Demand Uncertainty and Price Maintenance: Markdowns as Destructive Competition

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1 Introduction

The possibility of "destructive" or "ruinous" competition has long been offered as a potentially important defect of market systems. Early economists and policy makers alike were concerned that when firms were required to make up-front commitments prior to the resolution of demand uncertainty, in the event of slack demand "overproduction" could result in ruinous competition, impairing the very existence of the industry in question.[1] The first fundamental theorem of welfare economics, however, predisposes modern economists to be skeptical about arguments that too much competition can be destructive.[2] In the case of a monopolistic manufacturer selling to retailers, intuition might suggest that downstream competition minimizes retail profit margins and allows the manufacturer to manipulate the final retail price with its wholesale price, thereby extracting all available surplus. Even were downstream competition somehow to hurt the manufacturer, it would seem that consumers would be better off, so that competition could not truly be termed destructive. In this paper, we show that this intuition can be wrong: manufacturers may be better off by committing to a minimum resale price for their products sold through independent retailers. This guarantee of a stable market may induce retailers to order larger inventories than had retail markets been permitted to clear. The manufacturer may thus prefer to prevent unfettered retail competition, as it is "destructive" to inventory holdings and to expected sales. Surprisingly, we demonstrate that permitting manufacturers to set minimum resale prices can Pareto dominate retail market clearing. Consumers as well as manufacturers may prefer resale price maintenance (RPM) to retail market clearing, with retailers earning zero expected profits in both cases.

Destructive competition in this paper is not merely another "second-best" phenomenon. The retail market we model is perfectly competitive, with no externalities or other relevant

[1] See Hovenkamp (1989) for a history of such concerns. Leading economists such as Frank Taussig and John Maurice Clark credited ruinous competition as a serious threat to the market economy, as did influential jurists Oliver Wendell Holmes, Jr., and Louis Brandeis. Holmes in particular complained that in a market where information was imperfect, one needed to be concerned about "transitory cheapness unprofitable to the community as a whole." See Holmes' dissent in Dr. Miles Medical Co. v. John D. Park and Sons, Co., 220 U.S. 373 (1911), p. 412.

[2] One notable exception is the theory of contestable markets (Baumol, Panzar and Willig, 1982), which develops conditions on the cost function under which incumbents are vulnerable to hit-and-run entry, thereby causing competition to be unstable. Our approach does not rely on the indivisibilities that are at the heart of the (non)sustainability literature.

market failures. We consider the typical retail situation in which retailers must commit to inventories prior to the resolution of demand uncertainty. We show that the competitive scramble to sell inventories at very low prices should demand be low—as opposed to allowing the inventories to go to waste—can be destructive.[3] We also show, however, that when it is used to prevent deep discounting of unsold merchandise, RPM can, in some cases, be contrary to consumer interests. We thus can explain why consumer groups are often vociferous opponents of RPM.[4]

Our focus on manufacturer attempts to limit destructive competition leads naturally to consideration of resale price maintenance, for RPM has long been justified as necessary to limit such competition among distributors. During its long and contentious history, (Overstreet, 1984; Ippolito, 1988, 1991; Marvel 1994) RPM has been the focus of legislative and court disputes at both the state and federal levels as well as numerous articles in both professional journals and the popular press. Allegations of illegal RPM have been a feature of a very large number of antitrust cases, several of which have reached the Supreme Court. RPM was declared a per se violation of the Sherman Act by the Supreme Court in its famous *Dr. Miles* decision in 1911.[5] The Court noted that an agreement among retailers to set their prices would clearly be illegal, and concluded that if the manufacturer were permitted to control resale prices, the same result would obtain. There is, however, very little evidence that dealer cartels exist or that retailers possess the power to coerce their manufacturer-suppliers (Marvel 1994). Since the applicable section of the Sherman Act (§1) outlawed *combinations* in restraint of trade, the Court later recognized an exception to the rule of per se illegality for a manufacturer who *unilaterally* refused to deal with price-cutting retailers.[6] This exception was narrowly defined, however, and while in the 1930's, 42 states passed laws permitting RPM

[3]In a related paper (Deneckere, Marvel and Peck, 1994) we have have compared RPM to a game in which retailers must set prices prior to the resolution of demand uncertainty. In that model, as in the one presented below, RPM is preferred by manufacturers as it supports higher inventories and higher quantities.

[4]An upstream imperfection in the form of a monopoly manufacturer pricing above marginal cost in order to extract consumer surplus lies at the heart of our theory. Given, however, the large number of products whose production entails high fixed costs, some exercise of market power by manufacturers is unavoidable. Every example of the use of RPM of which we are aware involves a branded or unique product that can be expected to face downward-sloping demand.

[5]Dr. Miles Medical Co. v. John D. Park and Sons, Co., 220 U.S. 373 (1911).

[6]U.S. v. Colgate & Co., 250 U.S. 300 (1919).

for intrastate commerce, the practice remained illegal for most retail trade until 1937, when the Miller-Tydings Act[7] amended the Sherman Act to permit RPM contracts for interstate commerce where permitted by state law. RPM remained illegal in Texas, Missouri, Vermont, and the District of Columbia, while its status in the remaining states was generally legal, though sometimes constrained in ways too complicated to recount here. Over the next forty years, a number of state courts invalidated RPM statutes as unconstitutional, several states repealed their laws, and eventually, in 1975, Congress repealed the Miller-Tydings Act, making RPM again per se illegal.[8] In the 1980's, however, the Supreme Court provided an expansive treatment of its *Colgate* doctrine permitting unilateral manufacturer imposition of RPM programs, so that today, RPM has the curious status of being both per se illegal and widely practiced.[9] Manufacturers may suggest retail prices, receive dealer complaints, and terminate dealers who do not adhere to the manufacturer's desired prices so long as they do not reach agreements with their remaining dealers to establish prices. Manufacturers may not, however, establish policies of terminating dealers for setting discounted prices and reinstating those dealers who stop discounting, for then agreement would be inferred. State Attorney Generals, the Federal Trade Commission, and the Antitrust Division of the Department of Justice have all indicated strong hostility to RPM use, pursuing a number of investigations designed to limit the use of the practice (Marvel 1994).

While it is well understood (Telser, 1960; Marvel and McCafferty, 1984) that manufacturers may wish to suppress price competition to eliminate free-rider effects and thereby to create property rights to dealers providing pre-sale promotion,[10] our model does not rely on the existence of such services.[11] It is clear that early proponents of fair trade were more concerned

[7] 50 Stat. 693, 15 U.S.C.A. §1 (1937).

[8] Consumer Goods Pricing Act of 1975, Public Law 94-145, 89 Stat. 801 (1975).

[9] Business Elecs. Corp. v. Sharp Elecs. Corp., 485 U.S. 717 (1988).

[10] For example, suppose that a manufacturer invents a new appliance for food preparation, but that the uses of the appliance are not immediately obvious to potential consumers through inspection. It may be essential that retailers demonstrate the product's capabilities. Given that demonstration is a costly service, the retailer providing demonstrations must charge retail prices sufficient to cover demonstrations as well as the wholesale price paid to the manufacturer. Retailers not offering such demonstrations could thereby profit by undercutting the prices of demonstrating retail outlets. That is, a customer can visit a costly product demonstration, become convinced to buy the product, and then buy it elsewhere at a lower price. Otherwise identical retailers cannot survive if they provide demonstrations. Demonstrations will not, therefore, be provided, an inefficient outcome.

[11] Ippolito (1991) finds that less than half of litigated RPM cases from 1976-1982 involved complex products for which dealer efforts were important to product quality.

about "disorderly markets" than about product-specific dealer services. As a judge noted in an 1874 court opinion, "[t]he prohibition against selling below the trade price is a very common one between a manufacturer and those who buy of him to sell again, and is intended to prevent a ruinous competition between sellers of the same article."[12] The American Sugar Refinery Company, a near-monopoly trust, was a pioneer RPM user (Eichner, 1969; Zerbe, 1969). It marketed sugar through wholesale grocers dependent on such sugar for in excess of one-third of their business. Pre-sale retail services were not required, nor was the wholesaler required to attest to the purity of the product, for potential adulterants cost more than the sugar itself. In this and other contemporary examples, the manufacturer claimed to impose RPM to counter demand fluctuations that would otherwise destabilize the wholesale market. The desire to prevent disorderly price fluctuations under market clearing was claimed as the primary justification for a "fair trade" law enabling RPM by that law's leading proponent, Louis Brandeis, who argued that "There must be reasonable restriction upon competition else we shall see competition destroyed." (McCraw, p. 102, quoting Brandeis.) Our theory captures this emphasis on demand uncertainty and inventories without requiring either risk aversion or bankruptcy costs.[13]

Subsequent to the 1975 revocation of states' authority to permit RPM, retail markdowns have become an increasingly important feature of the U.S. retail distribution landscape. Defined by the National Retail Federation as "the dollar reduction from the originally set retail prices of merchandise…,"[14] markdowns grew modestly from 6.3% in 1966 to 8.9% in 1975, and then accelerated to 12% in 1981 and reached 24.7% in 1991.[15] In some instances, markdowns have proven fatal. Atari games vanished from store shelves following a period of slack demand and very substantial price cutting. Nintendo, successor to Atari as market leader in

[12]Nutter v. Wheeler, 18 F. Cas. 497 (C.C.D. Mass. 1874) (No. 10,334).

[13]Rey and Tirole (1986) consider vertical restraints under demand uncertainty. Their model permits manufacturers to employ two-part tariffs for sales to retailers, thereby imposing a fixed fee commitment on retailers. Rey and Tirole show that RPM will only be favored when retailers are very risk averse. Unlike our model, however, output is produced after demand is realized.

[14]National Retail Federation, *Merchandising and Operating Results: Fiscal 1991, Department and Specialty Stores,* 1992 Edition (New York: Business Services, National Retail Federation, 1992), p. 169.

[15]National Retail Federation, *Merchandising and Operating Results…,* ibid, various editions. This trend does not reflect changes in gross retail margins, which have hovered around 40% throughout the same time period. Gross margins for retail departments, stated as a percentage of retail department sales, were 37.4% in 1966, peaked at 42.9% in 1981, and had declined to 38.2% in 1991.

the electronic games market, has been charged with imposing RPM on its retailers. In section 5, we show that the electronic games experience is both incompatible with alternative explanations of RPM and is consistent with the predictions of our model.

We proceed as follows. First, in section 2, we illustrate our argument with a simple example of how a manufacturer can benefit by suppressing price competition among its retailers. We demonstrate that suppression of price competition can benefit not only manufacturers and society generally, but also consumers, even though such consumers are denied the chance to purchase goods at low prices in the event of slack demand. In section 3 we demonstrate with considerable generality that the manufacturer will often prefer to suppress retailer competition. In section 4 we provide sufficient conditions under which total surplus is higher under RPM than under flexible retail prices. We also provide conditions under which both profits and consumer surplus are higher under RPM than under flexible retail prices. When alternatives to market clearing yield higher total surplus, and in particular when that surplus accrues to both the manufacturer and consumers, the alternative of unrestrictive price competition can truly be considered destructive. Section 5 offers some concluding remarks.

2 The Model, and an Example

A risk-neutral monopoly manufacturer sells to a continuum of risk-neutral retailers, represented by an index $t \in [0,1]$. Retailers, in turn, sell to consumers. We assume throughout that inventories which are left unsold at the end of the demand period have no scrap value.[16] Retailers must order and take title to these inventories prior to the demand period so that unsold inventories are, to them, a sunk cost—the manufacturer does not offer to accept returns

[16]While we specify zero scrap value to simplify the analysis, our results merely require that retailers cannot recoup the full value of their investments in inventories. Such sunk investment costs can come about because inventories are costly to hold over to the next demand period or because the goods spoil or become outmoded. If inventory costs were negligible, our assumption would imply that manufacturers refuse to accept returns of unsold merchandise for full credit. Returns policies may be prohibitive either because of retailer moral hazard or the costs of administering such systems. (Returns are employed for books and magazines, but the costs of shipping are so large that for paperbacks, only the covers are returned, illustrating both the costs and potential moral hazard problems of such schemes.) Returns policies in the presence of demand uncertainty are analyzed in Marvel and Peck (1994).

of unsold merchandise.[17] We assume that other marginal costs of distribution are constant and, without loss of generality, normalize them equal to zero.[18] To compare RPM and retail market-clearing, we are interested in the subgame perfect equilibria of the following games:

The RPM Game The manufacturer must first announce its wholesale price, p^w, and the minimum retail price at which its product may be resold, $\bar{p}$. Next, retailers simultaneously choose how much inventory to hold, prior to the resolution of demand uncertainty. Once retailer inventories are in place, demand uncertainty is resolved. If the market clearing price exceeds $\bar{p}$, then market clearing determines the price and all transactions. Otherwise, the retail price is $\bar{p}$ and retailers find themselves burdened with excess inventories. In this case, we assume that consumers distribute themselves across the retailers so that the fraction of inventory sold is the same for all firms.

The Flexible-Price Game The manufacturer initially announces a wholesale price, p^w. Retailers then choose simultaneously how much inventory to hold, prior to the resolution of demand uncertainty. With inventories in place, demand is realized. The retail price is determined according to supply and demand.

An Example with Two Demand States and Linear Demand

Before presenting a model with general assumptions on demand and its distribution across different states of the world (see Section 3), we first provide a simple two-state, linear demand example illustrating the basic economic forces behind our results. In the example, RPM is

[17] A prototypical example of the type of market to which our model applies is that for "sell-through" videos, that is, movies that are sold, rather than rented, on video cassettes. Manufacturers of such products try to maintain minimum resale price through minimum advertised pricing (MAP) practices that deny advertising rebates to dealers failing to adhere to the manufacturer's preferred price. The Disney Company is an aggressive user of MAP for its animated videos such as "Snow White," "The Fox and the Hound," and "The Return of Jafar." The practice is also common for recorded music. For such products, the window of novelty in which they sell can be short and uncertain. In addition, as shown in Theorem 1, the low marginal cost of producing copies makes imposing RPM particularly attractive to the manufacturer. See "MAPS for Hot Vids are Hard to Read; Retail Price-Cutting Battles May Erupt," *Billboard,* July 16, 1994, p. 8.

[18] Suppose retailers face a constant marginal cost of inventories, denoted by c_1, as well as a constant marginal cost of providing sales, denoted by c_2. Inventory holding costs can simply be absorbed into the manufacturer's cost of producing for inventory, and the cost of providing sales can be accommodated by reinterpreting inverse demand as the willingness to pay above c_2. More precisely (using the notation of Section 3), retailer profits, manufacturer profits and consumer surplus in the model with inverse demand $P(q, \alpha)$, production cost $C(q)$ and positive distribution costs are identical to those in the model with inverse demand $P(q, \alpha) - c_2$, production cost $C(q) - c_1 q$, and no distribution costs.

always beneficial to the manufacturer and can (but need not) improve welfare in comparison to permitting prices to clear the market. Suppose that demand is given by

$$D(p,\theta) = \begin{cases} 1-p, & \text{with probability } 1/2 \quad \text{(low demand state)} \\ \theta(1-p), & \text{with probability } 1/2 \quad \text{(high demand state)} \end{cases} \tag{1}$$

Without loss of generality, we assume $\theta > 1$. Manufacturing costs are assumed to be zero.

Denote the equilibrium price floor as p^*. The equilibrium strategies along the subgame perfect equilibrium path of the RPM game are as follows:

$$\begin{aligned} p^w &= \frac{1+\theta}{4\theta}, \\ p^* &= \frac{1}{2}, \text{ and} \\ q(t) &\quad \text{is any integrable function satisfying } q^{RPM} = \int_0^1 q(t)dt = \frac{\theta}{2}. \end{aligned} \tag{2}$$

Note that the aggregate inventory holdings, q^{RPM}, are uniquely determined, but that inventory holdings of an individual retailer, denoted by $q(t)$, are not.[19] It follows from (2) that expected consumer surplus is $(1+\theta)/16$. Manufacturer profits, denoted Π^{RPM}, are given by $\Pi^{RPM} = (1+\theta)/8$.

To see why (2) defines an equilibrium, let us calculate profits of retailer t, denoted $\pi^{RPM}(t)$, under the assumption that the retail price, p^r, equals the price floor, $\bar{p}$, in the high demand state, so we have $q^{RPM} \geq \theta(1-\bar{p})$. In the low demand state, the probability of selling a unit is equal to $(1-\bar{p})/q^{RPM}$, and in the high demand state it is $\theta(1-\bar{p})/q^{RPM}$. Consequently,

$$\pi^{RPM}(t) = \left[\frac{\bar{p}(1-\bar{p})(1+\theta)}{2q^{RPM}} - p^w\right] q(t). \tag{3}$$

Since no individual retailer can affect q^{RPM},[20] retailer profits in equation (3) are linear in $q(t)$.

[19] The indeterminacy of the function $q(t)$ is an artifact of the continuum. Suppose instead that there were a finite number of retailers, where each retailer's strategy is to choose a level of inventory demand. As in our RPM game, these inventories are then inelastically supplied to the market at any price at least as high as the price floor. It is straightforward to check that the unique equilibrium of the RPM Game is symmetric and that as the number of retailers approaches infinity, p^w and p^* approach their values given by (2), and total orders approach $\theta/2$.

[20] If any retailer were able to affect q^{RPM}, that retailer would reduce its inventory holding in an effort to make the profit margin positive. A positive profit margin, however, is inconsistent with equilibrium as retailers with

Equilibrium then requires that the retail profit margin be zero, implying,

$$\bar{p}(1-\bar{p})(1+\theta)/2 = p^w q^{RPM}. \tag{4}$$

Since the strategies of expression (2) satisfy retailer t's zero profit condition, (4), for any choice of $q(t)$, retailer t has no profitable deviation. Therefore the retailer subgame is in equilibrium. Since the right side of (4) is manufacturer profits, these profits are maximized subject to (4) whenever the retail price floor is the monopoly price, $\bar{p} = 1/2$. The manufacturer is therefore optimally choosing $\bar{p} = p^* = 1/2$ and $p^w = (1+\theta)/4\theta$, so (2) constitutes an equilibrium.[21]

To analyze the Flexible-Pricing Game, we must consider two cases. If the wholesale price is low enough, retailers will order inventories sufficient to force the retail price to zero in the low demand state (case 1). If the wholesale price is higher, inventories will be lower, which leads to positive retail prices in both demand states (case 2). Because we have a continuum of retailers, the equilibrium must have retailers earning zero expected profits.

In case 1, retailers receive revenue only in the high demand state. We have $p^h = 1 - q^{FL}/\theta$, where q^{FL} is the quantity of inventory ordered and p^h is the retail price that clears the market in the high demand state. Since retailers only earn a positive margin half the time, their zero profit condition is $p^h = 2p^w$. When choosing p^w to induce $q^{FL} \geq 1$, the manufacturer therefore earns profits given by

$$\Pi^{FL} = \frac{1}{2}\left(1 - \frac{q^{FL}}{\theta}\right) q^{FL} \quad \text{(case 1)}. \tag{5}$$

In case 2, we have $p^l = (1 - q^{FL}) > 0$ and $p^h = 1 - q^{FL}/\theta$. The retailer zero expected profit condition is therefore given by $p^w = 1 - (1+\theta)q^{FL}/(2\theta)$. When choosing p^w to induce $q^{FL} \leq 1$, manufacturer profits are

$$\Pi^{FL} = \left[1 - \frac{(1+\theta)q^{FL}}{2\theta}\right] q^{FL} \quad \text{(case 2)}. \tag{6}$$

infinitesimal inventory holdings would then have an incentive to expand. Formally, the requirement that no individual retailer be able to affect q^{RPM} is reflected in the integrability requirement on $q(t)$.

[21] When $q^{RPM} < \theta(1-\bar{p})$ holds, equation (3) must be altered, but we still must have zero retail profits. With zero production costs, the manufacturer always prefers to induce full stocking, as in (2).

Since the expression in (6) exceeds the expression in (5) for all $q^{FL} \leq 1$, and since the optimum q^{FL} in case 2 is always strictly less than one, the manufacturer optimizes by inducing the level of inventory holdings that achieves the maximum profit among expressions (5) and (6).

For our example, when $\theta > 3$, maximal profits are obtained along branch (5), so that the manufacturer abandons state 1 revenues. The overall equilibrium then involves $q^{FL} = \theta/2$, $p^w = 1/4$, $p^h = 1/2$, $p^l = 0$, and $\Pi^{FL} = \theta/8$. Expected consumer surplus equals $1/4 + \theta/16$ and welfare equals $1/4 + 3\theta/16$.

When $\theta < 3$, case 2 is observed. The manufacturer then maximizes by choosing $q^{FL} = \theta/(1+\theta)$, yielding $p^w = 1/2$, $p^l = 1/(1+\theta)$, $p^h = \theta/(1+\theta)$, and $\Pi^{FL} = \dfrac{\theta}{2(1+\theta)}$. Equilibrium expected surplus and welfare respectively equal $\dfrac{\theta}{4(1+\theta)}$ and $\dfrac{3\theta}{4(1+\theta)}$. Finally, for $\theta = 3$, the manufacturer is indifferent between charging $p^w = 1/4$ (inducing $q^{FL} = 3/2$) and $p^w = 1/2$ (inducing $q^{FL} = 3/4$).

Comparing equilibrium profits between the RPM game and the flexible pricing game, we see that regardless of which case applies, *the manufacturer always prefers RPM to market clearing.* Indeed, setting a price floor of $1/2$ in the RPM Game shores up the retail price in the low demand state, so retailer revenues are greater than revenues in the Flexible-Price Game, holding inventories constant. This is because the lowest retail price in the Flexible-Price Game is strictly less than the monopoly price. In case 2, the RPM wholesale price remains equal to the Flexible-Price Game wholesale price of $1/2$, and retailers compete by increasing inventory holdings, yielding the manufacturer higher profits. In case 1, retailer inventories are already maximal in the Flexible-Price Game, but the manufacturer can extract the extra surplus by raising the wholesale price. In either case, the manufacturer's profits increase. In section 3, we provide general conditions under which manufacturer profits and inventories are strictly higher under RPM.

From the perspective of consumer surplus and total surplus, the preferred institution depends on which case applies, which in turn depends on the nature of the demand uncertainty. In case 1, consumer and total surplus are higher under flexible retail prices, since the manufacturer "gives up" on the low demand state and allows the retail price to fall to zero. The

manufacturer sets a wholesale price sufficient to induce inventory holdings that maximize receipts in the high demand state. This results in the same inventories as held under RPM. However, in the low demand state, the inventories are given away in the Flexible-Pricing Game, instead of being discarded, as in the RPM Game.

In case 2, consumer and total surplus are higher under RPM. The advantage to consumers of paying a low market-clearing price when demand is low is outweighed by the disadvantage of paying a high market-clearing price when demand is high. When demand fluctuations are not too great and when the low demand state is sufficiently likely (so that case 2 applies), retail market clearing is destructive in the sense that both consumers and the manufacturer would prefer RPM. Retailers must be compensated for their propensity to compete the price down when demand is low, giving rise to higher wholesale prices and lower quantity ordered than under RPM.

3 A General Demand Specification

In this section, we substantially weaken our assumptions on the demand process and the manufacturer's production costs in order to provide a more general theory of destructive retail competition. Because a price floor of zero is equivalent to unfettered market clearing, the manufacturer obviously cannot be hurt by the ability to set a minimum retail price for its product. Theorem 1 provides a necessary and sufficient condition for the manufacturer to strictly prefer to impose a price floor, that is, to enforce resale price maintenance. Theorem 2 shows that equilibrium inventories are always at least as high under RPM as under market clearing, and provides sufficient conditions for them to be strictly higher.

Let $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ denote the state of demand. We assume that higher α's are associated with higher levels of demand. For any Lebesgue-measurable subset $A \in [\underline{\alpha}, \bar{\alpha}]$, denote the probability-measure of A as $\mu(A)$. We do not place any restrictions on the distribution of α, represented by the distribution function, F. Thus, discrete, absolutely continuous, and mixed

distributions are allowed. The support of F is denoted by

$$\text{supp } F = \{x \in \Re : F(x+\epsilon) - F(x-\epsilon) > 0 \text{ for every } \epsilon > 0\}$$

Without loss of generality, we let $\underline{\alpha} = \inf \text{ supp } F$ and $\bar{\alpha} = \sup \text{ supp } F$, and assume $\underline{\alpha} < \bar{\alpha}$.

Let $D(p,\alpha)$ denote the final demand for the manufacturer's product in state α, and let $P(q,\alpha)$ denote the corresponding inverse demand. To make the problem interesting, we assume that for each α there exists q such that $P(q,\alpha) > 0$.[22] We assume that these functions are continuous everywhere, and twice continuously differentiable on the interior of the domain where they are strictly positive. The revenue functions, $R(q,\alpha)$ and $\bar{R}(p,\alpha)$, are defined by

$$\begin{aligned} R(q,\alpha) &= qP(q,\alpha) \text{ and} \\ \bar{R}(p,\alpha) &= pD(p,\alpha). \end{aligned}$$

The following additional notation will be useful. The choke quantity, or quantity for which the market-clearing price is zero, denoted as $\hat{q}(\alpha)$, is defined as

$$\hat{q}(\alpha) = \sup\{q : P(q,\alpha) > 0\},$$

and is assumed to be finite.[23]

Assumption 1 *For every α and for every $q \in [0, q(\alpha))$, we have $P_\alpha(q,\alpha) > 0$, $P_q(q,\alpha) < 0$, $2P_q(q,\alpha) + P_{qq}(q,\alpha) < 0$, and $P_{q\alpha}(q,\alpha) > 0$.*

Notice that Assumption 1 implies $R_{q\alpha}(q,\alpha) > 0$ and that $R_{qq}(q,\alpha) < 0$ whenever $q < \hat{q}(\alpha)$. Let the manufacturer's cost function be denoted as $C(q)$, and assume

Assumption 2 *$C(q) : \Re_+ \to \Re_+$ is a twice continuously differentiable function with $C(0) = 0$,*

[22] All of our results also hold in the limiting case where $P(q,\underline{\alpha}) = 0$ for all q, with trivial modifications to the proofs.

[23] All of our results go through if $\hat{q}(\alpha)$ is infinite, provided revenue is maximized at a finite quantity, yielding finite consumer surplus.

$C'(q) \geq 0$ *and* $C''(q) \geq 0$.

Consequently, the profit-maximizing quantity, $q^c(\alpha)$, the revenue-maximizing quantity, $q^m(\alpha)$, and the revenue-maximizing price, $p^m(\alpha)$, as defined below, are well-defined functions of α:

$$\begin{aligned} q^c(\alpha) &= \arg\max_q \{R(q,\alpha) - C(q)\} \\ q^m(\alpha) &= \arg\max_q R(q,\alpha) \\ p^m(\alpha) &= \arg\max_p \bar{R}(p,\alpha) \end{aligned}$$

Note that $R_{q\alpha} > 0$ implies that whenever they are nonzero, the functions $q^c(\alpha)$ and $q^m(\alpha)$ are strictly increasing in α. We also assume that

Assumption 3 $p^m(\alpha)$ *is nondecreasing in* α.

A sufficient condition for Assumption 3 to hold is that $\bar{R}_{p\alpha}(p^m(\alpha),\alpha) \geq 0$ for all α. We will maintain Assumptions 1-3 throughout the remainder of the paper.

3.1 The Flexible-Pricing Game

Let the aggregate quantity of inventories ordered by retailers be denoted by q. Since in the Flexible-Pricing Game, the price adjusts to clear all inventories from the market, aggregate retailer revenues in state α will be equal to $R(q,\alpha)$. Consequently, expected retailer revenue is given by the Stieltjes integral $R(q,\underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} R(q,z)dF(z)$. Since retailers compete by expanding their inventories up to the point where all profits are dissipated, the above expression also equals $p^w q$, the expected revenue received by the manufacturer from the retailers. Thus, manufacturer profit, as a function of the inventories retailers are induced to hold, $\Pi^{FL}(q)$, is given by

$$\Pi^{FL}(q) = R(q,\underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} R(q,z)dF(z) - C(q). \tag{7}$$

The equilibrium profit received by the manufacturer, Π^{FL}, is given by

$$\Pi^{FL} = \max_{q\geq 0} \Pi^{FL}(q). \tag{8}$$

It is important to note that the profit function in (7) need not be concave,[24] so that the solution to (8) is not necessarily unique (see the example in section 2). We let $Q^{FL} = \arg\max_{q \geq 0} \Pi^{FL}(q)$ denote the set of equilibrium inventory levels, and denote a generic element of Q^{FL} by q^{FL}.

3.2 The RPM Game

Suppose that aggregate inventories are q and the price floor in the RPM Game is $\bar{p}$. Then there will be a cutoff demand state, α, where the price floor is binding for states below α and the market clears at prices above $\bar{p}$ for states above α. Formally, this cutoff is defined by

$$\alpha = \inf\{z \in [\underline{\alpha}, \bar{\alpha}] : P(q, z) > \bar{p}\}.^{25}$$

For nontrivial price floors, we have $P(q, \alpha) = \bar{p}$ at the cutoff demand state, α. Thus, we can express the aggregate expected revenue of retailers as

$$\bar{R}(P(q, \alpha), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} \bar{R}[P(q, \alpha), z]dF(z) + \int_{\alpha}^{\bar{\alpha}} R(q, z)dF(z). \tag{9}$$

Because all retailers earn zero expected profits in equilibrium, expression (9) also equals $p^w q$, the revenues received by the manufacturer. Thus, we can express manufacturer profit, $\Pi(q, \alpha)$, as a function of induced inventories q and cutoff demand state α:

$$\Pi(q, \alpha) = \bar{R}(P(q, \alpha), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha} \bar{R}[P(q, \alpha), z]dF(z) + \int_{\alpha}^{\bar{\alpha}} R(q, z)dF(z) - C(q). \tag{10}$$

The manufacturer's equilibrium profit level, Π^{RPM}, is determined by

$$\Pi^{RPM} = \max_{q, \alpha} \Pi(q, \alpha). \tag{11}$$

[24] Assumption 1 guarantees that for each α the function $R(q, \alpha)$ is concave on $[0, \hat{q}(\alpha)]$. However, $R(q, \alpha)$ is not concave on all of $\Re_+$. Consequently, while $\Pi^{FL}(q)$ is concave on $[0, \hat{q}(\alpha)]$, it is not concave on the entire domain of potential maximizers.

[25] When $P(q, z) \leq \bar{p}$ for all $z \in [\underline{\alpha}, \bar{\alpha}]$, so that the inf expression is not well defined, let $\alpha = \bar{\alpha}$. This case cannot occur in equilibrium, unless the manufacturer chooses not to serve the market.

Any solution to (11) consists of a pair (q^{RPM}, α^*), where the corresponding equilibrium price floor is determined via $p^* = P(q^{RPM}, \alpha^*)$.

Our first theorem shows that the manufacturer will benefit from imposing a price floor if and only if costs are low enough for $q^{FL} > q^m(\underline{\alpha})$ to hold. The condition has intuitive appeal, for if $q^{FL} \leq q^m(\underline{\alpha})$, then imposing a floor cannot increase revenue in any state of the world, given inventories q^{FL}.

Theorem 1 *The manufacturer strictly prefers minimum RPM to flexible pricing if and only if there exists $q^{FL} \in Q^{FL}$ such that $q^{FL} > q^m(\underline{\alpha})$.*

Proof (Sufficiency): We will show that by keeping aggregate inventories at the level $q = q^{FL}$ and setting a price floor $\bar{p} = p^m(\underline{\alpha})$, the manufacturer can improve his profits above the level $\Pi^{FL} = \Pi(q^{FL}, \underline{\alpha})$. From (7), we have

$$\begin{aligned} \Pi(q^{FL}, \alpha) - \Pi(q^{FL}, \underline{\alpha}) &= [\bar{R}(P(q^{FL}, \alpha), \underline{\alpha}) - R(q^{FL}, \underline{\alpha})]F(\underline{\alpha}) \\ &\quad + \int_{\underline{\alpha}}^{\alpha} [\bar{R}(P(q^{FL}, \alpha), z) - R(q^{FL}, z)]dF(z). \end{aligned} \tag{12}$$

Let α_1 be determined by $P(q^{FL}, \alpha_1) = p^m(\underline{\alpha})$. We know that $q^{FL} > q^m(\underline{\alpha})$ holds, so $\alpha_1 > \underline{\alpha}$. Also, by Assumptions 1 and 3, $p^m(\alpha)$ is continuous and nondecreasing. Thus, for all $z \in A = [\underline{\alpha}, \alpha_1)$, the price floor $\bar{p} = p^m(\underline{\alpha})$ is binding and is below $p^m(z)$. Therefore, we have $\bar{R}(P(q^{FL}, \alpha_1), z) > R(q^{FL}, z)$ for all $z \in A$. Since $\mu(A) > 0$, it follows that (12) is strictly positive for $\alpha = \alpha_1$.

(Necessity): Suppose that the condition is false, so that for every $q^{FL} \in Q^{FL}$ we have $q^{FL} \leq q^m(\underline{\alpha}) < \hat{q}(\underline{\alpha})$. Since q^{FL} maximizes Π^{FL}, and since Π^{FL} is strictly concave on $[0, \hat{q}(\underline{\alpha})]$, we have

$$\frac{d\Pi^{FL}}{dq}(q^m(\underline{\alpha})) = \int_{\underline{\alpha}}^{\bar{\alpha}} R_q(q^m(\underline{\alpha}), z)dF(z) - C'(q^m(\underline{\alpha})) \leq 0. \tag{13}$$

Now $R_q(q^m(\underline{\alpha}), \underline{\alpha}) = 0$, so $R_{q\alpha} > 0$ implies $R_q(q^m(\underline{\alpha}), z) > 0$ for all $z \neq \underline{\alpha}$. Consequently, (13) implies

$$\int_{\alpha^*}^{\bar{\alpha}} R_q(q^m(\underline{\alpha}), z)dF(z) - C'(q^m(\underline{\alpha})) \leq 0. \tag{14}$$

Now let (q^{RPM}, α^*) be any solution to (11). Since the function $\int_{\alpha^*}^{\bar{\alpha}} R(q,z)dF(z) - C(q)$ is strictly concave on $[0, \hat{q}(\alpha^*)]$, and since $q^{RPM} < \hat{q}(\alpha^*)$ must satisfy

$$\int_{\alpha^*}^{\bar{\alpha}} R_q(q^{RPM}, z)dF(z) - C'(q^{RPM}) = 0, \tag{15}$$

we can conclude from (14) and (15) that $q^{RPM} \leq q^m(\underline{\alpha})$.[26] But then imposing a price floor does not improve the manufacturer's profits in any state, and so $\Pi^{RPM} = \Pi^{FL}$, contradicting the hypothesis that the manufacturer strictly prefers RPM. ■

We now return to the argument that unbridled retail competition can lead to inadequate inventories held by retailers. Whenever the condition of Theorem 1 holds, so that $P(q^{FL}, \underline{\alpha}) < p^*(\underline{\alpha})$, the manufacturer can increase retailer profitability by imposing a minimum retail price. Since retailers compete by increasing inventories, this will result in an outward shift of the demand faced by the manufacturer. In equilibrium, the manufacturer may very well respond by increasing his wholesale price, thereby possibly decreasing the level of equilibrium inventories. However, Theorem 2 below shows that our maintained assumptions are sufficient to guarantee that $q^{RPM} \geq q^{FL}$. If, in addition, RPM is strictly preferred by the manufacturer, and if F has full support, then we have $q^{RPM} > q^{FL}$.[27] When RPM is not strictly preferred, we must have $q^{FL} = q^{RPM}$.[28]

Theorem 2 *Let q^{FL} be any solution to (8) and let (q^{RPM}, α^*) be any solution to (11). Then $q^{FL} \leq q^{RPM}$. Furthermore, if the manufacturer strictly prefers RPM, and either F has full support or $q^c(\bar{\alpha}) \leq \hat{q}(\underline{\alpha})$, then $q^{FL} < q^{RPM}$.*

While the proof of Theorem 2 is quite complicated, and therefore relegated to the appendix, the underlying intuition is straightforward. When selecting the optimal inventory level, under

[26] When F is not absolutely continuous, $\Pi(q, \alpha)$ need not be differentiable. Nevertheless, we can show that left and right hand partial derivatives exist everywhere, and that at an optimum, the two must be equal. The derivation of these results is rather intricate, and for the sake of brevity, we have omitted the details.

[27] If F does not have full support, then it is possible that $q^{RPM} = q^{FL}$ even if the manufacturer strictly prefers RPM, as in the example in section 2 with $\theta > 3$.

[28] If the manufacturer does not strictly prefer RPM, then by Theorem 1 every solution to (8) must satisfy $q^{FL} \leq q^m(\underline{\alpha})$. Since Π^{FL} is strictly concave over the interval $[0, q^m(\underline{\alpha})]$, q^{FL} is unique. The proof of necessity part of Theorem 1 also shows that any solution to (11) must satisfy $q^{RPM} \in Q^{FL}$; we conclude that $q^{RPM} = q^{FL}$.

RPM the monopolist has only to consider the effect upon expected revenues in states where the price floor is not binding. Under flexible pricing, the manufacturer must also take into account the negative impact upon revenues in lower demand states, and hence selects a lower level of inventories. To understand why the effect is negative, note that the price floor tries to reach a compromise among the monopoly prices in the demand states where it binds, and hence cannot exceed the monopoly price at α^*. Consequently q^{RPM} (the demand at α^*) must be larger than the monopoly quantity at α^*. Under flexible pricing, marginally increasing inventories above q^{RPM} therefore lowers revenues in states below α^*. The reason the proof is complex is that neither q^{RPM} nor q^{FL} need be unique, and that when F does not have full support (or $q^c(\bar{\alpha}) > \hat{q}(\underline{\alpha})$) expected revenues in states below α^* may be zero.

4 Welfare under RPM and Flexible Pricing

The theorems in this section provide conditions under which allowing the manufacturer to set a price floor actually improves economic welfare, so that unfettered retail competition is destructive. Let (q^{RPM}, α^*) denote any solution to the RPM problem, let p^* be the corresponding price floor, and let q^{FL} denote any solution to the flexible pricing problem. We will now compare the welfare under these two solutions.[29]

Let $q^*(\alpha) = \min\{q^{RPM}, D(p^*, \alpha)\}$ represent the quantity actually sold under RPM in state α. Let $S(q, \alpha) = \int_0^q P(z, \alpha)dz$ represent the total surplus if the state is α and the quantity sold is q. Then welfare under the RPM solution, W^{RPM}, is given by

$$W^{RPM} = S(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} S(q^*(\alpha), \alpha)dF(\alpha) - C(q^{RPM}),$$

and welfare under the flexible-pricing solution, W^{FL}, is given by

$$W^{FL} = S(q^{FL}, \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} S(q^{FL}, \alpha)dF(\alpha) - C(q^{FL}).$$

[29] In order to ensure that the welfare comparisons are unambiguous, it must either be shown that the conditions in Theorems 3 and 4 hold for all possible solutions to (8) and (11) (as is done in Theorems 5 and 6), or conditions must be imposed to make the solutions unique (as in Theorem 7).

(Figure 1 about here)

Figure 1: The supply curves under RPM and flexible retail pricing

Obviously, consumer surplus under RPM and flexible pricing are defined by $CS^{RPM} = W^{RPM} - \Pi^{RPM}$ and $CS^{FL} = W^{FL} - \Pi^{FL}$.

Theorem 1 shows that as long as there is any demand uncertainty and costs are not too high, the manufacturer will always strictly prefer RPM to flexible pricing. Furthermore, Theorem 2 shows that when F has full support, consumers will be better off when demand is high, since $q^{RPM} > q^{FL}$. When demand is low, however, consumers will prefer the low prices produced under market clearing. Figure 1 illustrates the tradeoff. The net effect on consumer surplus and welfare is, in general, ambiguous, as was demonstrated by the example in section 2. Theorems 3 and 4, below, provide simple conditions for welfare to improve under RPM.

Theorem 3 *Suppose the expected quantity sold under RPM exceeds q^{FL}, so*

$$E(q^*) = q^*(\underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} q^*(\alpha)dF(\alpha) > q^{FL},$$

and suppose $C(q) = 0$. Then $W^{RPM} > W^{FL}$.

Proof: Note that $S(q, \alpha)$ is concave in q for every α. Consequently, for every α,

$$S(q^*(\alpha), \alpha) \geq S(q^{FL}, \alpha) + \frac{\partial S}{\partial q}(q^*(\alpha), \alpha)[q^*(\alpha) - q^{FL}]. \tag{16}$$

Now $\dfrac{\partial S}{\partial q}(q^*(\alpha), \alpha) = P(q^*(\alpha), \alpha)$, and so:

$$\begin{aligned} W^{RPM} - W^{FL} &\geq P(q^*(\underline{\alpha}), \underline{\alpha})[q^*(\underline{\alpha}) - q^{FL}]F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)[q^*(\alpha) - q^{FL}]dF(\alpha) \\ &= p^*[q^*(\underline{\alpha}) - q^{FL}]F(\underline{\alpha}) + p^* \int_{\underline{\alpha}}^{\alpha^*} [q^*(\alpha) - q^{FL}]dF(\alpha) \\ &\quad + \int_{\alpha^*}^{\bar{\alpha}} P(q^{RPM}, \alpha)[q^{RPM} - q^{FL}]dF(\alpha) \end{aligned}$$

$$
\begin{aligned}
&\geq p^*[q^*(\underline{\alpha}) - q^{FL}]F(\underline{\alpha}) + p^* \int_{\underline{\alpha}}^{\bar{\alpha}} [q^*(\alpha) - q^{FL}]dF(\alpha) \\
&= p^*[E(q^*) - q^{FL}] > 0
\end{aligned}
$$

The second to last inequality follows from the fact that $q^{RPM} \geq q^*(\alpha)$ for all α, the fact that $q^{RPM} \geq E[q^*] > q^{FL}$, and the fact that $P(q^{RPM}, \alpha) \geq p^*$ for all $\alpha \geq \alpha^*$. The last inequality follows from the fact that $q^{RPM} > q^{FL}$, so that $p^* > 0$. ■

Theorem 3 is reminiscent of a standard result, namely that if the total quantity sold under a single, uniform price is the same or higher than that under price discrimination, then price discrimination reduces total welfare.[30] Think of each state of nature as a separate market. RPM would correspond to the case of a uniform price, and flexible pricing to price discrimination, since prices vary across states. Note also that the applicability of Theorem 3 is not limited to the case where costs are sufficiently low that they can be safely ignored. Indeed it is straightforward to show that when costs are positive, the hypothesis in the Theorem can be replaced by $[E(q^*) - q^{FL}]p^* > C(q^{RPM}) - C(q^{FL})$.

Our welfare result offers the antitrust authorities guidance about which uses of RPM are most likely to harm consumers. Since RPM has long been illegal in the United States, any manufacturer employing the practice must strictly prefer it to the alternative of permitting the retail market to clear. Hence, by Theorem 2 we know that the manufacturer will ship larger inventories with RPM. Theorem 3 tells us that welfare is improved unless average inventories remaining unsold at the end of the demand period exceed the additional retailer orders for inventory that RPM supports. Excess inventories at the end of the demand period therefore indicate a source for antitrust concern. Note also that the standard test for the efficiency of vertical restraints, namely whether use of the restraint increases output, is flawed in this case. Even though output is always higher under RPM, welfare need not increase unless average sales rise as well.[31]

[30] See Tirole (1988), pp. 137–142.

[31] Since the increased sales in high demand states contribute more to expected welfare than the decreased sales in low demand states, welfare under RPM can be higher even if expected sales are lower. However, if $q^{FL} > q^m(\underline{\alpha})$ and the decline is too great in the sense that $E[(q^*(\alpha) - q^{FL})P(q^{FL}, \alpha)] \leq 0$, then $W^{RPM} < W^{FL}$. This assertion is

Theorem 3 is concerned with whether welfare on balance improves as a result of the imposition of RPM. Given that considerable opposition to RPM has been mounted by consumer groups, it is important to determine the effects of RPM use on consumers directly. Theorem 4 shows, surprisingly, that consumers can, in fact, benefit on balance from RPM, even though it denies them the ability to purchase at low market-clearing prices in the event of slack demand. This theorem has the additional advantage over Theorem 3 of not requiring any cost information.

Theorem 4 *Suppose that the average price under minimum RPM is strictly lower than the average price under flexible pricing. Then consumer surplus and manufacturer profits under RPM strictly exceed consumer surplus and manufacturer profits under flexible pricing. Hence RPM results in a Pareto improvement.*

Proof:

$$\begin{aligned} W^{RPM} - W^{FL} &= [S(q^*(\underline{\alpha}), \underline{\alpha}) - S(q^{FL}, \underline{\alpha})]F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} [S(q^*(\alpha), \alpha) - S(q^{FL}, \alpha)]dF(\alpha) \\ &\quad -C(q^{RPM}) + C(q^{FL}) \\ &\geq P(q^*(\underline{\alpha}), \underline{\alpha})[q^*(\underline{\alpha}) - q^{FL}]F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)[q^*(\alpha) - q^{FL}]dF(\alpha) \\ &\quad -C(q^{RPM}) + C(q^{FL}) \\ &= \Pi^{RPM} - q^{FL}\left[P(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)dF(\alpha)\right] + C(q^{FL}) \end{aligned}$$

The above inequality follows from the concavity of $S(q, \alpha)$. (See the proof of Theorem 3.)

proved as follows. Analogous to (16), we have:

$$S(q^{FL}, \alpha) \geq S(q^*(\alpha), \alpha) + P(q^{FL}, \alpha)(q^{FL} - q^*(\alpha)).$$

Furthermore, the inequality $q^{FL} > q^m(\underline{\alpha})$ implies $q^*(\underline{\alpha}) < q^{FL}$, so the above inequality is strict in a neighborhood of $\underline{\alpha}$. Consequently,

$$\begin{aligned} W^{FL} - W^{RPM} &= E[S(q^{FL}, \alpha) - S(q^*(\alpha), \alpha)] \\ &> E[P(q^{FL}, \alpha)(q^{FL} - q^*(\alpha))] \geq 0. \end{aligned}$$

Rearranging, we have

$$CS^{RPM} \geq W^{FL} - q^{FL}P(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) - q^{FL}\int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)dF(\alpha) + C(q^{FL}).$$

Therefore,

$$\begin{aligned} CS^{RPM} - CS^{FL} &\geq \Pi^{FL} + C(q^{FL}) - q^{FL}P(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) - q^{FL}\int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)dF(\alpha) \\ &= q^{FL}\left[P(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} P(q^{FL}, \alpha)dF(\alpha)\right. \\ &\quad \left. -P(q^*(\underline{\alpha}), \underline{\alpha})F(\underline{\alpha}) - \int_{\underline{\alpha}}^{\bar{\alpha}} P(q^*(\alpha), \alpha)dF(\alpha)\right] > 0. \end{aligned}$$

Thus, $CS^{RPM} > CS^{FL}$. To show that $\Pi^{RPM} > \Pi^{FL}$, observe that if $\Pi^{RPM} = \Pi^{FL}$, then by Theorem 1, we must have $q^{FL} = q^{RPM}$, so that (contrary to the hypothesis of the theorem) the average price under both solutions would have to coincide. ■

Remark 1 *The conclusions of Theorems 3 and 4 continue to hold if the strict inequalities in their hypotheses are replaced by weak inequalities, provided* $P(q^{FL}, \underline{\alpha}) > 0$.[32]

Theorem 4 can be used to evaluate the impact of forcing firms to desist from imposing RPM. If the average price rose as a result, both manufacturer and consumer welfare would be reduced. The converse of Theorem 4 is not true, however, for RPM could benefit consumers even if it resulted in a higher average price than under flexible pricing.

It might seem that RPM would typically increase average prices substantially. If prices increase too much in the sense that $E[(P(q^*(\alpha), \alpha) - P(q^{FL}, \alpha))q^*(\alpha)] > 0$, then we necessarily have $CS^{RPM} < CS^{FL}$. Indeed

$$\begin{aligned} CS^{FL} - CS^{RPM} &= W^{FL} - \Pi^{FL} - W^{RPM} + \Pi^{RPM} \\ &= E[S(q^{FL}, \alpha) - R(q^{FL}, \alpha) - S(q^*(\alpha), \alpha) + R(q^*(\alpha), \alpha)] \end{aligned}$$

[32] Indeed, if $q^*(\underline{\alpha}) < q^{FL}$, or if $\hat{q}(\underline{\alpha}) \geq q^*(\underline{\alpha}) > q^{FL}$, then since $S(q, \alpha)$ is strictly concave on $[0, \hat{q}(\alpha)]$, the weak inequality in (16) can be replaced with a strict inequality. If $q^*(\alpha) > \hat{q}(\alpha)$, then $\frac{\partial S(q^*(\alpha), \alpha)}{\partial q} = 0$, but strict inequality holds nevertheless.

$$\begin{aligned} &\geq E[P(q^{FL},\alpha)(q^{FL}-q^*(\alpha))-R(q^{FL},\alpha)+R(q^*(\alpha),\alpha)] \\ &= E[(P(q^*(\alpha),\alpha)-P(q^{FL},\alpha))q^*(\alpha)]>0. \end{aligned}$$

Theorems 5 and 6 provide demand conditions under which RPM does not raise average prices. These Theorems apply Theorem 4 and Remark 1 to show that for two extreme cases of demand uncertainty, additive and multiplicative, RPM raises consumer welfare. Both examples assume constant marginal costs and a condition ensuring that the solution to the flexible pricing problem is unique, and that the manufacturer strictly prefers RPM.

Theorem 5 *If $D(p,\alpha)=\alpha-p$, and $C(q)=cq$, then provided $c<E(\alpha)-\underline{\alpha}$ and $\bar{\alpha}\leq 2\underline{\alpha}$, any solution to (11) has $W^{RPM}>W^{FL}$ and $CS^{RPM}>CS^{FL}$.*

Proof: As can be seen from the calculations below, the condition $c<E(\alpha)-\underline{\alpha}$ ensures that $q^{FL}>q^m(\underline{\alpha})=\underline{\alpha}/2$. The condition $q^c(\bar{\alpha})\leq q^m(\bar{\alpha})=\bar{\alpha}/2\leq\underline{\alpha}=\hat{q}(\underline{\alpha})$ has two implications. First, Theorems 1 and 2 imply that $q^{RPM}>q^{FL}$. Second, since $q^{FL}\leq q^c(\bar{\alpha})$ the function $\Pi^{FL}(q)$ is strictly concave over its domain. We will now show that $E[P(q^{FL},\alpha)]=E[P(q^*(\alpha),\alpha)]$ (See Remark 1). The first-order condition for q^{FL} is

$$R_q(q^{FL},\underline{\alpha})F(\underline{\alpha})+\int_{\underline{\alpha}}^{\bar{\alpha}}R_q(q^{FL},\alpha)dF(\alpha)-c=0,$$

yielding $(\underline{\alpha}-2q^{FL})F(\underline{\alpha})+\int_{\underline{\alpha}}^{\bar{\alpha}}(\alpha-2q^{FL})dF(\alpha)-c=0$, or $q^{FL}=\frac{a-c}{2}$, where $a=E[\alpha]$. Since $P(q^{FL},\alpha)=\alpha-q^{FL}$, we obtain

$$E[P(q^{FL},\alpha)]=a-q^{FL}=\frac{a+c}{2}.$$

Now for the RPM case, observe that any solution to (11) must satisfy $\alpha\in(\underline{\alpha},\bar{\alpha})$.[33] Since $0<q^{RPM}<\hat{q}(\alpha^*)$, we have $P_q(q^{RPM},\alpha^*)<0$ and $P_\alpha(q^{RPM},\alpha^*)>0$. Equations (19) and (20)

[33] Since the manufacturer strictly prefers RPM, it must be that $\alpha^*>\underline{\alpha}$. Furthermore, since $q^{RPM}\leq q^c(\bar{\alpha})\leq q^m(\bar{\alpha})$, if $\alpha^*=\bar{\alpha}$ we would have $p^*=P(q^{RPM},\bar{\alpha})\geq p^m(\bar{\alpha})$. But then since $p^m(z)$ is strictly increasing in z, by lowering the price floor below $p^m(\bar{\alpha})$, the manufacturer can increase his expected revenues in states below $\bar{\alpha}$ without affecting his receipts in state $\bar{\alpha}$, a contradiction.

in the Appendix then imply that the following first order conditions must be satisfied:

$$(\underline{\alpha} - 2p^*)F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha^*} (\alpha - 2p^*)dF(\alpha) = 0$$

and

$$\int_{\alpha^*}^{\bar{\alpha}} (\alpha - 2q^{RPM})dF(\alpha) = c.$$

Since $P(q^*(\alpha), \alpha) = p^*$ for $\alpha \leq \alpha^*$ and $P(q^*(\alpha), \alpha) = P(q^{RPM}, \alpha)$ for $\alpha \geq \alpha^*$, any solution to (11) must satisfy:

$$\begin{aligned} 2E[P(q^*(\alpha), \alpha)] &= 2p^*F(\alpha^*) + 2\int_{\alpha^*}^{\bar{\alpha}} (\alpha - q^{RPM})dF(\alpha) \\ &= \underline{\alpha}F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha^*} \alpha dF(\alpha) + \int_{\alpha^*}^{\bar{\alpha}} \alpha dF(\alpha) + \int_{\alpha^*}^{\bar{\alpha}} (\alpha - 2q^{RPM})dF(\alpha) \\ &= a + c, \end{aligned}$$

where the second and third inequality follow from the first-order condition for RPM. We conclude that $E[P(q^*(\alpha), \alpha)] = E[P(q^{FL}, \alpha)]$. Note that $P(q^{FL}, \underline{\alpha}) > 0$, for $q^{FL} = \hat{q}(\underline{\alpha}) = q^m(\underline{\alpha})$ would imply $\lim_{q \uparrow q^{FL}} d\Pi^{FL}/dq < 0$. Hence by Remark 1, $W^{RPM} > W^{FL}$ and $CS^{RPM} > CS^{FL}$.

■

Theorem 6 *Suppose $D(p, \alpha) = \alpha(1 - p)^{\frac{1}{\beta}}$ for some $\beta > 0$ and $C(q) = cq$. Suppose also that $c < 1 - \int_{\underline{\alpha}}^{\bar{\alpha}} \left(\frac{\underline{\alpha}}{\alpha}\right)^{\beta} dF(\alpha)$ and $\bar{\alpha} \leq (1 + \beta)^{\frac{1}{\beta}}\underline{\alpha}$. Then any solution to (11) has $W^{RPM} > W^{FL}$ and $CS^{RPM} > CS^{FL}$.*

Proof: As in the proof of Theorem 5, the condition $c < 1 - \int_{\underline{\alpha}}^{\bar{\alpha}} \left(\frac{\underline{\alpha}}{\alpha}\right)^{\beta} dF(\alpha)$ ensures that $q^{RPM} > q^{FL}$. Similarly, $\bar{\alpha} \leq (1 + \beta)^{\frac{1}{\beta}}\underline{\alpha}$ guarantees that the solution to (8) is unique, and that any solution to (11) has $q^{RPM} > q^{FL}$. Now $P(q, \alpha) = 1 - \left(\frac{q}{\alpha}\right)^{\beta}$, so that $R_q(q, \alpha) = 1 - (1+\beta)\left(\frac{q}{\alpha}\right)^{\beta}$. Consequently, q^{FL} satisfies

$$R_q(q^{FL}, \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} R_q(q^{FL}, \alpha)dF(\alpha) = 1 - (1 + \beta)(q^{FL})^{\beta}[\underline{\alpha}^{-\beta}F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} \alpha^{-\beta}dF(\alpha)] = c.$$

Observe now that

$$E[P(q^{FL},\alpha)] = 1-(q^{FL})^{\beta}[\underline{\alpha}^{-\beta}F(\underline{\alpha})+\int_{\underline{\alpha}}^{\bar{\alpha}}\alpha^{-\beta}dF(\alpha)] = 1-\frac{1-c}{1+\beta} = \frac{\beta+c}{1+\beta}.$$

Since at every price all demand curves have the same elasticity, the optimum minimum RPM price must be equal to the monopoly price, that is, $p^* = p^m = \frac{\beta}{1+\beta}$. Now Equation (21) in the Appendix shows that any solution to (11) must satisfy

$$\int_{\alpha^*}^{\bar{\alpha}} R_q(q^{RPM},\alpha)dF(\alpha) = [1-F(\alpha^*)]-(1+\beta)(q^{RPM})^{\beta}\int_{\alpha^*}^{\bar{\alpha}}\alpha^{-\beta}dF(\alpha) = c.$$

Consequently, since $p^m = \beta/(1+\beta)$, we have:

$$\begin{aligned}
E[P(q^*(\alpha),\alpha)] &= p^m F(\alpha^*)+\int_{\alpha^*}^{\bar{\alpha}} P(q^{RPM},\alpha)dF(\alpha)\\
&= p^m F(\alpha^*)+(1-F(\alpha^*))-(q^{RPM})^{\beta}\int_{\alpha^*}^{\bar{\alpha}}\alpha^{-\beta}dF(\alpha)\\
&= p^m F(\alpha^*)+(1-F(\alpha^*))-\left[\frac{1-F(\alpha^*)-c}{1+\beta}\right]\\
&= \frac{\beta+c}{1+\beta}.
\end{aligned}$$

As in the proof of Theorem 5, we conclude that $E[P(q^*(\alpha),\alpha)] = E[P(q^{FL},\alpha)] = \frac{\beta+c}{1+\beta}$, so that $W^{RPM} > W^{FL}$ and $CS^{RPM} > CS^{FL}$. ■

Without a functional form for demand, the change in the average price is harder to sign, but we may still apply Theorem 3 to to show that total surplus is lower under market clearing than under minimum RPM. Theorem 7 provides one set of sufficient conditions. We assume that demand uncertainty is multiplicative, so $D(p,\alpha)$ can be expressed as $\alpha D(p)$, $P(q,\alpha)$ can be expressed as $\mathcal{P}\left(\frac{q}{\alpha}\right)$, $q^m(\alpha)$ can be expressed as αq^m, and $\hat{q}(\alpha)$ can be expressed as $\alpha\hat{q}$. Also let revenue, $R(q,\alpha)$, be denoted as $\alpha\mathcal{R}\left(\frac{q}{\alpha}\right)$.[34]

Theorem 7 *Suppose $D(p,\alpha) = \alpha D(p)$, $C(q) = 0$, and F is nondegenerate. Then if $\bar{\alpha}q^m \le \underline{\alpha}$, and $2\mathcal{R}''(z)+z\mathcal{R}'''(z) < 0$, we have $W^{RPM} > W^{FL}$.*

[34] Since $R(q,\alpha) = \mathcal{P}(q/\alpha)q = \alpha[\mathcal{P}(q/\alpha)q/\alpha]$, we can express revenue as $\alpha\mathcal{R}(q/\alpha)$, where $\mathcal{R}(z) = z\mathcal{P}(z)$.

Proof: Since $R_{q\alpha}(q,\alpha) > 0$, it follows that $\mathcal{R}''(z) < 0$. Let $h(\alpha; q^{FL}) \equiv \mathcal{R}'\left(\frac{q^{FL}}{\alpha}\right)$. The conditions $\mathcal{R}''(z) < 0$ and $2\mathcal{R}''(z) + z\mathcal{R}'''(z) < 0$ imply h is strictly increasing and strictly concave in α.

Let G be a distribution such that F is a mean-preserving spread of G and $F \neq G$. Then $q^{FL} \leq q^m(\bar{\alpha}) \leq \hat{q}(\underline{\alpha})$ and since h is strictly concave and strictly increasing, we have:

$$\mathcal{R}'\left(\frac{q^{FL}}{\underline{\alpha}}\right)F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} \mathcal{R}'\left(\frac{q^{FL}}{\alpha}\right)dF(\alpha) < \mathcal{R}'\left(\frac{q^{FL}}{\underline{\alpha}}\right)G(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} \mathcal{R}'\left(\frac{q^{FL}}{\alpha}\right)dG(\alpha), \qquad (17)$$

where q^{FL} is the equilibrium inventory in the Flexible-Price Game with the distribution F. Since the left side of inequality (17) must be zero, and since $\mathcal{R}''(z) < 0$, the equilibrium inventory in the Flexible-Price Game with the distribution G, $q^{FL}(G)$, must satisfy $q^{FL}(G) > q^{FL}$.[35] Let $a \equiv \underline{\alpha}F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} \alpha dF(\alpha)$, and consider the degenerate distribution, G, in which all mass is placed on the single point, a. It follows, since F is not degenerate, that $q^{FL}(G) > q^{FL}$. Also, the first-order condition implies $q^{FL}(G)$ solves $\mathcal{R}'\left(\frac{q^{FL}(G)}{a}\right) = 0$. Therefore $q^{FL}(G)/a = q^m$, so we have $q^{FL}(G) = aq^m$. Since the family of demand curves $D(p,\alpha)$ is isoelastic at the same price, and since costs are zero, the minimum RPM solution satisfies $p^* = p^m$ and $q^{RPM} = q^m(\bar{\alpha})$. Consequently, $E(q^*) = aq^m = q^{FL}(G) > q^{FL}$. Invoking Theorem 3, we conclude $W^{RPM} > W^{FL}$.

■

Theorem 4 provides a sufficient condition for retail competition to be destructive because competition hurts not just overall welfare but even consumer surplus, as compared to allowing the manufacturer to set a price floor. When will market clearing result in a higher expected retail price than minimum RPM? Remember that inventories are always weakly higher under RPM (see Theorem 2). When $q^{RPM} > q^{FL}$, two effects are at work. First, when demand is low, the price floor set with minimum RPM results in higher prices than under flexible pricing. Second, when demand is high enough, the price will be lower under RPM, due to the higher inventories. Consumers benefit on average when the latter effect dominates sufficiently.

If there is "too much" demand uncertainty, as represented by the distribution F and the

[35] A similar argument is given in Rothschild and Stiglitz (1971).

demand function $D(p, \alpha)$, the retail price in the Flexible-Price Game will be zero in low demand states. When the manufacturer considers inducing a higher quantity of inventories to be demanded, revenues are not hurt in low demand states. In effect, the manufacturer ignores the low demand states, focusing on the high demand states. Under RPM, the price floor binds and causes the retail price to be well above zero in low demand states. Since inventories will be discarded when demand is low, the inventory decision is determined by the high demand states. Induced inventories and prices would be almost the same under market clearing and RPM in high demand states, so overall expected prices would be higher under RPM. Notice that this case requires extremely low demand to be unlikely enough for the manufacturer to allow the retail price to be zero, but requires extremely low demand and a binding price floor to be likely enough to make the average price under RPM higher than that under market clearing. Thus, retail competition is most likely to be "destructive"—Pareto-dominated by the alternative of minimum RPM—when demand fluctuations are moderate. If demand fluctuations are so great that "fire sales" are likely to occur unless prohibited by RPM, consumers will prefer flexible pricing. If demand fluctuations are sufficiently small, then the manufacturer will not impose a binding price floor, so flexible pricing and RPM yield the same outcome.

When fixed production costs are introduced, retail competition can be destructive for a new reason. Since manufacturer profits, not including fixed costs, are higher under RPM, the manufacturer may be willing to serve the market under RPM but not under flexible pricing.

Corollary 1 ***(to Theorem 1):*** *Extend both games by having the manufacturer first choose whether or not to pay a fixed cost and enter the market. Suppose that F is not a degenerate distribution and that when fixed costs are zero we have $q^{FL} > q^m(\underline{\alpha})$. Then there is a range of fixed costs for which the manufacturer will enter the market when minimum RPM is allowed but will not enter the market under flexible pricing.*

This Corollary corresponds to the ultimate destructive competition. Under flexible pricing, fixed costs are too high to support nonnegative profits for the manufacturer, so the market is not served and no surplus is generated. If instead the manufacturer is allowed to set a

price floor, then the market is profitable, the manufacturer serves the market, and everyone benefits.[36]

This paper has shown that imposing a retail price floor may yield higher profits to the manufacturer and higher surplus to consumers compared to market clearing. One potential explanation for the inferiority of market clearing is that markets are incomplete. Under flexible prices, consumers face price risk, and there are no markets before the realization of demand on which consumers could trade contingent claims. However, assuming that consumers have quasilinear utility (thereby justifying our use of consumer surplus as an indicator of their economic welfare), it is not hard to show that in the absence of market power, market clearing is Pareto optimal. The intuition is that, under the quasi-linear specification, consumers are risk neutral with respect to numeraire consumption. If we completed the markets by allowing income to be traded across states of nature, no one would be better off.[37]

The nonoptimality of market clearing therefore stems from the presence of market power, or more precisely, the upstream manufacturer's unwillingness or inability to employ marginal cost pricing. In our model, this is a consequence of our assumption that the manufacturer is a monopolist manipulating the wholesale price. Suppose instead that the manufacturer had declining average costs and was required to serve all demand by retailers at a wholesale price equal to average cost. In this context as well, imposing a retail price floor could improve welfare over a flexible retail price.

5 Summary and Conclusions

We have shown that a monopoly manufacturer selling to uncertain consumer demand through a competitive retail sector will often wish to impose resale price maintenance on its retailers in preference to permitting the retail market to clear ex post. Surprisingly, consumers also

[36] While we have modeled the manufacturer as a monopolist, the inefficiency we identify persists in the presence of competition between manufacturers as long as the wholesale price remains above marginal cost. Hence there will be a *unilateral* incentive to introduce RPM when manufacturers compete, particularly when the manufacturers' brand names are well regarded by consumers.

[37] This argument has probably been made by other authors, and is formalized in an earlier version of this paper. After a careful specification of the states of nature, the result is immediate.

may benefit from the manufacturer's ability to impose this vertical restraint, as they see lower prices and greater product availability in the event of high demand. The primary requirement for RPM to be desirable to all parties is that demand fluctuations not be so great as to cause catastrophic (to the manufacturer and its retailers) consequences in the event of slack demand. But even when consumers prefer flexible prices, RPM is desirable to the manufacturer because it prevents large price fluctuations which would otherwise have impaired inventory holding.

Our model applies best to products satisfying the following criteria:

i. competitive retailers must order inventories prior to the resolution of significant demand uncertainty;

ii. when RPM is not imposed, retail prices adjust quickly to demand shocks, thereby clearing the market; and

iii. the products in question have little scrap value, and are expensive to hold for future demand periods.

For products satisfying criteria i-iii, our model makes the following predictions:

a. Manufacturers prefer RPM to market clearing. Other things equal, orders for inventory are higher under RPM than under market clearing.

b. Under market-clearing pricing, retail prices and markups are correlated positively with shocks to demand conditions and retail sales revenues.[38]

The experience of Nintendo, a leading manufacturer of video game players and cartridges, provides a particularly apt illustration of our model. Nintendo introduced its game player and games to the U.S. in the mid-1980's, a very difficult time to induce retailers to stock such products.[39] Retailers had incurred very large losses on a previous generation of video games.

[38]While it might seem obvious that an unexpectedly good holiday season should lead to high retail prices, or fewer markdowns, Julio J. Rotemberg and Garth Saloner (1986) have argued in booms, implicit collusion is harder to maintain, so that lower prices prevail. They cite evidence to suggest that markups are countercyclical. Our model predicts that for products satisfying criteria i-iii, markups should be higher when the market in question experiences a boom.

[39]The discussion of Nintendo's experience is based on a detailed account of the history of Nintendo in David Sheff (1994).

In the early 1980's, video games had enjoyed phenomenal success, with sales rising from about $200 million in 1978 to one billion dollars in 1981 and further to three billion dollars in 1982. Excessive inventories resulted in massive price cutting soon after, so that in 1983, sales fell precipitously to $100 million and the leading manufacturer of games and game players, Atari, collapsed. The decline in revenues was entirely due to price cutting—unit sales actually increased. Because retailers ended up holding worthless inventory, they were obviously reluctant to stock the products of an untested (in the U.S.) video game manufacturer.[40]

The characteristics of the video game market fit our model well. Games are ordered for inventory in the late winter and delivered to retailers in the following summer. Nintendo deviated from the standard toy market practice of offering December 10 invoice dating (payment for goods added to inventories in summer was deferred until cash came in during the holidays) by demanding payment within ninety days of shipment, thereby ensuring that the cost of inventories to retailers was sunk. Most sales occur in the winter holiday season. The Atari experience proved how sensitive prices were to an excess of available supplies over the quantity demanded. Finally, since new games were introduced each year, scrap value was small. Accordingly, we should find that Nintendo tried to protect retail prices in order to promote adequate inventory holding. Initially, Nintendo prevented price cutting by accepting returns of unsold merchandise (Sheff, 1994, p. 166), but once it obtained a retail foothold, it relied on controlling inventories and cutting off dealers who cut suggested prices to keep price cutting from breaking out. These practices resulted, in 1991, in the signing of a consent decree under which Nintendo agreed not to maintain resale prices of any of its products. Under investigation for RPM, Nintendo had been unable to enforce resale prices during the 1990 holiday

[40] Consider the following, from Sheff (1994, p.158–9):

> "The reason I have this terrific job," a buyer for the toy company began, "is that the guy before me was fired after he lost so much in video games. Do you think there is any way *I'm* going to make that mistake?"
>
> Throughout 1984, Arakawa [a Nintendo executive] heard variations on that theme over and over when he met with toy- and department-store representatives to tell them he was considering entering the home video-game business. They thought he was nuts.
>
> Arakawa marveled at the intensity of the hostility toward video games—even the phrase was taboo. In the horror stories about the industry, hyperbole was unnecessary…

Nintendo's early efforts to introduce its machines, already very successful in Japan, into the U.S. market failed because retailers would not stock inventories (Sheff, 1994, p. 191ff.).

season. Influenced by uncertainties surrounding events in Iraq and Kuwait, the 1990 holiday season was a slack demand period. The result of slack demand combined with Federal and state scrutiny of potential RPM was sharp declines in prices of some of Nintendo's most popular titles.[41]

Previous RPM theories do not seem applicable to Nintendo's use of RPM. Nintendo was not involved in a manufacturer cartel, and given the range of retail outlets carrying Nintendo products, a dealer cartel seems distinctly implausible. Efficiency explanations for RPM, such as pre-sale services (Telser, 1960) and quality certification (Marvel and McCafferty, 1984), also appear inapplicable.[42] We believe that our theory of destructive competition is capable of explaining RPM use for products for which pre-sale, free-rideable services are difficult to discern.[43] But even for products for which efficiency theories can be applied, our theory provides a complementary explanation.[44]

Our goal has been to show that market-clearing pricing need not dominate minimum RPM when a monopoly manufacturer selling through a competitive retail sector must produce and ship its product prior to the resolution of demand uncertainty. Our results provide surprising support for the common claim that RPM is a device desirable for its ability to suppress destructive competition. At the same time, we have shown that the use of RPM will be most attractive to manufacturers when demand uncertainty leads to the prospect of deep price cutting in the event of slack demand. It is no surprise, then, that consumer and producer groups have been deeply divided over the merits of RPM.

Appendix

Let q^{FL} be any solution to (8). Define $\alpha_0 \equiv \inf\{z \in [\underline{\alpha}, \bar{\alpha}] : \hat{q}(z) \geq q^{FL}\}$. Note that α_0 is well

[41] See William G. Flanagan and Evan McGlinn, "The Sunny Side of the Recession," *Forbes*, January 7, 1991, p. 298: "Nintendo's popular Zelda game is $19.95 this year at Toys 'R' Us, versus $40 last Christmas."

[42] Note that leading Nintendo retailers did not offer pre-sale services. In 1991, Toys 'R' Us captured 22% of the U.S. toy market. Nintendo sales were approximately 20% of its revenues. K-Mart and Wal-Mart, discount retailers, captured about 10% each of the toy market and were also leading Nintendo retailers. None of these retailers appears to offer the package of services and reputation that in other instances has resulted in RPM use.

[43] For catalogs of products to which RPM has been applied, see Overstreet (1983) and Ippolito (1988).

[44] Our model of minimum RPM is consistent with Ippolito's (1991) result that empirically, RPM consists predominantly of enforcement of minimum resale prices.

defined, since $q^{FL} \leq q^m(\bar{\alpha}) < \hat{q}(\bar{\alpha})$. Also, let $\Delta F(\alpha_0) = F(\alpha_0) - \lim_{z \uparrow \alpha_0} F(z)$ and define

$$\varphi(q) = R(q, \alpha_0)\Delta F(\alpha_0) + \int_{\alpha_0}^{\bar{\alpha}} R(q, z)dF(z) - C(q).$$

Given q^{FL}, α_0 is the lowest demand state above which the market-clearing price is positive, and $\varphi(q)$ represents profits in those states not less than α_0. We first establish the following auxiliary result, which will be used in the proof of Theorem 2.

Lemma 1 *The function $\varphi(q)$ is strictly concave on $[0, \hat{q}(\alpha_0)]$, and $q^{FL} = \arg\max_{q \leq \hat{q}(\alpha_0)} \varphi(q)$.*

Proof: Suppose there existed $q \leq \hat{q}(\alpha_0)$ such that $\varphi(q) > \varphi(q^{FL})$. Then we would have $\Pi^{FL}(q) \geq \varphi(q) > \varphi(q^{FL}) = \Pi^{FL}(q^{FL})$, contradicting the fact that q^{FL} solves (8). Next, observe that for all $q \in (0, \hat{q}(\alpha_0))$ and all $z \in [\alpha_0, \bar{\alpha}]$, we have $R(q, z) \geq R(q, \alpha_0) > 0$. Thus, φ is a strictly concave function on $[0, \hat{q}(\alpha_0)]$, and q^{FL} uniquely maximizes $\varphi(q)$. ■

Proof of Theorem 2: If $q^{FL} = 0$, then the result is obvious. Hence suppose that $q^{FL} > 0$, and that contrary to the statement of the theorem, there exist solutions to (8) and (11) for which $q^{RPM} < q^{FL}$.

Case 1: $\alpha_0 < \alpha^*$. We have

$$\varphi'(q^{RPM}) = R_q(q^{RPM}, \alpha_0)\Delta F(\alpha_0) + \int_{\alpha_0}^{\bar{\alpha}} R_q(q^{RPM}, z)dF(z) - C'(q^{RPM}). \tag{18}$$

We claim that $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$. Since we are in case 1, we know that $\alpha^* > \underline{\alpha}$. Furthermore, for q^{RPM} to be an equilibrium inventory for the RPM Game, $q^{RPM} \leq q^m(\bar{\alpha})$ must hold. If $\alpha^* = \bar{\alpha}$, we would have $p^* \geq P(q^{RPM}, \alpha^*) \geq p^m(\bar{\alpha})$. The last inequality cannot be strict, for then the price floor would exceed the revenue-maximizing price in every state. But $q^{RPM} = q^m(\bar{\alpha})$ implies $q^{FL} > q^m(\bar{\alpha})$, contradicting the optimality of q^{FL}. Thus, $\alpha^* \in (\underline{\alpha}, \bar{\alpha})$.

While $\Pi(q, \alpha)$ is not necessarily differentiable, it can nevertheless be shown that if $\alpha \in (\underline{\alpha}, \bar{\alpha})$, we must have

$$\frac{\partial \Pi}{\partial q}(q^{RPM}, \alpha^*) = \{\bar{R}_p(P(q^{RPM}, \alpha^*), \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha^*} \bar{R}_p[P(q^{RPM}, \alpha^*), z]dF(z)\}P_q(q^{RPM}, \alpha^*)$$

$$+\int_{\alpha^*}^{\bar{\alpha}} R_q(q^{RPM},z)dF(z) - C'(q^{RPM}) = 0 \tag{19}$$

$$\frac{\partial \Pi}{\partial \alpha}(\alpha^*, q^{RPM}) = \{\bar{R}_p(P(q^{RPM},\alpha^*),\underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha^*} \bar{R}_p[P(q^{RPM},\alpha^*),z]dF(z)\}P_\alpha(q^{RPM},\alpha^*) = 0 \tag{20}$$

If $q^{RPM} = 0$, then $\Pi^{FL} = 0$, so $q = 0$ is a solution to the Flexible-Price Game. Since $q^{FL} > 0$ is also a solution, this contradicts the fact that q^{FL} uniquely maximizes $\varphi(q)$ on the interval $[0, \hat{q}(\alpha_0)]$. Thus, $0 < q^{RPM} < \hat{q}(\alpha^*)$, which implies $P_q(q^{RPM},\alpha^*) < 0$ and $P_\alpha(q^{RPM},\alpha^*) > 0$. Hence (19) and (20) yield

$$\int_{\alpha^*}^{\bar{\alpha}} R_q(q^{RPM},z)dF(z) - C'(q^{RPM}) = 0. \tag{21}$$

For p^* to be an equilibrium price floor, $p^* = P(q^{RPM},\alpha^*) \le p^m(\alpha^*)$ holds, and so $R_q(q^{RPM},\alpha^*) \le 0$. Since $R_{q\alpha} > 0$, we have $q^m(\underline{\alpha}) \le q^m(\alpha^*) < q^{RPM} < q^{FL} \le \hat{q}(\alpha_0)$ so that $R_q(q^{RPM},z)$ is well defined for all $z \in [\alpha_0,\alpha^*]$ and satisfies $R_q(q^{RPM},z) \le 0$ for all $z \le \alpha^*$. Thus (18) and (21) imply

$$\varphi'(q^{RPM}) = R_q(q^{RPM},\alpha_0)\Delta F(\alpha_0) + \int_{\alpha_0}^{\alpha^*} R_q(q^{RPM},z)dF(z) \le 0. \tag{22}$$

Since $q^{RPM} < q^{FL}$, and φ is strictly concave on $0, \hat{q}(\alpha_0)]$, this contradicts the optimality of q^{FL}.

Case 2: $\alpha_0 \ge \alpha^*$**.** We have

$$\begin{aligned}\Pi^{FL} &\ge R(q^{RPM},\underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\bar{\alpha}} R(q^{RPM},z)dF(z) - C(q^{RPM}) \\ &\ge \int_{\alpha^*}^{\bar{\alpha}} R(q^{RPM},z)dF(z) - C(q^{RPM}) \\ &\ge \int_{\alpha^*}^{\bar{\alpha}} R(q^{FL},z)dF(z) - C(q^{FL}) = \Pi^{FL}\end{aligned} \tag{23}$$

The third inequality in (23) follows from the fact that q^{RPM} maximizes $\int_{\alpha^*}^{\bar{\alpha}} R(q,z)dF(z) - C(q)$. The equality in (23) holds because $R(q^{FL},z) = 0$ for $z \le \alpha_0$. Therefore, all inequalities in (23)

must hold as equalities, and so:

$$\int_{\alpha^*}^{\bar{\alpha}} R(q^{RPM}, z)dF(z) - C(q^{RPM}) = \int_{\alpha_0}^{\bar{\alpha}} R(q^{FL}, z)dF(z) - C(q^{FL}). \tag{24}$$

Now $p^* = P(q^{RPM}, \alpha^*) \leq p^m(\alpha^*)$, for otherwise the price floor would strictly exceed the revenue-maximizing price in every state in which the price floor binds, which would be inconsistent with equilibrium. Furthermore, the price floor must strictly bind in some states, for otherwise $\Pi^{RPM} = \Pi^{FL}$, and hence by Theorem 1 we would have $q^{RPM} = q^{FL}$ (see note 28).

Consider the following deviation in the RPM Game. Keep the price floor at p^*, but raise inventories from q^{RPM} to q^{FL}. Let α^{**} be defined by $\alpha^{**} = \sup\{z \in [\underline{\alpha}, \bar{\alpha}] : P(q^{FL}, z) < p^*\}$. Since $p^* > 0$, we have $\alpha^{**} > \alpha_0 \geq \alpha^*$. Under this deviation, the manufacturer's profit would be:

$$\begin{aligned}
\bar{R}(p^*, \underline{\alpha})F(\underline{\alpha}) \quad &+ \quad \int_{\underline{\alpha}}^{\alpha^{**}} \bar{R}(p^*, z)dF(z) + \int_{\alpha^{**}}^{\bar{\alpha}} R(q^{FL}, z)dF(z) - C(q^{FL}) \\
&\geq \quad \bar{R}(p^*, \underline{\alpha})F(\underline{\alpha}) + \int_{\underline{\alpha}}^{\alpha_0} \bar{R}(p^*, z)dF(z) + \int_{\alpha_0}^{\bar{\alpha}} R(q^{FL}, z)dF(z) - C(q^{FL}) \\
&= \quad \int_{\alpha^*}^{\alpha_0} \bar{R}(p^*, z)dF(z) + \Pi^{RPM}.
\end{aligned} \tag{25}$$

The inequality in (25) follows from the fact that for $z \in (\alpha^*, \alpha^{**}]$, we have $P(q^{FL}, z) \leq p^* \leq p^m(\alpha^*) \leq p^m(z)$ and so $R(q^{FL}, z) \leq \bar{R}(p^*, z)$. The equality in (25) holds because of (24).

If the last integral in (25) is strictly positive, we will have contradicted the optimality of q^{RPM}. To show this, first note that $\bar{R}(p^*, z) > 0$ for $z \in (\alpha^*, \alpha_0]$. If $\mu((\alpha^*, \alpha_0]) = 0$, then (24) would imply that q^{RPM} and q^{FL} both maximize $\varphi(q)$, contradicting Lemma 1. Therefore $\mu((\alpha^*, \alpha_0]) > 0$, proving the contradiction. We conclude that $\alpha_0 \leq \alpha^*$ is also inconsistent with $q^{RPM} < q^{FL}$.

Furthermore, suppose that the manufacturer strictly prefers RPM, and that either F has full support or $q^c(\bar{\alpha}) \leq \hat{q}(\underline{\alpha})$, but that contrary to the hypothesis of the theorem,

$q^{RPM} = q^{FL}$. Note that since $q^{FL} \leq q^c(\bar{\alpha})$, the condition $q^c(\bar{\alpha}) \leq \hat{q}(\underline{\alpha})$ implies $\alpha_0 = \underline{\alpha}$.

Since the manufacturer strictly prefers RPM, the price floor must be strictly binding, so

that $p^* = P(q^{RPM}, \alpha^*) > P(q^{FL}, \underline{\alpha}) \geq 0$, implying $\alpha^* > \alpha_0 \geq \underline{\alpha}$. Now if $\alpha^* = \bar{\alpha}$, we would have $q^{RPM} = q^m(\bar{\alpha})$, as shown in Case 1 above. Since $R_{q\alpha} > 0$ we would then have $R_q(q^{RPM}, z) < R_q(q^{RPM}, \bar{\alpha}) = 0$ for all $z \in [\alpha_0, \bar{\alpha})$. Consequently, since either $\alpha_0 = \underline{\alpha}$ or F has full support, it follows from (18) that $\varphi(q^{RPM}) < 0$, contradicting $q^{RPM} = q^{FL}$. SImilarly, if we had $\alpha^* < \bar{\alpha}$, then since $\alpha^* > \alpha_0 \geq \underline{\alpha}$, it follows from (22) that $\varphi'(q^{RPM}) < 0$, again contradicting $q^{RPM} = q^{FL}$. ■

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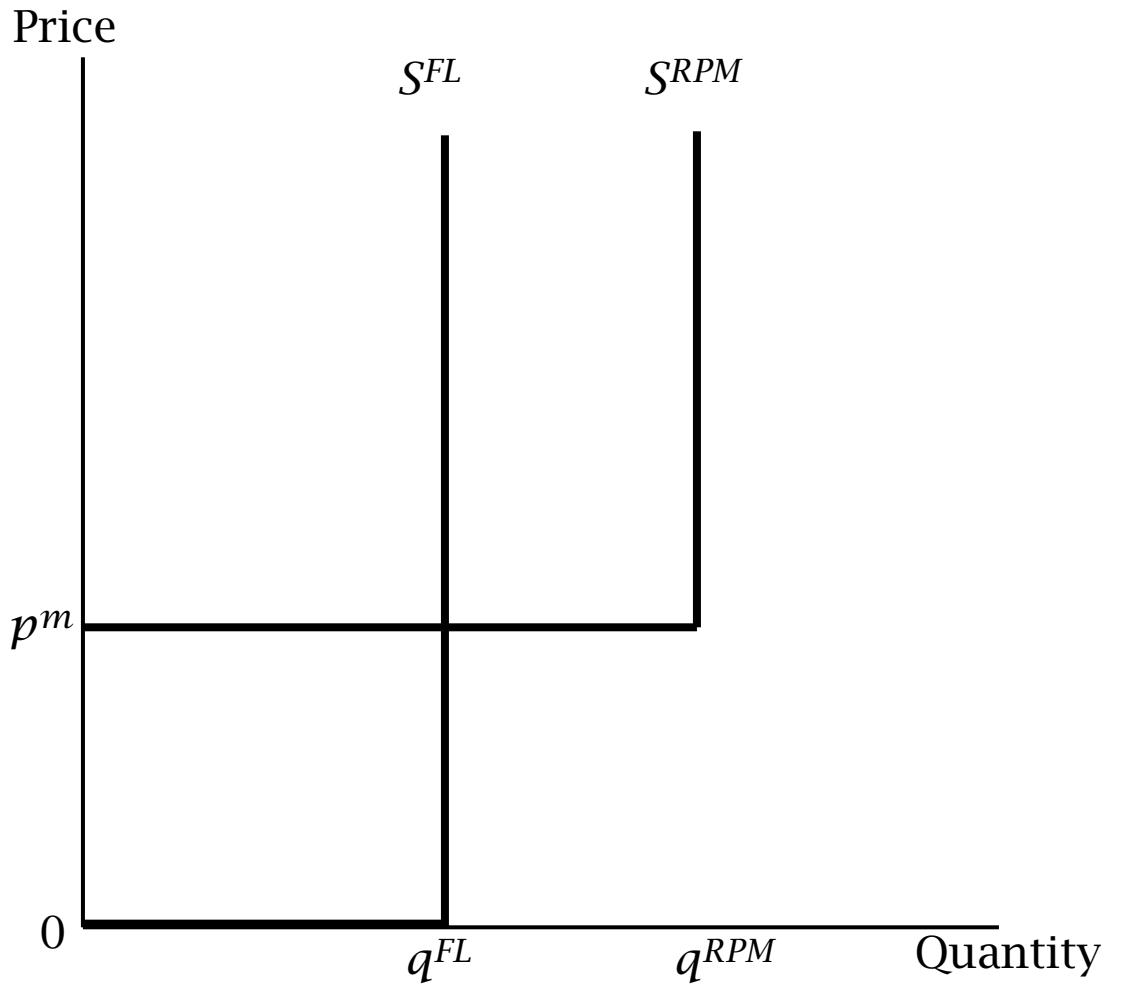
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Price
S^{FL}
S^{RPM}
p^m
0
q^{FL}
q^{RPM}
Quantity