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## Peer reviewed journal articles

- 2020 Classifying module categories for generalized Temperley-Lieb-Jones  $\ast$ -2-Categories (with Hernández-Palomares), *Internat. J. Math.* **31** (2020), no. 04, 2050027, [MR4098904](#), [arXiv:1905.00471](#)
- 2020 Harmonic gradients on higher dimensional Sierpinski gaskets (with Brown, Mograby, Rogers, and Sangam), *Fractals* **28** (2020), no. 06, 2050108, [arXiv:1908.10539](#)
- ★ 2022 Gray-categories model algebraic tricategories, *Theor. Appl. Categ.* **38** (2022), no. 29, pp. 1136–1155, [arXiv:2203.03748](#)
- 2023 3D Koch-type crystals (with Vélez-Santiago), *J. Fractal Geometry* **10** (2023), no. 1/2, pp. 109–149, [arXiv:2302.10628](#)

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## ArXiv preprints (under review)

- 2024 Dagger  $n$ -categories (with Hungar, Johnson-Freyd, Krulewski, Müller, Nivedita, Penneys, Reutter, Scheimbauer, Stehouwer, Vuppulury), [arXiv:2403.01651](#)
- ★ 2024 Foundations for operator algebraic tricategories, [arXiv:2404.05193](#)
- ★ 2024 Manifestly unitary higher Hilbert spaces (with Chen, Hungar, Penneys, Sanford), [arXiv:2410.05120](#)

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## Manuscripts (in preparation)

- 2025 Unitary Quantum Symmetries Lite (UQSL) textbook (with Kawagoe and Penneys), [available online](#)
- 2025 Bases for 3-Hilbert spaces and higher unitary duality (with Hungar, Penneys, Wesley)
- 2025 The homotopy 3-type of abelian  $C^*$ -algebras (with Faurot)
- 2025 Models for higher Hilbert spaces as homotopy fixed points (with Müller, Penneys, Stehouwer)
- 2025 Knot theoretic model for  $\text{Rep}(\mathcal{U}_q(\mathfrak{sl}_4))$ -webs arising from HOMFLY-PT (with Lu and Poudel)
- 2025 A duality theory in the spirit of functional analysis (with Circele)

Here we describe the relation between this work and our *Higher Functional Analysis* research program.

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## Past accomplishments

Our past work is focused on the finite dimensional theory of *higher functional analysis*.

**Manifestly unitary higher Hilbert spaces.** In joint work with Chen, Hungar, Penneys, and Sanford, we define *finite dimensional* 2-operator algebras and 3-Hilbert spaces. We then describe a formal process of constructing 3-Hilbert spaces from 2-Hilbert spaces, known as *unitary condensation*. In particular, this is a *manifestly unitary* formulation of the work of Gaiotto and Johnson-Freyd for higher vector spaces.

**Foundations for operator algebraic tricategories.** In single author work, we provide foundational results for the theory of 3-operator algebras and 4-Hilbert spaces. More technically, we define and prove both coherence and concreteness results for so-called *operator algebraic tricategories*. We then show there is an operator algebraic tricategory  $E_2^\dagger(\text{Hilb})$  consisting of commutative operator algebras and their 3-categorical quantum symmetries.

**Dagger  $n$ -categories.** In joint work with Hungar, Johnson-Freyd, Krulewski, Muller, Nivedita, Penneys, Reutter, Scheimbauer, Stehouwer, and Vuppulury, we establish a theory of *dagger  $n$ -categories*, one of the key ingredients in the future formulation of  $n$ -Hilbert spaces and  $n$ -operator algebras for arbitrary  $n = 1, 2, 3, \dots$

**Classifying module categories for generalized Temperley-Lieb-Jones  $\ast$ -2-categories.** In joint work with Hernandez-Palomares, we provide a universal construction for a family of finite dimensional 3-Hilbert spaces related to the Jones polynomial and classify their *representations*.

**Gray-categories model algebraic tricategories.** In single author work, we prove the Grothendieck's *homotopy hypothesis*, as popularized by Baez, for *algebraic trigroupoids*.

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## Current work

Our current work is focused on furthering these results in the direction of homotopy theory.

**Bases for 3-Hilbert spaces and higher unitary duality.** Building on work from *Manifestly unitary higher Hilbert spaces*, joint with Hungar, Penneys, and Wesley. We define and characterize unitary dual 2-functors on finite dimensional 3-operator algebras.

**The homotopy 3-type of commutative operator algebras.** Building on work from *Foundations for operator algebraic tricategories*, joint with Faurot. We describe an equivalent topological model to the Morita operator algebraic tricategory  $E_2^\dagger(\text{Hilb})$  of commutative operator algebras and their quantum symmetries, and provide a tricategorical Gelfand duality, simultaneously extending classical Gelfand duality, the Serre-Swan theorem, and the Dauns-Hoffman theorem. We then describe the underlying homotopy 3-type of this 3-category in terms of its Postnikov data.

**Models for higher Hilbert spaces as homotopy fixed points.** Building on work from *Dagger  $n$ -categories*, joint with Müller, Penneys, and Stehouwer. We systematically describe the different theories of operator algebras ( $C^*$ -algebras,  $W^*$ -algebras,  $H^*$ -algebras) in terms of homotopy fixed points for different subgroups of  $O(2)$ .

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## Broader impacts and outreach

**REU Research.** Joint work with Poudel, mentored undergraduate student Lu during our quantum symmetries REU summer program. We established a skein-theoretic immersed curve model for  $\text{Rep}(\mathcal{U}_q(\mathfrak{sl}_4))$ , the representation theory of the quantum group associated to  $SL_4$ . In particular, this project relates  $\text{Rep}(\mathcal{U}_q(\mathfrak{sl}_4))$  to work of Kauffman and Vogel on the HOMFLY-PT knot invariant.

**Undergraduate Research.** Mentoring undergraduate student Circe as part of the CYCLE program at OSU. Our project is focused on establishing a duality theory in the spirit of functional analysis. In particular, we generalize the notion of rigidity in a monoidal category to account for infinite dimensional spaces and provide a formal calculus for Hilbert spaces internal to a monoidal category under suitable conditions.

**REU Expository Work.** Mentored undergraduate student Kolt, producing an expository video on the *Synoptic Chart of Tensor Categories* with the goal of disseminating the geometric aspects of tensor categories to a broader audience.

**Textbook.** Writing a coauthored book *Unitary Quantum Symmetries Lite* (UQSL) with Kawagoe and Penneys. Our primary goal is to make the tools of higher category theory accessible to a large audience, in order to bridge the gap between mathematicians and physicists working on unitary tensor categories and topological phases of matter. Our main theme is the aforementioned higher functional analysis program in *finite dimensions*.