K.K. Gan
Physics 416
Problem Set 2
Due Thursday, October 18, 2007

1) Assuming a Gaussian probability distribution answer the following questions (Use Tables in *Taylor Appendix A and/or B)*:
   a) What is the probability of a value lying more than 1.5σ from the mean?
   b) What is the probability of a value lying $\geq 1.5\sigma$ above the mean?
   c) What is the probability of a value lying $\leq 1.5\sigma$ below the mean?
   d) What is the probability of a value, y, lying in the range $\mu - \sigma \leq y \leq \mu + 2\sigma$?
   e) What is the probability of a value, y, lying in the range $\mu + \sigma \leq y \leq \mu + 2\sigma$?

For this problem μ is the mean of the Gaussian and σ is its standard deviation.

2) Taylor, Problem 5.12, page 156.

3) The sun emits an enormous number of neutrinos. Assume that 10^6 solar neutrinos uniformly pass through a square with an area of 1 m^2 each μsec. Inside the square is a neutrino detector with an area of 1 mm^2. Assume Poisson statistics for this problem.
   a) What is the average number of neutrinos going through the detector each μsec?
   b) What is the probability that no neutrinos go through the detector in a μsec?
   c) What is the probability that ≥ 2 neutrinos go through the particle detector in a μsec?
   d) How big should the detector be (in mm^2) if we want ≥ 2 particles per μsec to pass through the detector with a probability of 95%?

4) Suppose a missile defense system destroys an incoming missile 95% of the time.
   a) If an evil country launches 20 missiles what is the probability that the missile defense system will destroy all of the incoming missiles?
   b) How many missiles have to be launched to have a 50% chance of at least one missile making it through the defense system?

Note: this problem can be done using either binomial or Poisson statistics.

5) According to quantum mechanics, the position (x) of a particle in a one dimensional box with dimensions $-L/2 \leq x \leq L/2$ (L constant) can be described by the following probability distribution function p(x):

$$p(x) = A\cos^2[\pi x/L] \text{ for } -L/2 \leq x \leq L/2, \text{ and 0 for all other x.}$$

a) Find the normalization constant A in terms of L.
b) Find the mean, mode, and median position of the particle in the box.
c) Show that the variance (σ^2) of x is given by:

$$\sigma^2 = \left(\frac{L}{\pi}\right)^2 \frac{\pi^2 - 6}{12}$$

d) What is the probability of finding the particle in the region: $L/4 \leq x \leq L/2$?

6) In the Bohr theory of the structure of the hydrogen atom the energies of the various quantum states are given by:

$$E_n = -\frac{me^4}{2N^2\hbar^2}$$

With: m the mass of the electron
e the electric charge of the electron
$\hbar$ Planck's constant divided by 2π

If: $\sigma_m/m = 0.1\%$ (i.e. the mass is known to 0.1%)
$\sigma_e/e = 0.2\%$ (i.e. the charge is known to 0.2%)
$\sigma_\hbar / \hbar = 0.1\%$

a) Calculate σ_E/E for arbitrary N.

b) If the precision of σ_E/E is to be improved which of the three quantities should be determined more precisely?

7) Suppose 100 six sided dice are tossed. Assume that the faces are labeled by one through six dots. Let Y_i be the number of dots on the ith (i =1 to 100) die.

a) What is the average number of dots expected for a single dice?

b) What is the variance of the numbers of dots expected for a single dice?

c) Use the Central Limit Theorem to estimate the probability that the sum of the Y_i's exceeds 400.

8) A Central Limit Theorem problem. When a certain chemical product is prepared the amount of a certain impurity is a random variable with a mean of 4 grams and a standard deviation of 2 grams. If 100 independent batches of the chemical are produced what is the (approximate) probability of the average amount of the impurity in the 100-batch sample being more that 4.5 grams?