Lecture 1: Introduction

Some Definitions:

- Current (I): Amount of electric charge (Q) moving past a point per unit time
  - $I = dQ/dt$ = Coulombs/sec
  - units = Amps (1 Coulomb = 6×10^{18} electrons)

- Voltage (V):
  - Work needed to move charge from point a to b

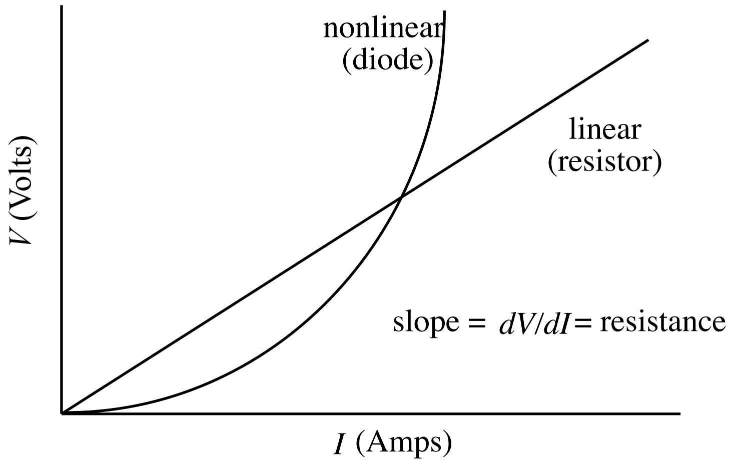
    Work = $V \cdot Q$

    ☞ Volt = Work/Charge = Joules/Coulomb
  - Voltage is always measured with respect to something
  - "ground" is defined as zero Volts

- Direct Current (DC): In a DC circuit the current and voltage are constant as a function of time

- Power (P): Rate of doing work
  - $P = dW/dt$
  - units = Watts

- Ohms Law: Linear relationship between voltage and current
    - $V = I \bullet R$
    - R = Resistance (Ω)
    - units = Ohms



- Joules Law: When current flows through a resistor energy is dissipated

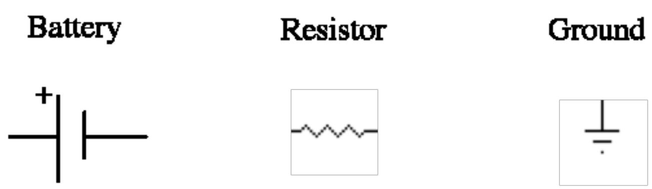
    $W = QV$

    $P = dW/dt = VdQ/dt + QdV/dt$

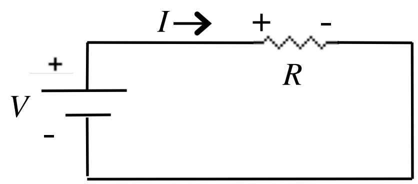
    - $dV/dt = 0$ for DC circuit and averages to 0 for AC
    - ☞ Power = $VdQ/dt = V \bullet I$
    - Using Ohms law
        - ☞ $P = I^2 R = V^2 / R$
        - 100 Watts = 10 V and 10 Amps or 10 V through 1 Ω

Simple Circuits

- Symbols:

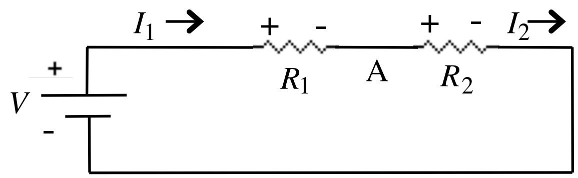


- Simple(st) Circuit:



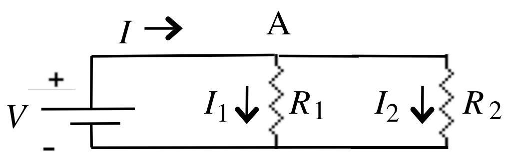
  - Convention: Current flow is in the direction of <u>positive</u> charge flow
    - When we go across a battery in direction of current (- ➔ +)
      - ☞ $+V$
    - Voltage drop across a resistor in direction of current (+ ➔ -)
      - ☞ $-IR$
      - **Conservation of Energy: sum of potential drops around the circuit should be zero**
        - ☞ $V - IR = 0$ or $V = IR$!!

- Next simple(st) circuit: two resistors in series



  - **Conservation of charge:** $I_1 = I_2 = I$ **at point A**
    - $V = I(R_1+R_2) = IR$
    - $R = R_1 + R_2$
      - ***Resistors in Series Add:*** $R = R_1 + R_2 + R_3 \ldots$
  - What's voltage across R_2?
    - $V_2 = I_2R_2 = VR_2/(R_1 + R_2)$ "**Voltage Divider Equation**"

- Two resistors in parallel

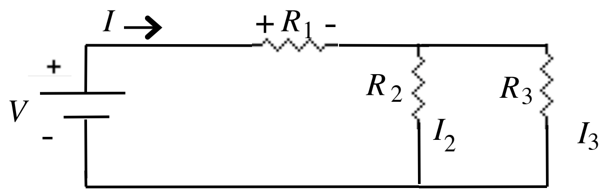


  - $I = I_1 + I_2 = V/R_1 + V/R_2 = V/R$
    - $1/R = 1/R_1 + 1/R_2$

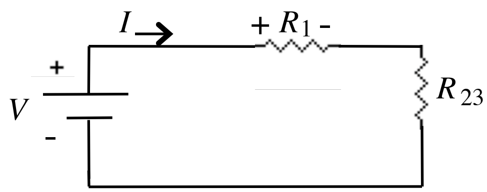
      $$\therefore\ R = \frac{R_1R_2}{R_1 + R_2}$$

      - ***Parallel Resistors add like:*** $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

- In a circuit with 3 resistors (series and parallel), what's $I_2 = V_2/R_2$?



  - reduce to a simpler circuit:



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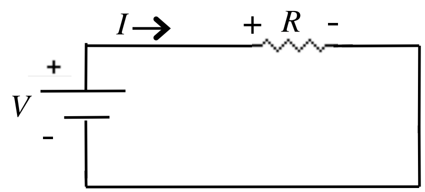


  - $I = V/R = V/(R_1 + R_{23})$

$$R_{23} = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_2 = IR_{23}$$

$$= \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2 R_3}{R_2 + R_3}$$

$$= \frac{V R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

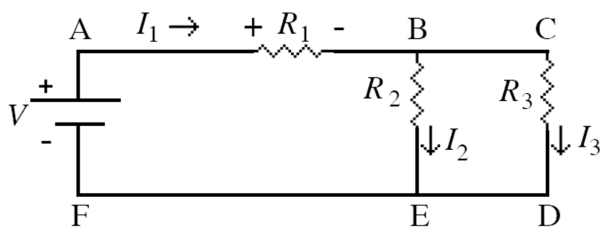
$$I_2 = \frac{V_2}{R_2}$$

$$= \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

If $R_3 \to \infty$ then $I_2 = I = V/(R_1 + R_2)$ as expected!

Kirchoff's Laws

- We can formalize and generalize the previous examples using Kirchoff's Laws:
  1. $\Sigma I = 0$ at a node: conservation of charge
  2. $\Sigma V = 0$ around a closed loop: conservation of energy
  - example



- node B: $I_1 = I_2 + I_3$ ➔ $I_1 - I_2 - I_3 = 0$
- loop ABEF: $V - I_1R_1 - I_2R_2 = 0$
- loop ACDF: $V - I_1R_1 - I_3R_3 = 0$
  - ☞ 3 linear equations with 3 unknowns: I_1, I_2, I_3
  - ☞ always wind up with as many linear equations as unknowns!
- use matrix methods to solve these equations:

$$\boldsymbol{V} = \boldsymbol{RI}$$

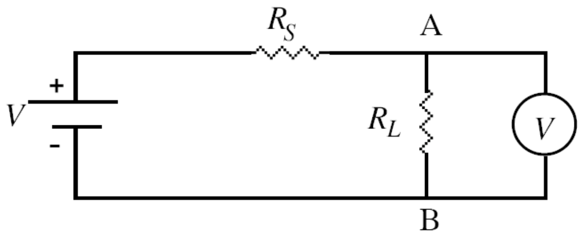
$$\begin{bmatrix} V \\ V \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_2 = \frac{\det\begin{bmatrix} R_1 & V & 0 \\ R_1 & V & R_3 \\ 1 & 0 & -1 \end{bmatrix}}{\det\begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

☞ the *same* solution as in page 5!

Measuring Things

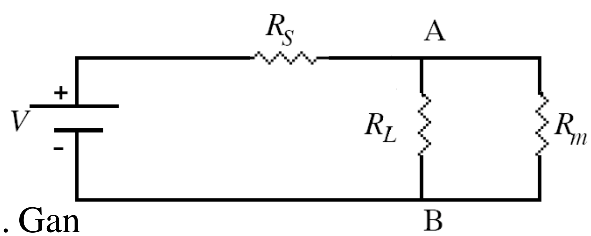
- Voltmeter: **Always** put in **parallel** with what you want to measure



- If no voltmeter we would have:

$$V_{AB} = \left[\frac{R_L}{R_S + R_L}\right]V$$

- If the voltmeter has a finite resistance R_m then circuit looks like:



- From previous pages we have:

$$V^*_{AB} = \left[\frac{R_m \| R_L}{R_S + R_m \| R_L}\right] V$$

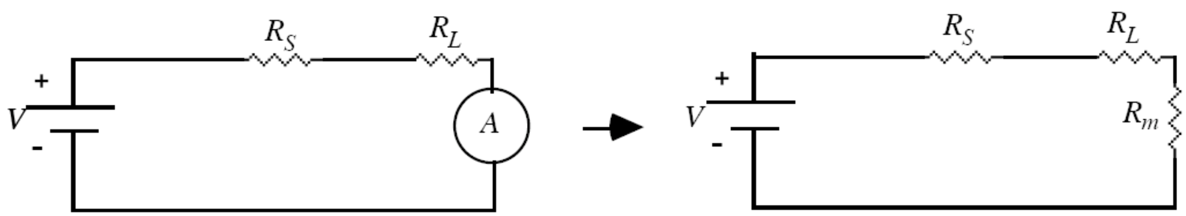
$$= \frac{V R_m R_L}{R_S R_L + R_m R_L + R_S R_m}$$

$$= \frac{V R_L}{R_L + R_S + \dfrac{R_S R_L}{R_m}}$$

$$\cong V_{AB} \qquad \text{if } R_L << R_m$$

  - ☞ good voltmeter has high resistance (> $10^6\ \Omega$)

- Ammeter: measures current
  - **Always** put in **series** with what you want to measure

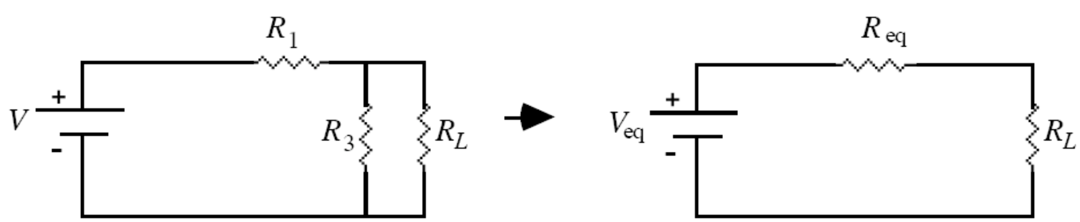


  - Without meter: $I = V/(R_S + R_L)$
  - With meter: $I^* = V/(R_S + R_L + R_m)$
    - ☞ good ammeter has $R_m << (R_m + R_L)$, i.e. low resistance (0.1-1 Ω)

Thevenin's Equivalent Circuit Theorem

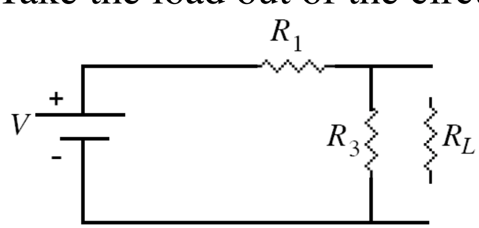
- Any network of resistors and batteries having 2 output terminals may be replaced by a series combination of resistor and battery
  - Useful when solving complicated (!?) networks
  - Solve problems by finding V_{eq} and R_{eq} for circuit without load, then add load to circuit.
  - Use basic voltage divider equation:

$$V_L = \frac{V_{eq} R_L}{R_L + R_{eq}}$$



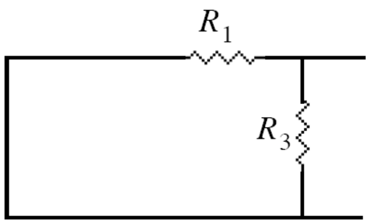
- Two rules for using Thevenin's Thereom:

1. Take the load out of the circuit to find V_{eq}:



$$V_{eq} = \frac{V R_3}{R_1 + R_3}$$

2. Short circuit all power supplies (batteries) to find R_{eq}:



$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3}$$

- Can now solve for I_L as in previous examples:

$$I_L = \frac{V_{eq}}{R_{eq} + R_L}$$

$$= \left[\frac{VR_3}{R_1 + R_3}\right] \times \frac{1}{\frac{R_1 R_3}{R_1 + R_3} + R_L}$$

$$= \frac{VR_3}{R_1 R_L + R_1 R_3 + R_L R_3}$$

☞ Same answer as previous examples!