K.K. Gan

Physics 416
Problem Set 3
Due Thursday, November 1, 2007

1) The probability density function (*pdf*) (often called a Maxwellian Distribution) that describes the speed v of molecules in an ideal gas is given by:

$$p(v) = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

Here m is the mass of the molecule, k is the Boltzmann's constant, and T is the temperature.

a) Show this is a properly normalized *pdf*.
b) Find the most probable speed.
c) Find the average speed.
d) Find the variance of the speed.

2) A Central Limit Theorem problem. When a batch of a certain type of superconductor is produced the amount of an impurity in the material (considered to be a random variable) must be tightly controlled. The amount of the impurity in a single batch of superconductor has a mean of 4 micrograms and a standard deviation of 2 micrograms. If 100 independent batches of the superconductor material are produced and then combined what is the probability for the total amount of the impurity in the combined batch being more that 450 micrograms?

3) Taylor P3.22, page 83.

4) Taylor P3.24, page 83.

5) Taylor P3.28, page 85.

6) Taylor P3.46, page 90.

7) The decay of an unstable particle is described by the following probability density function in terms of the decay time (t) and the particle's lifetime (λ).

$$p(t,\lambda) = \frac{e^{-\frac{t}{\lambda}}}{\lambda}$$

Three measurements of t ($t_1 = 7$ sec, $t_2 = 3$ sec, $t_3 = 4$ sec) are made.

a) Write down the likelihood function for this problem.
b) Use the Maximum Likelihood Method to calculate the value of λ for this data set.