 0.028$, which implies a targeting rule that places a larger weight on inflation stabilization than on the output gap (in annual inflation terms, the ratio output gap to inflation stabilization

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\(^{21}\) Appendix III describes how $\lambda$ and the loss function are derived from first principles.

\(^{22}\) When the NKPC is flat, a large change in output is required to affect inflation.
is $\sqrt{0.022 \times 4} = 0.66$).\footnote{The traditional Taylor rule that places equal weights on output stabilization and inflation stabilization would imply $\lambda = 1/16$ with quarterly inflation.} Note that the focus of policy is very sensitive to the degree of price stickiness. Setting $\theta = 0.5$ results in $\lambda = 0.138$ and a policy that sets a larger weight on output gap stabilization ($\sqrt{0.138 \times 4} = 1.48$).

< Figure 3 >

BG07 argue that the optimal policy choice depends crucially on the degree of real wage stickiness. Figure 4 verifies this claim by letting the degree of real wage stickiness vary between $\eta = 0$ and $\eta = 0.9$. The larger the real wage stickiness, the larger is the cost-push shock but the larger is the sacrifice ratio as a relatively larger drop in labor demand and output is necessary to engineer the required drop in real wages that stabilizes the real marginal cost and inflation. The central bank will tend to be more concerned with inflation stabilization and $\lambda$ will be smaller when $\eta$ is high. Assuming $\eta = 0.9$ and our baseline calibration, Figure 4 shows that $\lambda = 0.002 (\sqrt{0.002 \times 4} = 0.18)$.

< Figure 4 >

4.2 Analyzing the trade-off

How different is the transmission of an oil price shock under the optimal precommitment policy in a timeless perspective characterized by the targeting rule (see Woodford, 2003a and Appendix IV): 

$$x_t = x_{t-1} - \frac{k_y}{\lambda} \pi_{y,t},$$

which holds for all $t = 0, 1, 2, 3, \ldots$, from strict inflation targeting which replicates the
FPWE solution? Figure 5 shows that, assuming $\delta = \chi = 0.3$, the latter implies an increase in real interest rates (which corresponds to the expected growth of future consumption), while optimal policy recommends a temporary drop for one year following the shock. The output drop on impact is more than three times larger in the FPWE allocation, which is the price for stabilizing core inflation perfectly.

< Figure 5 >

Figure 6 shows how acute the policy trade-off is by displaying the differences in both the welfare relevant output gaps and inflation reactions to a 1-percent increase in the price of oil under optimal policy and strict inflation targeting. The "oil-in-production-only" case (dotted line) is compared with the case where energy is an input to both consumption and production (solid line). In both cases, optimal policy lets inflation increase and the welfare relevant output gap decrease. But the difference with strict inflation targeting is three times as large when oil is both an input to production and consumption, as could be inferred from Section 3.

< Figure 6 >

5 Simple rules

Welfare-based optimal policy plans may not be easy to communicate as they typically rely on the real-time calculation of the welfare relevant output gap, an abstract, non-observable theoretical construct. Accountability issues can be raised, which may cast doubt on the overall credibility of the precommitment assumption that underlies the whole analysis. As an alternative, some authors (McCallum 1999, Söderlind 1999,
Dennis 2004) have advocated the use of simple optimal interest rate rules. Those rules should approximate the allocation under the optimal plan but should not rely on an overstretched information set.

In what follows, a simple rule that is equivalent to the optimal plan is derived from first principles. I show that it must be based on core inflation and on current and lagged deviations of output and the real price of oil from the steady state. As the mere notion of steady-state can also be subject to uncertainty in real-time policy exercises, I also show that the optimal simple rule can be approximated by a 'speed limit' interest rate rule (Walsh 2003, Orphanides and Williams 2003) that relies only on the rate of change of the variables, i.e., on current core inflation, oil price inflation, and the growth rate of output; this rule remains close to optimal even when real wages are sticky.

5.1 The optimal precommitment simple rule

Using the minimal state variable (MSV) approach pioneered by McCallum, one can conjecture the no-bubble solution to the dynamic system formed by i) the optimal targeting rule under the timeless perspective optimal plan and ii) the NKPC equation to get.\textsuperscript{24}

\begin{equation}
\pi_{y,t} = \alpha_{11} x_{t-1} + \alpha_{12} \mu_t, \tag{12}
\end{equation}

\begin{equation}
x_t = \alpha_{21} x_{t-1} + \alpha_{22} \mu_t, \tag{13}
\end{equation}

where \(\alpha_{ij}\) for \(i, j = 1, 2\) are functions of \(\beta, k_y,\) and \(\lambda.\)

\textsuperscript{24}See McCallum (1999b) for a recent exposition of the MSV approach.
Combining (12) and (13) with the Euler equation for consumption, one can solve for \( r_t \), the nominal interest rate, and derive the optimal simple rule consistent with the optimal plan (see Appendix V):

\[
r_t = \Phi \alpha_{11}^{-1} \pi_{yt,t} + \Omega y_{t-1} - \Gamma y_{t-1} + (\Xi + \Psi \Omega) p o_t - \Psi \Gamma p o_{t-1},
\]

for \( \Phi \equiv \rho_o - \sigma \alpha_{22} \alpha_{12}^{-1} (1 - \rho_o), \Omega \equiv \alpha_{11} + \sigma \alpha_{21}, \Gamma \equiv \Phi + \sigma \alpha_{21} \) and \( \Xi \equiv (\rho_o - 1) (\tilde{\omega}_{oc} / (1 - \tilde{\omega}_{oc}) - \Psi) \).

The optimal interest rate rule is a function of core inflation and current and lagged output and real oil price, all taken as log deviations from their respective steady states. Its parameters are functions of households preferences, technology, and nominal frictions.

For a permanent shock, \( \rho_o = 1 \), the rule simplifies to

\[
r_t = \alpha_{11}^{-1} \pi_{yt,t} + \Omega (y_{t-1} - \Gamma \Omega^{-1} y_{t-1}) + \Psi \Omega (p o_t - \Gamma \Omega^{-1} p o_{t-1}),
\]

as \( \Phi = 1 \), and \( \Xi = 0 \). Looking at \( \Gamma \) and \( \Omega \) shows that the closer \( \alpha_{11} \) is to 1, the more precisely a speed limit policy (a rule based on the rate of growth of the variables) replicates optimal policy as in this case \( \Gamma = \Omega \).

In Section 6, I show that for \( \rho_o = 0.95 \), a degree of persistence which corresponds closely to the 1979 oil shock, the speed limit policy approximates almost perfectly the optimal feedback rule despite a value of \( \alpha_{11} \) clearly below 1.

### 5.2 Optimized simple rules

Analytical solutions to the kind of problem described in Section 5.1 rapidly become intractable when the number of shocks and lagged state variables is increased (e.g., by allowing for the possibility of real wage rigidity).
An alternative is to rely on numerical methods in order to estimate a simple rule mimicking the optimal plan’s allocation along all relevant dimensions. The following distance minimization algorithm is defined over the $n$ impulse response functions of $m$ variables of interest to the policymakers and searches the parameter space of a simple interest rate rule that minimizes the distance criterion:

$$\arg \min_{\vartheta} (IRF_{SR}(\vartheta) - IRF_{O})' (IRF_{SR}(\vartheta) - IRF_{O}),$$

where $IRF_{SR}(\vartheta)$ is an $mn \times 1$ vector of impulses under the postulated simple interest rate rule, and $IRF_{O}$ is its counterpart under the optimal plan.$^{25}$ The algorithm matches the responses of eight variables (output, consumption, hours, headline inflation, core inflation, real marginal costs, and nominal and real interest rates) over a 20-quarter period using constrained versions of the following general specification of the simple interest rate rule derived from equation (14):

$$r_t = g_{\pi} \pi_{y,t} + g_y y_t + g_{y1} y_{t-1} + g_{po} po_t + g_{po1} po_{t-1} + g_{w1} w_{t-1}, \quad (15)$$

where $\vartheta = (g_{\pi}, g_y, g_{y1}, g_{po}, g_{po1}, g_{w1})'$. 

I start with a version of the model that assumes perfect real wage flexibility and run the minimum distance algorithm on an unconstrained version of equation (15), and on a ’speed limit’ version where $g_y + g_{y1} = 0$, $g_{po} + g_{po1} = 0$ and $g_{w1} = 0$. Figure 7 shows the response to a 1 percent shock to oil prices under the optimal precommitment policy (solid line), the optimized simple rule (OR, dotted line) and the speed limit rule (SLR,

$^{25}$Another possibility is to search within a predetermined space of simple interest rate rules for the one that minimizes the central bank loss function (Söderlind 1999, Dennis 2004). However as different combinations of output gaps and inflation variability could in principle produce the same welfare loss, I rely on IRFs instead.
dashed line). The responses under the OR stand exactly on top of the ones under the optimal policy, which is not surprising given that a closed-form solution to the problem can be derived (see previous sub-section). More remarkable, however, is how well the SLR (dashed line) is able to match the optimal precommitment policy (solid line). For most variables they are almost indistinguishable.

< Figure 7 >

How robust are these findings to the introduction of real wage stickiness or to alternative definitions of target inflation? Assuming \( \eta = 0.9 \) and \( \pi_{y,t} \) a four quarter moving average of core inflation, Figure 8 shows that, again, the OR (dotted line) is almost on top of the optimal plan benchmark (solid line).

< Figure 8 >

The optimal rules’ coefficients are reported in Table 1. They are quite large compared to the coefficients typically found for Taylor-type interest rate rules. Monetary authorities react strongly to both inflation and the output gap, but also to changes in oil prices. This can be interpreted as meaning that the optimal rule tends towards perfect price stability in the case of a demand shock that pushes inflation and the output gap in the same direction, but that it acknowledges the policy trade-off in the case of an oil price shock.26

< Table 1 >

26 It can also be mentioned that although our analysis assumes that oil prices are exogenous events, the optimal rule suggests that policy will be swiftly tightened in the case of a demand-driven increase in oil prices as growth would accelerate and core inflation would increase beyond its steady-state level. Kilian (2009) discusses at length the importance of disentangling demand and supply shocks in the oil market.
6 Oil price shocks and US monetary policy

All US recessions since the end of World War II — and the latest vintage is no exception — have been preceded by a sharp increase in oil prices and an increase in interest rates (Hamilton 2009). But are US recessions really caused by oil shocks, or should the monetary policy responses to the shocks be blamed for this outcome? Empirical evidence seems to suggest a role for monetary policy (Bernanke et al. 2004), but its importance remains difficult to assess. One major stumbling block is the role of expectations. To evaluate the effect of different monetary policies in the event of an oil price shock one has to take into account the effect of those policies on the agents’ expectations, which is typically not feasible using reduced-form time series models whose estimated parameters are not invariant to policy (see Lucas 1976 and Bernanke et al. 2004 for a discussion in the context of an oil shock).

The alternative approach is to rely on a structural, microfounded model to simulate counterfactual policy experiments. I start by describing the dynamic behavior of the economy under different monetary policies during the 1979 oil price shock and compute the welfare costs associated with suboptimal policies. I have chosen to focus on this episode for the oil shock was clearly exogenous to economic activity (Iranian revolution) and as such corresponds to the model definition of an oil price shock. Clearly, oil prices, like any prices, are formed endogenously in reality. Assuming exogenous oil prices might be seen as a limitation of the analysis (Kilian 2009) but it is a convenient shortcut that allows to focus on the intra- and inter-temporal allocative consequences of alternative policy responses. Future work should try to assess the robustness of the findings to the
explicit modeling of the oil market.

In the second part of the Section, I compare the optimal rule to standard alternatives during the 2006-2008 oil price rally and find significant differences with the policy conducted by the Federal Reserve.

6.1 1979 oil price shock

Figure 9 shows that the pattern of real oil prices between 1979 and 1986 can be well replicated by an $AR(1)$ process $p_{o_t} = \rho_o p_{o_{t-1}} + \varepsilon_{o,t}$ for $\rho_o = 0.95$. I will thus rely on this shock process to perform all simulations and compute welfare losses.

Figure 10 compares the IRFs under optimal policy (OR henceforth, solid line), the traditional Taylor rule (HTR, dashed line) and a Taylor rule based on core inflation (CTR henceforth, dotted line).

Under optimal policy, the central bank credibly commits to a state-contingent path for future interest rates that involves holding real interest rates positive in the next five years despite negative headline inflation and close to zero core inflation. In doing so it is able to dampen inflation expectations without having to resort to large movements in real interest rates and it attains superior stabilization outcome in the short to medium run. At the peak, output falls twice as much and core inflation is five times larger under HTR than under OR. Because inflation never really takes off under OR, nominal interest rates remain practically constant over the whole period. This suggests that if monetary policy had been conducted according to OR during the oil shock of 1979, the recession would not have been averted but it would have been much milder with

25
almost no increase of core inflation beyond steady-state inflation. CTR leads to less output gap fluctuations than OR but at the cost of much higher core inflation.

\[< \text{Figure 10}>\]

Some authors (Bernanke et al. 1999) have argued that monetary policy should be framed with respect to a \textit{forecast} of inflation rather than \textit{realized} inflation. And, indeed, many inflation-targeting central banks communicate their policy by referring to an explicit goal for their forecast of inflation to revert to some target within a specified period. Like BEG08, I define a forecast-based rule as a Taylor-type rule where realized inflation has been replaced by a one-quarter-ahead forecast of core or headline inflation; the parameters remain the same with $g_\pi = 1.5$ and $g_y = 0.5$. Figure 11 shows that forecast-based rules fulfill their goal of stabilizing both headline and core inflation in the long run but appear too accommodative in the short run.\textsuperscript{27}

\[< \text{Figure 11}>\]

But how costly is it to follow suboptimal rules? Table 2 summarizes the main results. The first column shows the cumulative welfare loss from following alternative policies between 1979 Q1 and 1983 Q4 expressed as a percent of one year steady-state consumption. The second and third columns report the $\lambda$-weighted decomposition of the loss arising from volatility in the output gap or in core inflation.

The numbers seem to be unusually large. They are about 100 times larger than the ones reported by Lucas (1987), for example. One has to keep in mind, however, that

\textsuperscript{27}Under a temporary oil price shock, oil price inflation is negative next period, pushing down headline inflation. A Taylor rule based on a forecast of headline inflation completely eliminates the output consequence of the 1979 oil shock, as can be seen in the upper left panel, but with dire effects on inflation in the short to medium run.
our calculation refers to the cumulative welfare loss associated with one particularly painful episode and not the average cost from garden variety oil price shocks. Indeed, Galí et al. (2007) report that the welfare costs of recessions can be quite large. Their typical estimate for the cumulative cost of a 1980-type recession is in the range of 2 to 8 percent of one year steady-state consumption, depending on the elasticities of labor supply and intertemporal substitution.\(^\text{28}\)

Table 2 shows that despite very good performances in terms of stabilizing the welfare relevant output gap, forecast-based HTR ranks last among the rules considered because of much higher core inflation. Taylor rules based on contemporaneous headline inflation are also quite costly if there is no inertia in interest rate decisions. The results also suggest that having followed a policy close to the benchmark Taylor rule (HTR) during the 1979 oil shock instead of the optimal policy may have cost the equivalent of 2.1 percent of one year steady-state consumption to the representative household (or about 200 billions of 2008 dollars).\(^\text{29}\) The overall cost would have been 40 percent smaller if monetary policy had been based on an inertial interest rate rule such as CTR or HTR with \(\rho = 0.8\).

As mentioned above, our utility-based welfare metric tends to weigh heavily inflation deviations as a source of welfare costs.\(^\text{30}\) This notwithstanding, the results suggest that

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\(^{28}\) They also acknowledge that their estimates are probably a lower bound as they ignore the costs of price and wage inflation resulting from nominal rigidities.

\(^{29}\) Admittedly, HTR is only a rough approximation of the actual Federal Reserve behavior, but it seems sufficiently accurate to describe how US monetary policy was conducted during the oil shock of 1979. See Orphanides (2000) for a detailed analysis using real-time data.

\(^{30}\) Assuming \(\theta = 0.75\) and \(\eta = 0.9\) amounts to setting \(\lambda\) to 0.02, which means that the central bank attributes about twice as much importance to inflation stabilization as to output gap stabilization when inflation is expressed in annual terms.
welfare losses under the perfect price stability policy remain three times as large as under optimal policy and amount to 1.8 percent of one year steady-state consumption because of disproportionately large fluctuations in the welfare relevant output gap.\textsuperscript{31}

6.2 US monetary policy 2006-2008

How does the optimal rule (SLR) compare to actual policy in the US and to usual benchmarks during the last run up in oil prices, from 2006 to 2008? This episode is of great interest as some recent empirical evidence tends to show that the policy rule followed by the Federal Reserve was different in the post-Volcker period from the one followed during the 1979 oil price shock (Herrera and Pesavento 2009, Kilian and Lewis 2010). Figure 12 shows that in the period 2000-2005 the SLR is not very different from a classical Taylor rule based on headline (HTR) or core inflation (CTR) : All rules tend to suggest higher interest rates than the actual 3 months market rates.

Things become more interesting during the 2006-2008 oil price rally (shaded area). In this period, the Federal reserve accommodated the oil price increase by dropping interest rates in the second half of 2007. This is also what would have been recommended by CTR, whereas HTR, reacting to increases in headline inflation would have supported further interest rate hikes until mid-2008. SLR, on the contrary, because it takes into account the detrimental impact of higher oil prices on consumption, would have suggested to start dropping interest rates a year and a half before the Fed did. Following the SLR, the Fed would also have started to tighten in 2009 already in an effort to keep inflation and inflation expectations in check.

\textsuperscript{31}See appendix VII for a sensitivity analysis to alternative values for the elasticities of substitution in production and in consumption.
7 Time-varying elasticities of substitution

It is a well-known empirical fact that the demand for energy is almost unrelated to changes in its relative price in the short run. In the long run, however, persistent changes in prices have a significant bearing on the demand for energy.\footnote{Pindyck and Rotemberg (1983), for example, report a cross-section long-run price elasticity of oil demand close to one.}

How are the result of the precedent sections affected by the possibility of time-varying elasticities of substitution? Is the short run monetary trade-off after an oil price shock the mere reflection of some CES-related speciﬁcity, or is it a more general argument related to low short-term substitutability in a distorted economy?\footnote{Here, optimal monetary policy is also derived under the timeless perspective assumption. Since we are only interested in the dynamic response of variables to an oil price shock under optimal policy, we do not compute the LQ solution. We directly solve the non-linear model for the Ramsey policy that would maximize utility under the constraint of our model using Andrew Levin’s Matlab code (Levin 2004 and 2005).}

To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production as in Bodenstein et al. (2008) (see Appendix VI). Figure 13 shows impulse responses to a 1 percent shock to the price of oil and compares the flex-price equilibrium allocation with the optimal precommitment policy when a fiscal transfer is available to neutralize the steady-state ineﬃciency due to monopolistic competition.\footnote{To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production as in Bodenstein et al. (2008) (see Appendix VI). Figure 13 shows impulse responses to a 1 percent shock to the price of oil and compares the flex-price equilibrium allocation with the optimal precommitment policy when a fiscal transfer is available to neutralize the steady-state ineﬃciency due to monopolistic competition.}\footnote{Here, optimal monetary policy is also derived under the timeless perspective assumption. Since we are only interested in the dynamic response of variables to an oil price shock under optimal policy, we do not compute the LQ solution. We directly solve the non-linear model for the Ramsey policy that would maximize utility under the constraint of our model using Andrew Levin’s Matlab code (Levin 2004 and 2005).}

Because of adjustment costs — which add two state variables to the problem — the IRFs are not exactly similar to the ones obtained under CES production. However, the message remains the same: Price stability is the optimal policy when the economy’s
steady-state is efficient.

Figure 14 performs the same exercise but allows for the same degree of monopolistic competition distortion in steady-state as in previous sections (a 20 percent steady-state markup of core prices over marginal costs). It shows that allowing for time-varying elasticities of substitution does not affect the paper’s main finding: In a distorted equilibrium, an oil price shock introduces a significant monetary policy trade-off if the oil cost share is allowed to vary in the short-run.

< Figure 13 >

< Figure 14 >

8 Conclusion

Most inflation targeting central banks understand their mandate to be ensuring long-term price stability. Following an oil price shock, however, none of them would be ready to expose the economy to the type of output and employment drops recommended by standard New Keynesian theory for the sake of stabilizing prices in the short term. This paper argues that policies which perfectly stabilize prices entail significant welfare costs, explaining the reluctance of policymakers to enforce them.

Interestingly, I find that the optimal monetary policy response to a persistent increase in the oil price indeed resembles the typical response of inflation targeting central banks: While long-term price stability is ensured by a credible commitment to keep inflation and inflation expectations in check, short-term real rates drop right after the shock to help dampen real output fluctuations. By managing expectations efficiently,
central banks can improve on both the flexible price equilibrium solution and the recommendation of simple Taylor rules. Using standard welfare criteria, I calculate that following a standard Taylor rule in the aftermath of the 1979 oil price shock may have cost the US household about 2% of one year consumption.

Those findings are based on the assumptions that monetary policy is perfectly credible and transparent and that agents and central banks have the right (and the same) model of the economy. Further work should explore how sensitive the policy conclusions are to the incorporation of imperfect information and learning into the analysis. Further research should also establish the robustness of the simple optimal rule derived in this paper to the incorporation of more shocks into a larger, empirically more relevant DSGE model. A related issue concerns the incorporation of open economy dimensions into the analysis. Bodenstein et al. (2008), for example, have shown that the effect of an oil price shock on the terms of trade, the trade balance and consumption depends on the assumption made about financial market risk-sharing arrangements.

Finally, another potential limitation of the analysis is that oil price shocks are treated as exogenous events here. It can be argued, however, that the optimal monetary policy response to oil price changes might differ if the latter are considered a consequence of changing world aggregate demand (Kilian 2009, Kilian and Lewis 2010). An extension of this work should focus on the optimal response to the underlying shocks driving the price of oil instead of the response to the price of oil itself.\footnote{An important step in that direction is Nakov and Pescatori (2007) who explicitly model the oil market in general equilibrium.} I leave these important considerations for future research.
References


Appendix I: The model

A1.1 Households

There exists a unit mass continuum of infinitely lived households indexed by \( j \in [0, 1] \), which maximize the discounted sum of present and expected future utilities defined as follows

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{t-s} \left\{ \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \nu \frac{H_t(j)^{1+\phi}}{1+\phi} \right\},
\]

(A1.1)

where \( C_t(j) \) is the consumption goods bundle, \( H_t(j) \) is the (normalized) quantity of hours supplied by household of type \( j \), the constant discount factor \( \beta \) satisfies \( 0 < \beta < 1 \) and \( \nu \) is a parameter calibrated to ensure that the typical household works eight hours a day in steady state.

In each period, the representative household \( j \) faces a standard flow budget constraint

\[
B_t(j) + P_tC_t(j) = R_{t-1}B_{t-1}(j) + W_tH_t(j) + \tilde{\Pi}_t(j) + T_t(j),
\]

(A1.2)

where \( B_t(j) \) is a non-state-contingent one period bond, \( R_t \) is the nominal gross interest rate, \( P_t \) is the CPI, \( \tilde{\Pi}_t(j) \) is the household \( j \) share of the firms’ dividends and \( T_t(j) \) is a lump sum fiscal transfer to the household of the profits from sovereign oil extraction activities.

Because the labor market is perfectly competitive, I drop the index \( j \) such that \( \Pi_t \equiv \Pi_t(j) = \int_0^1 H_t(j) \, dj \), and I write the consumption goods bundle\(^{35} \) \( C_t \) as a CES aggregator of the core consumption goods basket \( C_{Y,t} \) and the household’s demand for oil \( O_{C,t} \)

\[
C_t = \left( (1 - \omega_{oc}) C_{Y,t}^{\chi_{Y,t}} + \omega_{oc} O_{C,t}^{\chi_{C,t}} \right)^{\frac{1}{\chi_{C,t}}} ,
\]

(A1.3)

where \( \omega_{oc} \) is the oil quasi-share parameter and \( \chi \) is the elasticity of substitution between oil and non-oil consumption goods.

Households determine their consumption, savings, and labor supply decisions by maximizing (A1.1) subject to (A1.2). This gives rise to the traditional Euler equation

\[
1 = \beta \mathbb{E}_t \left\{ \left( \frac{C_t}{C_{t+1}} \right)^\sigma R_t \frac{\Pi_t}{\Pi_{t+1}} \right\},
\]

(A1.4)

which characterizes the optimal intertemporal allocation of consumption and where \( \Pi_t \) represents headline inflation.

Allowing for real wage rigidity (which may reflect some unmodeled imperfection in the labor market as in BG07), the labor supply condition relates the marginal rate of

\(^{35}\)The consumption basket can be regarded as produced by perfectly competitive consumption distributors whose production function mirrors the preferences of households over consumption of oil and non-oil goods.
substitution between consumption and leisure to the geometric mean of real wages in periods \( t \) and \( t-1 \).

\[
(C_t^\omega H_t^\varphi)^{(1-\eta)} = \frac{W_t}{P_t} \left( \frac{W_{t-1}}{P_{t-1}} \right)^{-\eta}.
\] (A1.5)

In the benchmark calibration, i.e., unless stated otherwise, \( \eta = 0 \); real wages are perfectly flexible and equal to the marginal rate of substitution between labor and consumption in all periods.

Finally, households optimally divide their consumption expenditures between core and oil consumption according to the following demand equations:

\[
C_{Y,t} = P_{y,t} (1 - \omega_{oc})^x C_t,
\] (A1.6)

\[
O_{C,t} = P_{o,t} \omega_{oc} C_t,
\] (A1.7)

where \( P_{y,t} \equiv \frac{P_{Y,t}}{K} \) is the relative price of the core consumption good and \( P_{o,t} \equiv \frac{P_{O,t}}{P_t} \) is the relative price of oil in terms of the consumption good bundle and where

\[
P_t = \left( (1 - \omega_{oc})^x P_{Y,t}^{1-x} + \omega_{oc} P_{O,t}^{1-x} \right)^{\frac{1}{1-x}}
\] (A1.8)

represents the overall consumer price index (CPI).

### A1.2 Firms

#### Core goods producers

I assume that the core consumption good is produced by a continuum of perfectly competitive producers indexed by \( c \in [0,1] \) that use a set of imperfectly substitutable intermediate goods indexed by \( i \in [0,1] \). In other words, core goods are produced via a Dixit-Stiglitz aggregator

\[
Y_t(c) = \left( \int_0^1 Y_t(i,c)^{\frac{\varepsilon - 1}{x-1}} \, di \right)^{\frac{x}{x-1}},
\] (A1.9)

where \( \varepsilon \) is the elasticity of substitution between intermediate goods. Given the individual intermediate goods prices, \( P_{Y,t}(i) \), cost minimization by core goods producers gives rise to the following demand equations for individual intermediate inputs:

\[
Y_t(i,c) = \left( \frac{P_{Y,t}(i)}{P_{Y,t}} \right)^{-\varepsilon} Y_t(c),
\] (A1.10)

where \( P_{Y,t} = \left( \int_0^1 P_{Y,t}(i)^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}} \) is the core price index.

Aggregating (A1.10) over all core goods firms, the total demand for intermediate goods \( Y_t(i) \) is derived as a function of the demand for core consumption goods \( Y_t \)

\[
Y_t(i) = \left( \frac{P_{Y,t}(i)}{P_{Y,t}} \right)^{-\varepsilon} Y_t,
\] (A1.11)

using the fact that perfect competition in the market for core goods implies \( Y_t(c) = Y_t = \int_0^1 Y_t(c) \, dc \).
Intermediate goods firms

Each intermediate goods firm produces a good \( Y_t(i) \) according to a constant returns-to-scale technology represented by the CES production function

\[
Y_t(i) = \left( (1 - \omega_{oy}) \left( \mathcal{H}_t H_t(i) \right)^{\delta_{1}} + \omega_{oy} \left( O_{Y,t}(i) \right)^{\delta_{2}} \right)^{\frac{1}{\delta}}, \tag{A1.12}
\]

where \( \mathcal{H}_t \) is the exogenous Harrod-neutral technological progress whose value is normalized to one. \( O_{Y,t}(i) \) and \( H_t(i) \) are the quantities of oil and labor required to produce \( Y_t(i) \) given the quasi-share parameters, \( \omega_{oy} \), and the elasticity of substitution between labor and oil, \( \delta \).

Each firm \( i \) operates under perfect competition in the factor markets and determines its production plan so as to minimize its total cost

\[
TC_t(i) = \frac{W_t}{P_{Y,t}} H_t(i) + \frac{P_{O,t}}{P_{Y,t}} O_{Y,t}(i), \tag{A1.13}
\]

subject to the production function (A1.12) for given \( W_t \), \( P_{Y,t} \), and \( P_{O,t} \). Their demands for inputs are given by

\[
H_t(i) = \left( \frac{W_t}{MC_t(i) P_{Y,t}} \right)^{-\delta} (1 - \omega_{oy})^{\delta} Y_t(i) \tag{A1.14}
\]

\[
O_{Y,t}(i) = \left( \frac{P_{O,t}}{MC_t(i) P_{Y,t}} \right)^{-\delta} \omega_{oy}^{\delta} Y_t(i), \tag{A1.15}
\]

where the real marginal cost in terms of core consumption goods units is given by

\[
MC_t(i) \equiv MC_t = \left( (1 - \omega_{oy})^{\delta} \left( \frac{W_t}{P_{Y,t}} \right)^{1-\delta} + \omega_{oy}^{\delta} \left( \frac{P_{O,t}}{P_{Y,t}} \right)^{1-\delta} \right)^{\frac{1}{1-\delta}}. \tag{A1.16}
\]

Price setting

Final goods producers operate under perfect competition and therefore take the price level \( P_{Y,t} \) as given. In contrast, intermediate goods producers operate under monopolistic competition and face a downward-sloping demand curve for their products, whose price elasticity is positively related to the degree of competition in the market. They set prices so as to maximize profits following a sticky price setting scheme à la Calvo. Each firm contemplates a fixed probability \( \theta \) of not being able to change its price next period and therefore sets its profit-maximizing price \( \bar{P}_{Y,t}(i) \) to solve

\[
\arg \max_{\bar{P}_{Y,t}(i)} \left\{ \mathbb{E}_{t} \sum_{n=0}^{\infty} \theta^n D_{t,t+n} \bar{\Pi}_{t,t+n}(i) \right\},
\]

where \( D_{t,t+n} \) is the stochastic discount factor defined by \( D_{t,t+n} = \beta^n \left( \frac{C_{t+n}}{C_t} \right)^{-\sigma} \frac{P_{t+n}}{P_{t+n}} \) and profits are

\[
\bar{\Pi}_{t,t+n}(i) = \bar{P}_{Y,t}(i) Y_{t+n}(i) - MC_{t+n} P_{t+n} Y_{t+n}(i).
\]
The solution to this intertemporal maximization problem yields

\[
\frac{P_{Y,t}(i)}{P_{Y,t}} = \frac{K_t}{F_t},
\]

where

\[K_t \equiv \left(\frac{\varepsilon}{\varepsilon - 1}\right) \mathbb{E}_t \sum_{n=0}^{\infty} (\beta \theta)^n \left(X_{t+n}^Y\right)^{-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^\sigma}\right) \left(\frac{P_{t+n}^Y}{P_{t+n}}\right) MC_{t+n},\]

and

\[F_t \equiv \mathbb{E}_t \sum_{n=0}^{\infty} (\beta \theta)^n \left(X_{t+n}^Y\right)^{1-\varepsilon} \left(\frac{Y_{t+n}}{C_{t+n}^\sigma}\right) \left(\frac{P_{t+n}^Y}{P_{t+n}}\right).\]

Since only a fraction \((1 - \theta)\) of the intermediate goods firms are allowed to reset their prices every period while the remaining firms update them according to the steady-state inflation rate, it can be shown that the overall core price index dynamics is given by the following equation

\[(P_{Y,t})^{1-\varepsilon} = \theta (P_{Y,t-1})^{1-\varepsilon} + (1 - \theta) \left(\frac{P_{Y,t}}{P_{Y,t}}(i)\right)^{1-\varepsilon} \tag{A1.18}\]

Following Benigno and Woodford (2005), I rewrite equation (A1.18) in terms of the core inflation rate \(\Pi_{Y,t}\)

\[\theta (\Pi_{Y,t})^{\varepsilon-1} = 1 - (1 - \theta) \left(\frac{K_t}{F_t}\right)^{1-\varepsilon}, \tag{A1.19}\]

for

\[K_t = \frac{\varepsilon}{\varepsilon - 1} \left(\frac{Y_t}{C_t^\sigma}\right) \frac{P_{Y,t}}{P_t} MC_t + \beta \theta \mathbb{E}_t \left\{ (\Pi_{Y,t+1})^{\varepsilon} K_{t+1} \right\},\]

and

\[F_t = \frac{Y_t}{C_t^\sigma} \frac{P_{Y,t}}{P_t} + \beta \theta \mathbb{E}_t \left\{ (\Pi_{Y,t+1})^{\varepsilon-1} F_{t+1} \right\}.\]

### A1.3 Government

To close the model, I assume that oil is extracted with no cost by the government, which sells it to the households and the firms and transfers the proceeds in a lump sum fashion to the households. I abstract from any other role for the government and assume that it runs a balanced budget in each and every period so that its budget constraint is simply given by

\[T_t = P_{O,t} O_t,\]

for \(O_t\) the total amount of oil supplied.
AI.4 Market clearing and aggregation

In equilibrium, goods, oil, and labor markets clear. In particular, given the assumption of a representative household and competitive labor markets, the labor market clearing condition is

\[ H_t^D \equiv \int_0^1 H_t(i)\, di = \int_0^1 H_t(j)\, dj \equiv H_t. \]

Because I assume that the real price of oil \( P_{o,t} \) is exogenous in the model, the government supplies all demanded quantities at the posted price. The oil market clearing condition is then given by

\[ \int_0^1 O_{C,t}(j)\, dj + \int_0^1 O_{Y,t}(i)\, di = O_t, \]

for \( O_t \) the total amount of oil supplied.

As there is no net aggregate debt in equilibrium,

\[ \int_0^1 B_t(j)\, dj = B_t = 0, \]

we can consolidate the government’s and the household’s budget constraints to get the overall resource constraint

\[ C_{Y,t} = Y_t. \]

Finally, Calvo price setting implies that in a sticky price equilibrium there is no simple relationship between aggregate inputs and aggregate output, i.e., there is no aggregate production function. Namely, defining the efficiency distortion related to price stickiness \( P_t^* \equiv \frac{P_{t}^{\text{disp}}}{P_t} \) for \( P_t^{\text{disp}} \equiv \left( \int_0^1 (P_{Y,t}(i))^{-\varepsilon} \, di \right)^{-\frac{1}{\varepsilon}} \), I follow Yun (1996) and write the aggregate production relationship

\[ Y_t = \left( (1 - \omega_{oy}) H_t^{\frac{\varepsilon-1}{\varepsilon}} + \omega_{oy} O_{Y,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} P_t^*, \tag{A1.20} \]

where price dispersion leads to an inefficient allocation of resources given that

\[ P_t^* : \begin{cases} \leq 1 & P_{Y,t}(r) = P_{Y,t}(s), \text{ all } r = s. \end{cases} \]

The inefficiency distortion \( P_t^* \) is related to the rate of core inflation \( \Pi_{Y,t} \) by making use of the definition

\[ P_t^* = \left( \theta \left( P_{t-1}^{\text{disp}} \right)^{-\varepsilon} + (1 - \theta) \left( \frac{P_{Y,t}(i)}{P_{t}^{*}} \right)^{-\varepsilon} \right)^{-\frac{1}{\varepsilon}}, \]

and equations (A1.19) and (A1.17) to get

\[ P_t^* = \left( (1 - \theta) \left( \frac{K_t}{F_t} \right)^{-\varepsilon} + \theta (\Pi_{Y,t})^\varepsilon \right)^{-1}. \]
Appendix II: log-linearized economy

The allocation in the decentralized economy can be summarized by the following five equations. Log-linearizing the labor supply equation (A1.5) (and setting \( \eta = 0 \) for flexible real wages), the labor demand equation (A1.14), and the real marginal cost (A1.16), gives equations (A2.1), (A2.2), and (A2.3). Substituting out oil consumption (A1.7) in (A1.3) and making use of the overall resource constraint gives (A2.4). Finally, equation (A2.5) is the log-linear version of (A1.8) and describes the evolution of the ratio of core to headline price indices as a function of the real price of oil in consumption units. Lowercase letters denote the percent deviation of each variable with respect to their steady states (e.g., \( c_t \equiv \log \left( \frac{c_t}{c_s} \right) \)):

\[
\begin{align*}
w_t &= \phi h_t + \sigma c_t \quad \text{(A2.1)} \\
h_t &= y_t - \delta (w_t - mc_t - py_t) + \Delta_t \quad \text{(A2.2)} \\
mc_t &= (1 - \omega_{oy}) (w_t - py_t) + \omega_{oy} (po_t - py_t) \quad \text{(A2.3)} \\
c_t &= -\chi \omega_{oc} po_t + y_t \quad \text{(A2.4)} \\
py_t &= -\frac{\omega_{oc}}{1 - \omega_{oc}} po_t \quad \text{(A2.5)}
\end{align*}
\]

where \( w_t = \log \left( \frac{W_t P'}{P W} \right) \) is the consumption real wage, \( po_t = \log \left( \frac{P_{o,t}}{P_{o,s}} \right) \) is the real oil price in consumption units, \( py_t = \log \left( \frac{P_{y,t}}{P_{y,s}} \right) \) is the relative price of the core goods in terms of consumption goods, \( \omega_{oy} \equiv \omega_{oy} \left( \frac{P_{o}}{P_{c} P_{y}} \right)^{1-\delta} \) is the share of oil in the real marginal cost, \( \omega_{oc} \equiv \omega_{oc} P_{o}^{1-\chi} \) is the share of oil in the CPI, and \( sy \equiv (1 - \omega_{oc}) \left( \frac{P_{y}}{P_{c}} \right)^{\chi-1} \) is the share of the core good in the consumption goods basket.

Also, the real marginal cost is equal to the inverse of the desired gross markup in the steady state, itself determined by the degree of monopolistic competition as measured by the elasticity of substitution between goods \( \varepsilon \). So \( MC = \frac{1}{\varepsilon} \) in the steady state and \( MC \to 1 \) when \( \varepsilon \to \infty \) in the perfect competition limit.

Appendix III: Deriving a quadratic loss function

The policy problem originally defined as maximizing households utility can be rewritten in terms of a quadratic loss function defined over the welfare relevant output gap \( y_t - y^*_t \) and core inflation \( \pi_{c,t} \) as shown in Benigno and Woodford (2005).
AIII.1 Second-order approximation of the model supply side

Starting with the labor market, labor demand can be rewritten as:

\[ H_t = \left( \frac{W_t}{MC_t P_{yt,t}} \right)^{-\delta} (1 - \omega_{oy})^\delta y_t \frac{Y_t}{P_t^*}, \]

and the labor supply as:

\[ C_t \sigma^{(1-\eta)} H_t \phi^{(1-\eta)} = \frac{W_t}{P_t} \left( \frac{W_{t-1}}{P_{t-1}} \right)^{-\eta} \]

Rewriting them in log-deviations from steady state:

\[ h_t = y_t - \delta (w_t - mc_t - py_t) + \Delta_t \]

\[ w_t = \eta w_{t-1} + (1 - \eta) (\phi h_t + \sigma c_t) \]

where \( \Delta_t \) is the log deviation of the price dispersion measure \( \frac{1}{P_t} \) from its steady state and measures the distortion due to inflation. Note that these two log-linear equations are exact transformations of the nonlinear equations.

Combining the labor demand and supply with a second-order approximation of the real marginal cost

\[ mc_t = (1 - \tilde{\omega}_{oy}) [w_t - py_t] + \tilde{\omega}_{oy} [p_0 t - py_t] + \frac{1}{2} \tilde{\omega}_{oy} (1 - \tilde{\omega}_{oy}) (1 - \delta) [w_t - p_0 t]^2 + O (||\xi||^3) \]

and a first-order approximation of the demand for consumption (where the demand for energy consumption has been substituted out)

\[ c_t = -\chi \frac{\tilde{\omega}_{ce}}{8y} p_{o,t} + y_t + O (||\xi||^2), \quad (A3.1) \]

we obtain a second-order accurate equilibrium relation linking total hours to output and the real price of oil

\[ h_t = (1 - D (\sigma + \phi)) y_t + \frac{D}{(1 - \eta)} M p_{o,t} + \frac{W}{(1 - \omega_{oy})} \Delta_t + \frac{1}{2} \frac{D}{(1 - \eta)} W^2 \frac{(1 - \delta)}{(1 - \omega_{oy})} [J y_t + L p_0 t]^2 + O (||\xi||^3) + t.i.p \quad (A3.2) \]

where

\[ M \equiv \left[ \frac{\tilde{\omega}_{oc} (1 - \eta) \sigma \chi}{8y} - \frac{\tilde{\omega}_{oc} \phi \chi}{8y} + B \right] \]

\[ B \equiv 1 - W - W (1 + (1 - \eta) \delta) \tilde{\omega}_{oc} (1 + (1 - \eta) A) - (1 - \eta) A \]

\[ A \equiv \frac{\phi \delta}{(1 + (1 - \eta) \phi)} \left( \frac{\tilde{\omega}_{oc}}{1 - \omega_{oc}} \right) + \frac{\chi \sigma}{(1 + (1 - \eta) \phi)} \left( \frac{\tilde{\omega}_{oc}}{8y} \right) \]

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\[ W \equiv \frac{(1-\omega_0)}{1+\omega_0(1-\eta)\phi}, \]
\[ 1 - W \equiv \frac{\omega_0(1+(1-\eta)\phi\delta)}{1+\omega_0(1-\eta)\phi}, \]
\[ J \equiv (1 - \eta) (\sigma + \phi), \]
\[ L \equiv \frac{(1-\eta)\phi\delta}{1+(1-\eta)\phi\delta}B - (1 + \omega_0 (1 - \eta) \phi \delta) (1 + (1 - \eta) A), \]
\[ D = (1 - \eta) \delta W \frac{\omega_0}{(1-\omega_0)}. \]

As the price dispersion measure can be written as
\[ \mathcal{P}_t^* = \left( (1 - \theta) \left( \frac{K_t}{F_t} \right)^{-\varepsilon} + \theta (\Pi_{t+1})^{\varepsilon} \right)^{-1}, \]

Benigno and Woodford (2004) demonstrate that \( \Delta_t \) — the log deviation of the price dispersion measure — has a second-order approximation that depends only on second-order inflation terms and lagged dispersion
\[ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta_t = f (\Delta_{t_0-1}) + \frac{1}{2} \varepsilon \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\pi_y^2}{2} + O \left( \| \xi \|^3 \right). \] (A3.3)

### AIII.2 Second-order approximation to NKPC

In this section I derive a second-order approximation to the NKPC, which can be used to substitute out the term linear in \( y_t \) in the second-order approximation to utility when the steady state is distorted.

I start by writing a second-order approximation to the model inflation/marginal cost nexus. For convenience, I rewrite from the main text
\[
K_t = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \left( \frac{Y_t}{C_t} \right) P_{y,t}M_{C_t} + \beta \theta E_t (\Pi_{t+1})^{\varepsilon} K_{t+1}
\]
\[
F_t = \left( \frac{Y_t}{C_t} \right) P_{y,t} + \beta \theta E_t (\Pi_{t+1})^{\varepsilon - 1} F_{t+1}
\]
\[
K_t

Taking a second-order approximation of the three preceding equations, I follow Benigno and Woodford (2005), Castillo et al. (2006) and Montoro (2007) and express the NKPC as
\[
V_t = kmc_t + \frac{1}{2} kmc_t \left[ 2(y_t - \sigma c_t + p_{y,t}) + mc_t \right] + \frac{1}{2} \varepsilon n_t^2 + \beta E_t V_{t+1} + O \left( \| \xi \|^3 \right) \] (A3.4)

where I define the auxiliary variable \( V_t \)
\[
V_t = \Pi_{y,t} + \frac{1}{2} \left( \frac{\varepsilon - 1}{1 - \theta} + \varepsilon \right) (\Pi_{y,t})^2 + \frac{1}{2} (1 - \theta \beta) \Pi_{y,t} z_t
\]
and the linear expansion of $z_t$

$$z_t = 2(y_t - \sigma c_t + p_{y,t}) + mc_t + \theta\beta E_t \left( \frac{2\varepsilon - 1}{1 - \theta\beta} \Pi_{Y,t+1} + z_{t+1} \right).$$

Using the first-order\textsuperscript{36} approximation of $c_t$ and $p_{y,t}$ and a second-order approximation of $mc_t$, I write

$$mc_t = \mathcal{W}\eta w_{t-1} + (1 - \eta) (\sigma + \phi) \mathcal{W}y_t$$
$$+ (1 - \eta) \phi \mathcal{W}\Delta_t + \mathcal{B} p_0 t$$
$$+ \frac{1}{2} \frac{(1 - \delta)}{(1 - \omega_{ay})} \mathcal{W}^2 \left( (1 - \mathcal{W}) \left[ (1 - \eta) (\sigma + \phi) y_t + \mathcal{L} p_{0,t} \right]^2 \right. $$
$$+ O \left( \|\xi\|^3 \right) + t.i.p.$$

which I substitute in (A3.4) to get

$$V_t = k_y y_t + k_p P_{o,t} + k \mathcal{W} \phi \Delta_t$$
$$+ \frac{1}{2} k \left( c_{yy} y_t^2 + 2 c_{yp} y_t p_{o,t} + c_{pp} P_{0,t}^2 \right)$$
$$+ \frac{1}{2} \varepsilon \pi_{y,t}^2 + \beta E_t V_{t+1} + O \left( \|\xi\|^3 \right)$$

(A3.5)

for

$$k_y \equiv k (1 - \eta) (\sigma + \phi) \mathcal{W}$$
$$k_p \equiv k \mathcal{B}$$
$$c_{yy} \equiv F (1 - \eta)^2 (\sigma + \phi)^2 + 2 (1 - \eta) (\sigma + \phi) (1 - \sigma) \mathcal{W} + (1 - \eta)^2 (\sigma + \phi)^2 \mathcal{W}^2$$
$$c_{yp} \equiv (1 - \eta) (\sigma + \phi) \mathcal{W} (\Sigma + \mathcal{B}) - F (\sigma + \phi) (1 - \eta) \mathcal{L} + \mathcal{B} (1 - \sigma)$$
$$c_{pp} \equiv F \mathcal{L}^2 + 2 \Sigma \mathcal{B} + \mathcal{B}^2$$
$$F \equiv \frac{1 - \delta}{1 - \omega_{ay}} \mathcal{W}^2 (1 - \mathcal{W})$$
$$\Sigma \equiv \sigma \chi \frac{\omega_{ay}}{\sigma} - \frac{\omega_{ay}}{1 - \omega_{ay}}.$$

Note that the natural level of output can be found from the preceding equation by rewriting it as

$$V_t = k_y \left\{ y_t + k_y^{-1} k_p P_{o,t} + k_y^{-1} k \mathcal{W} \phi \Delta_t + \frac{1}{2} k_y^{-1} k \left( c_{yy} y_t^2 + 2 c_{yp} y_t p_{o,t} + c_{pp} P_{0,t}^2 \right) + \frac{1}{2} k_y^{-1} \varepsilon \pi_{y,t}^2 \right\}$$
$$+ \beta E_t V_{t+1} + O \left( \|\xi\|^3 \right)$$

and ignoring all second-order terms.

\textsuperscript{36}A second-order approximation is not necessary here as these two variables enter multiplicatively with $mc_t$. 

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Using the law of iterated expectation and (A3.3), equation (A3.5) can be rewritten as an infinite discounted sum

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{k_y} (V_{t_0} - f (\Delta_{t_0-1}))$$

$$- \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ k_y^{-1} p_0, t + \frac{1}{2} k_y^{-1} k (c_{yy} y_t^2 + 2 c_{yp} y_t p_{o,t} + c_{pp} p_{o,t}^2) \right\} + O \left( \|\xi\|^3 \right).$$

(A3.6)

AIII.3 Second-order approximation to utility

I take a second-order approximation of the representative household utility function in \( t_o \)

$$U_{t_o} = E_{t_o} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ u(C_t) - v(H_t) \right\}. \quad \text{(A3.7)}$$

The second-order approximation of the first term is given by

$$u(C_t) = C u_c \left\{ c_t + \frac{1}{2} (1 - \sigma) c_t^2 \right\} + O \left( \|\xi\|^3 \right) + t.i.p. \quad \text{(A3.8)}$$

Substituting (A3.1) and its square into (A3.8), I get

$$u(C_t) = C u_c \left\{ u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yp} y_t p_{o,t} \right\} + O \left( \|\xi\|^3 \right) + t.i.p. \quad \text{(A3.9)}$$

for \( u_y \equiv 1, \ u_{yy} \equiv 1 - \sigma, \ u_{yp} \equiv -\frac{\sigma}{s_y} (1 - \sigma) \) and \( t.i.p \) stands for terms independent of policy.

The second term in household utility is approximated by

$$v(H_t) = H v_h \left\{ h_t + \frac{1}{2} (1 + \phi) h_t^2 \right\} + O \left( \|\xi\|^3 \right) + t.i.p. \quad \text{(A3.10)}$$

Substituting (A3.2) and its square in (A3.10) and getting rid of variables independent of policy, I obtain

$$v(H_t) = H v_h \left\{ v_y y_t + v_{\Delta} \Delta_t + \frac{1}{2} v_{yy} y_t^2 + v_{yp} p_{o,t} y_t \right\} + O \left( \|\xi\|^3 \right) + t.i.p. \quad \text{(A3.11)}$$

for
\[ v_y \equiv 1 - D (\phi + \sigma), \]
\[ v_\Delta \equiv \frac{\nu}{1 - \omega_{\text{opt}}}, \]
\[ v_{yy} \equiv \frac{D}{1 - \eta} \mathcal{W}^2 \left( \frac{1 - \delta}{1 - \omega_{\text{opt}}} \right) \mathcal{J}^2 + (1 + \phi) (1 - D (\phi + \sigma))^2, \]
\[ v_{yp} \equiv \frac{D}{1 - \eta} \left[ (1 + \phi) (1 - D (\phi + \sigma)) \mathcal{M} + \mathcal{W}^2 \left( \frac{1 - \delta}{1 - \omega_{\text{opt}}} \right) \mathcal{J} \right]. \]

Now, since the technology is constant returns to scale, the share of labor in total cost is equivalent to its share in marginal cost and the following equilibrium relationship at the steady state

\[ Y u_c M C (1 - \bar{\omega}_{\text{opt}}) = H v_h, \]

which can be used to rewrite total utility \( U_{t_0} \) by substituting (A3.11) and (A3.9) into (A3.7), to get

\[
U_{t_0} = (Y u_c) E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ u_y y_t + \frac{1}{2} u_{yy} y_t^2 + u_{yp} p_o y_t + \frac{1}{2} u_{\pi} \pi_t^2 \right\} + O (||\xi||^3) + \text{t.i.p.},
\]

(A3.12)

where

\[
\begin{align*}
  u_y &= \frac{C}{Y} - MC (1 - \bar{\omega}_{\text{opt}}) v_y, \\
  u_{yy} &= \frac{C}{Y} (1 - \sigma) - MC (1 - \bar{\omega}_{\text{opt}}) v_{yy}, \\
  u_{yp} &= -\frac{C}{Y} \left( \frac{\bar{\omega}_{\text{opt}}}{\omega_{\text{opt}}} \right) \chi (1 - \sigma) - MC (1 - \bar{\omega}_{\text{opt}}) v_{yp}, \\
  u_\Delta &= -MC (1 - \bar{\omega}_{\text{opt}}) v_\Delta = -MC W, \\
  u_\pi &= \frac{g}{k} u_\Delta = \frac{g}{k} MC W.
\end{align*}
\]

For the last step, substituting the expression (A3.6) for \( \sum_{t=t_0}^{\infty} \beta^{t-t_0} y_t \) in (A3.12) obtains

\[
U_{t_0} = -Y u_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left( u_y k_y^{-1} k c_{yy} - u_{yy} \right) y_t^2 \\
+ \frac{1}{2} (2 u_y k_y^{-1} k c_{yy} - 2 u_{yy}) p_o y_t \\
+ \frac{1}{2} (u_y k_y^{-1} \varepsilon (1 + \phi W) - u_\pi) \pi_t^2 + O (||\xi||^3) + \text{t.i.p.},
\]

that can be rewritten equivalently as

\[
U_{t_0} = -Y u_c E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left( u_y k_y^{-1} k c_{yy} - u_{yy} \right) [y_t - y_t^*]^2 \\
+ \frac{1}{2} (u_y k_y^{-1} \varepsilon (1 + \phi W) - u_\pi) \pi_t^2 + O (||\xi||^3) + \text{t.i.p.}
\]

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which ends up as the central banks’s loss function to minimize

$$U_t = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \lambda x_t^2 + \pi_t^2 \right\} + O \left( \| \xi \|^3 \right) + t.i.p. \quad \text{(A3.13)}$$

for $\Upsilon \equiv \frac{1}{2} \lambda_{\pi_y} \Psi_{uc}$, $\Psi \equiv \frac{k_{\pi_y} k_{pp}^{-1} c_{pp} - u_{pp}}{k_{\pi_y} k_{pp}^{-1} c_{pp} - u_{pp}}$, and where $\lambda \equiv \frac{\lambda_y}{\lambda_{\pi_y}}$ for $\lambda_y = u_y k_{\pi}^{-1} c_{yy} - u_{yy}$ and $\lambda_{\pi_y} = u_y k_{\pi}^{-1} c (1 + \phi W) - u_{\pi}$. The output gap $x_t = y_t - y_t^*$ is now the percent deviation of output with respect to the welfare relevant output $y_t^*$ itself defined as

$$y_t^* \equiv - \frac{k u_y k_{\pi}^{-1} c_{yp} - u_{yp}}{k u_y k_{\pi}^{-1} c_{yy} - u_{yy}} p_{o,t} = - \Psi p_{o,t}. \quad \text{(A3.13)}$$

The values of $\lambda_y$ and $\lambda_{\pi_y}$ are functions of the model parameters and describe the weights assigned by the central bank to stabilize the welfare relevant output gap and core inflation. In what follows I summarize this information with $\lambda \equiv \frac{\lambda_{\pi_y}}{\lambda_y}$, which determines how concerned about the output gap a central bank should be after an oil price shock. Typically, $\lambda$ decreases with the sacrifice ratio and the degree of price stickiness.

**Appendix IV: Characterizing optimal policy**

As shown in Section 3, acknowledging the low level of short-term substitutability between oil and other factors gives rise to a cyclical distortion coming from the interaction between the steady-state efficiency distortion and the oil price shock. In terms of the model equations, this cyclical distortion is translated into a cost-push shock that enters the New Keynesian Phillips curve (NKPC henceforth). Taking a log-linear approximation of equation (A1.19) around the zero inflation steady-state yields the standard result that (core) inflation is a function of next period inflation and this period real marginal cost: the NKPC

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k m c_t, \quad \text{(A4.1)}$$

where $m c_t$ is the log-deviation of real marginal cost from its (distorted) steady state and $k = \left( \frac{1 - \theta}{\eta} \right) (1 - \theta \beta)$ is the elasticity of inflation to the real marginal cost.

Substituting the labor market clearing level of the real wage into the real marginal cost equation (A1.16), we can rewrite (A4.1) as

$$\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k y g a p_t, \quad \text{(A4.2)}$$

where the output gap $gap_t = y_t - y_t^N$ measures the deviation between current output and the natural level of output, and where

$$y_t^N = - \frac{k \mathcal{B}}{k_y} p_{o,t}, \quad \text{(A4.3)}$$

for $\mathcal{B}$ a decreasing function of $\delta$ and $\chi$, the oil production and consumption elasticities of substitution (see Appendix III).
But (A4.2) can be equivalently rewritten as

\[
\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t, \tag{A4.4}
\]

for \( \mu_t = k_y (y_t^* - y_t^N) \) the cost-push shock that arises as a direct function of the cyclical wedge between the natural and the welfare maximizing level of output. and \( x_t = y_t - y_t^* \) as defined in Appendix III.

Obviously, the divine coincidence obtains when \( y_t^* = y_t^N \), which is the case for \( \chi = \delta = 1 \), as shown in Section 3.

Following Benigno and Woodford’s (2005) linear-quadratic approach, I circumvent the usual time consistency issues associated with fully optimal monetary policies by assuming that the central bank is able to commit with full credibility to an optimal policy plan which specifies a full set of state-contingent sequences \( \{x_t, \pi_{y,t}\}_{t=0}^{\infty} \) that minimize

\[
\gamma \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \lambda \pi_t^2 + \pi_{y,t}^2 \right\}
\]

subject to the following sequence of constraints

\[
\pi_{y,t} = \beta \mathbb{E}_t \pi_{y,t+1} + k_y x_t + \mu_t \tag{A4.5}
\]

and a constraint on the initial inflation rate

\[
\pi_{y,t_0} = \pi_{y,t_0}, \tag{A4.6}
\]

where \( \pi_{y,t_0} \) is defined as the inflation rate in time \( t_0 \) that is consistent with optimal policy in a "timeless perspective" or, in other words, the inflation rate that would have been chosen a long time ago and which is consistent with the optimal precommitment plan.

Solving this problem under the timeless perspective gives rise to the following set of first-order conditions

\[
x_t = x_{t-1} - \frac{k_y}{\lambda} \pi_{y,t}, \tag{A4.7}
\]

which are supposed to hold for all \( t = 0, 1, 2, 3, \ldots \) and which characterize the central bank’s optimal policy response.

**Appendix V : Deriving an optimal simple rule**

Following McCallum (1999b) and McCallum and Nelson (2004), the no bubble MSV solution to the system formed by equations (A4.4) and (A4.7) can be written as:

\[
\pi_{y,t} = \alpha_{11} x_{t-1} + \alpha_{12} \mu_t \tag{A5.1}
\]

\[
x_t = \alpha_{21} x_{t-1} + \alpha_{22} \mu_t, \tag{A5.2}
\]
where $\alpha_{ij}$ for $i, j = 1, 2$ are functions of $\beta$, $k_y$ and $\lambda$.

Because the supply of oil is supposed perfectly elastic at a given exogenous real price, one can write the following definitions:

$$c_t - c_t^* = y_t - y_t^*$$

$$\pi_{c,t} = \pi_{y,t} + \frac{\omega_{oc}}{1 - \omega_{oc}} (p_{ot} - p_{ot-1})$$

that are used to rewrite the consumption Euler equation in deviation from efficient consumption as follows:

$$x_t = \frac{1}{\sigma} \left( r_t - \mathbb{E}_t \pi_{y,t+1} - \frac{\omega_{oc}}{1 - \omega_{oc}} \mathbb{E}_t (p_{ot+1} - p_{ot}) - r_t^* \right) + \mathbb{E}_t x_{t+1}$$  \hspace{1cm} (A5.3)

for $rr_t^* = \sigma \mathbb{E}_t \{ \Delta y_{t+1}^* \} = -\Psi \sigma (1 - \rho_o) p_{ot}$.

Combining (A5.1), (A5.2), and (A5.3) leads to:

$$\alpha_{21} x_{t-1} + \alpha_{22} \mu_t = -\frac{1}{\sigma} \left( r_t - \alpha_{11} x_t - \alpha_{12} \rho_o \mu_t \right) + \left( \rho_o - 1 \right) \left( \frac{\omega_{oc}}{1 - \omega_{oc}} - \Psi \sigma \right) p_{ot}$$

which can be solved for $r_t$:

$$r_t = (\alpha_{11} + \sigma \alpha_{21}) x_t - (\sigma \alpha_{22} - \alpha_{12} \rho_o - \sigma \alpha_{22} \rho_o) \mu_t$$

$$+ \left( \rho_o - 1 \right) \left( \frac{\omega_{oc}}{1 - \omega_{oc}} - \Psi \sigma \right) p_{ot}$$

From (A5.1), $\mu_t = \frac{\pi_{y,t} - \alpha_{11} x_{t-1}}{\alpha_{12}}$, so that the above equation can be rewritten as:

$$r_t = \frac{\Phi}{\alpha_{11}} \pi_{y,t} + (\alpha_{11} + \sigma \alpha_{21}) x_t - (\Phi + \sigma \alpha_{21}) x_{t-1} + \Xi p_{ot},$$

for $\Phi = (\rho_o - \sigma \alpha_{22} \alpha_{12}^{-1} (1 - \rho_o)) \alpha_{11}$.

Finally, substituting $y_t^* = -\Psi p_{ot}$ in the above yields:

$$r_t = \frac{\Phi}{\alpha_{11}} \pi_{y,t} + (\alpha_{11} + \sigma \alpha_{21}) (y_t + \Psi p_{ot})$$

$$- (\Phi + \sigma \alpha_{21}) (y_{t-1} + \Psi p_{ot-1}) + \Xi p_{ot},$$

so:

$$r_t = \Phi \alpha_{11}^{-1} \pi_{y,t} + \Omega y_t - \Gamma y_{t-1} + (\Xi + \Psi \Omega) p_{ot} - \Psi \Gamma p_{ot-1}.$$
Appendix VI : Time varying elasticities

To allow for time-varying elasticities of substitution, I transform the production processes of Section 2 by introducing a convex adjustment cost of changing the input mix in production and redefine equations (A1.12) and (A1.3), such that

$$Y_t = \left( (1 - \omega_{oy}) H_t^{\frac{k_y}{k}} + \omega_{oy} \left[ \frac{\varphi_{OY,t} O_{Y,t}}{O_{Y,t-1}/H_{t-1}} \right]^{\frac{\delta}{\delta-1}} \right)^{\frac{1}{\delta-1}}, \quad (A6.1)$$

and

$$C_t = \left( (1 - \omega_{oc}) C_{Y,t}^{\frac{k_c}{k}} + \omega_{oc} \left[ \frac{\varphi_{OC,t} O_{C,t}}{O_{C,t-1}/C_{Y,t-1}} \right]^{\frac{\chi}{\chi-1}} \right)^{\frac{1}{\chi-1}}. \quad (A6.2)$$

The variables $\varphi_{OY,t}$ and $\varphi_{OC,t}$ represent the costs of changing the oil intensity in the production of the core good and the consumption basket, and are supposed to take the following quadratic form

$$\varphi_{OY,t} = \left[ 1 - \frac{\varphi_{OY}}{2} \left( \frac{O_{Y,t}/H_t}{O_{Y,t-1}/H_{t-1}} - 1 \right)^2 \right], \quad (A6.3)$$

$$\varphi_{OC,t} = \left[ 1 - \frac{\varphi_{OC}}{2} \left( \frac{O_{C,t}/C_{Y,t}}{O_{C,t-1}/C_{Y,t-1}} - 1 \right)^2 \right]. \quad (A6.4)$$

This specification allows for oil demand to respond quickly to changes in output and consumption, while responding slowly to relative price changes. In the long-run, the elasticity of substitution is determined by the value of $\delta$ and $\chi$. Although somewhat ad hoc, this form of adjustment costs introduces a time-varying elasticity of substitution for oil, an important characteristic of putty-clay models such as in Atkeson and Kehoe (1999) or Gilchrist and Williams (2005).37

The presence of adjustment costs transforms the static cost-minimization problem of the representative intermediate firms and final consumption goods distributors into forward-looking dynamic ones. They can be regarded as choosing contingency plans for $O_{Y,t}$, $H_t$, $O_{C,t}$, and $C_Y$, that minimize their discounted expected cost of producing $Y_t$ and $C_t$ subject to the constraints represented by equations (A6.1) to (A6.4).

I calibrate $\varphi_{OY}$ and $\varphi_{OC}$ such that the instantaneous price elasticities of demand for oil correspond to the baseline calibration chosen in the CES setting of the previous sections. The de facto short-term elasticities are then set to 0.3. In the long run, I assume a unitary elasticity of substitution ($\delta = \chi = 1$) such that (A6.1) and (A6.2) are de facto Cobb-Douglas functions when $t \to \infty$.37

37In putty-clay models of energy use, a large variety of types of capital goods are combined with energy in different fixed proportions, making the short-term elasticity of substitution low. In the longer run, the elasticity goes up as firms invest in capital goods with different fixed energy intensities.
Appendix VII: Welfare cost and elasticities

The analysis in Sections 3 and 4 shows that the optimal response of policy to an oil price shock is a function of the economy’s dependence on oil. Small elasticities of substitution lead to large trade-offs as the wedge between natural and efficient output increases (see Figures 1 and 2). But low elasticity also tends to decrease lambda, the relative weight of output fluctuations in the central bank’s loss function (see Figures 3 and 4 and Section 4). The net effect is therefore uncertain.

This appendix computes the sensitivity of the welfare costs to different values for the elasticity of oil (energy) in use and in production. Like in Section 6, the costs will be expressed in percentage of steady state consumption. Table 3 reports the loss differences with respect to optimal policy when policy focuses on stabilizing core prices (PPS), when it follows a Taylor rule based on core inflation (CTR) or a Taylor rule based on headline inflation (HTR).

The first result is that the welfare costs under sub-optimal policies are larger the more dependent on oil is the economy. Both the elasticities of substitution for energy in production (δ) and in consumption (χ) play a role. The second insight is that welfare losses are highly non-linear in the elasticities, which is in line with Figures 1 and 2 which show the response of the gap between natural and efficient output (driving the cost-push shock). Policies do not differ much when the elasticities of substitution in use and production are both higher than .6. Differences become extremely large, however, for elasticities below .2. If we take the median estimates from Kilian and Murphy (2010) of χ = .2 and δ = .4 (in bold), we obtain that perfectly stabilizing core prices would have cost .6% of steady-state consumption when compared to optimal policy during the 1979 oil price shock. If the authorities had followed a Taylor rule based on core inflation (CTR), the cost would have been a more substantial 6.3%. Finally, assuming HTR, the cost would have been even larger at 7.7%.
Tables

Table 1: Optimized simple rule (OR) and speed limit rule (SLR)

<table>
<thead>
<tr>
<th>Simple rule</th>
<th>( g_z )</th>
<th>( g_y )</th>
<th>( g_{y1} )</th>
<th>( g_{po} )</th>
<th>( g_{po1} )</th>
<th>( g_{w1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OR</td>
<td>5.123</td>
<td>4.742</td>
<td>-4.731</td>
<td>-0.007</td>
<td>0.014</td>
<td>-</td>
</tr>
<tr>
<td>SLR</td>
<td>5.101</td>
<td>4.742</td>
<td>-4.742</td>
<td>-0.008</td>
<td>0.008</td>
<td>-</td>
</tr>
<tr>
<td>OR_w (( \eta = 0.9 ))</td>
<td>5.134</td>
<td>8.708</td>
<td>-7.884</td>
<td>-0.276</td>
<td>0.240</td>
<td>0.088</td>
</tr>
<tr>
<td>SLR_w (( \eta = 0.9 ))</td>
<td>2.054</td>
<td>3.404</td>
<td>-3.404</td>
<td>-0.096</td>
<td>0.096</td>
<td>-</td>
</tr>
</tbody>
</table>

*note*: all coefficients consistent with annualized interest rates and inflation

Table 2: Welfare costs under alternative policies (percent of annual consumption)

<table>
<thead>
<tr>
<th>Policy</th>
<th>Total Loss</th>
<th>( \pi_y ) Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>core</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strict inflation target</td>
<td>1.8</td>
<td>0</td>
</tr>
<tr>
<td>CTR</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>CTR inertia (( \rho = 0.8 ))</td>
<td>1.7</td>
<td>1.5</td>
</tr>
<tr>
<td>Forecasting CTR</td>
<td>4.4</td>
<td>4.2</td>
</tr>
<tr>
<td>headline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HTR</td>
<td>2.7</td>
<td>0.4</td>
</tr>
<tr>
<td>HTR inertia (( \rho = 0.8 ))</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Forecasting HTR</td>
<td>9.4</td>
<td>9.3</td>
</tr>
</tbody>
</table>

*note*: all simulations with baseline calibration and \( \eta = 0.9 \)

Table 3: Welfare loss under suboptimal policy for a 1979-like oil price shock; different elasticities

<table>
<thead>
<tr>
<th>Loss under</th>
<th>( \chi = 0.2 )</th>
<th>( \chi = 0.4 )</th>
<th>( \chi = 0.6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPS (core)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta = 0.2 )</td>
<td>( \delta = 0.4 )</td>
<td>( \delta = 0.6 )</td>
<td>( \delta = 0.2 )</td>
</tr>
<tr>
<td>4.1</td>
<td>.6</td>
<td>.4</td>
<td>1.6</td>
</tr>
<tr>
<td>CTR</td>
<td>20.4</td>
<td>6.3</td>
<td>5.2</td>
</tr>
<tr>
<td>HTR</td>
<td>43</td>
<td>7.7</td>
<td>5.3</td>
</tr>
</tbody>
</table>

*note*: welfare loss differences with respect to optimal policy; percentage points of steady-state consumption
Figure 1: Response of the gap between natural (YN) and efficient (Ystar) output to a 1-percent increase in oil price as a function of the production and the consumption elasticity of substitution (CHI); baseline calibration with 2% oil share of output, 6% energy component of consumption and 20% steady-state markup
Figure 2: Response of the gap between natural (YN) and efficient (Ystar) output to a 1-percent increase in oil price as a function of the degree of monopolistic competition; baseline calibration with 2% oil share of output and 6% energy component of consumption.
Figure 3: Weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of price stickiness (teta)
Figure 4: Weight (Lambda) assigned to output gap stabilization as a function of the elasticities of substitution (Delta) and the degree of real wage rigidity (eta)
Figure 5: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with flexible price equilibrium; CES technology with low elasticity ($\delta = \chi = 0.3$); baseline calibration
Figure 6: Trade-off magnification effect; difference between optimal policy and FPWE when oil is an input to production only (dashed line) and when oil is an input to both production and consumption (solid line); baseline calibration
Figure 7: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy; CES technology with low elasticity ($\delta = \chi = 0.3$); baseline calibration
Figure 8: Impulse response functions to a 1-percent increase in oil price; comparison of optimal precommitment monetary policy with optimized simple rule and speed limit policy based on four quarters moving average of core inflation; CES technology with low elasticity ($\delta = \chi = 0.3$); real wage rigidity ($\eta = 0.9$)
Figure 9: 1979 oil price shock and comparable AR(1) exogenous process for the real price of oil; log scale
Figure 10: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple Taylor rules based on four quarters moving average values of core or headline inflation; baseline calibration; real wage stickiness ($\eta = 0.9$)
Figure 11: Impulse response functions to a 1979-like 100% log-increase in oil price; comparison of optimal precommitment monetary policy with simple forecast based Taylor rules; baseline calibration; real wage stickiness ($\eta = 0.9$)
Figure 12: Comparison of simple monetary policy rules in the US during the 2006-2008 oil prices rally (shaded area); all rules hp-filtered (hp coef. = 10)
Figure 13: Impulse response functions to a 1-percent log-increase in oil price; time-varying elasticities; comparison of optimal precommitment monetary policy with FPWE; undistorted equilibrium
Figure 14: Impulse response functions to a 1-percent log-increase in oil price; time varying elasticities; comparison of optimal precommitment monetary policy with FPWE; distorted equilibrium (markup 20%)