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Physics 416
Problem Set 2

Due October 19, 2010

1) Assuming a Gaussian probability distribution answer the following questions

(Use Tables in *Taylor Appendix A and/or B*):

- a) What is the probability of a value lying more than 1.5σ from the mean?
- b) What is the probability of a value lying $\geq 1.5\sigma$ above the mean?
- c) What is the probability of a value lying $\leq 1.5\sigma$ below the mean?
- d) What is the probability of a value, y , lying in the range $\mu - \sigma \leq y \leq \mu + 2\sigma$?
- e) What is the probability of a value, y , lying in the range $\mu + \sigma \leq y \leq \mu + 2\sigma$?

For this problem μ is the mean of the Gaussian and σ is its standard deviation.

2) Taylor, Problem 5.12, page 156.

3) The sun emits an enormous number of neutrinos. Assume that 10^6 solar neutrinos uniformly pass through a square with an area of 1 m^2 each μsec . Inside the square is a neutrino detector with an area of 1 mm^2 . Assume Poisson statistics for this problem.

- a) What is the average number of neutrinos going through the detector each μsec ?
- b) What is the probability that no neutrinos go through the detector in a μsec ?
- c) What is the probability that ≥ 2 neutrinos go through the particle detector in a μsec ?
- d) How big should the detector be (in mm^2) if we want ≥ 2 particles per μsec to pass through the detector with a probability of 95%?

4) Suppose a missile defense system destroys an incoming missile 95% of the time.

- a) If an evil country launches 20 missiles what is the probability that the missile defense system will destroy all of the incoming missiles?
- b) How many missiles have to be launched to have a 50% chance of at least one missile making it through the defense system?

Note: this problem can be done using either binomial or Poisson statistics.

5) According to quantum mechanics, the position (x) of a particle in a one dimensional box with dimensions - $L/2 \leq x \leq L/2$ (L constant) can be described by the following probability distribution function $p(x)$:

$$p(x) = A \cos^2[\pi x/L] \text{ for } -L/2 \leq x \leq L/2, \text{ and } 0 \text{ for all other } x.$$

- a) Find the normalization constant A in terms of L .
- b) Find the mean, mode, and median position of the particle in the box.
- c) Show that the variance (σ^2) of x is given by:

$$\sigma^2 = \left(\frac{L}{\pi}\right)^2 \frac{\pi^2 - 6}{12}$$

d) What is the probability of finding the particle in the region: $L/4 \leq x \leq L/2$?

6) Suppose 100 six sided dice are tossed. Assume that the faces are labeled by one through six dots. Let Y_i be the number of dots on the i th ($i=1$ to 100) die.

- a) What is the average number of dots expected for a single dice?
- b) What is the variance of the numbers of dots expected for a single dice?

c) Use the Central Limit Theorem to estimate the probability that the sum of the Y_i 's exceeds 400.

7) A Central Limit Theorem problem. When a certain chemical product is prepared the amount of a certain impurity is a random variable with a mean of 4 grams and a standard deviation of 2 grams. If 100 independent batches of the chemical are produced what is the (approximate) probability of the average amount of the impurity in the 100-batch sample being more than 4.5 grams?