Lecture 7: Transistors and Amplifiers

Hybrid Transistor Model for small AC:

- The previous model for a transistor used one parameter ($\beta$, the current gain) to describe the transistor.
  - doesn't explain many features of three common forms of transistor amplifiers (common emitter etc.)
  - e.g. could not calculate the output impedance of the common emitter amp.

- Very often in electronics we describe complex circuits in terms of an equivalent circuit or model.
  - need a model that relates the input currents and voltages to the output currents and voltages.
  - the model needs to be linear in the currents and voltages.
    - For a transistor this condition of linearity is true for small signals.

- The most general linear model of the transistor is a 4-terminal “black box”.

  ![Diagram of 4-terminal black box]

  - In this model we assume the transistor is biased on properly and do not show the biasing circuit.
  - Since a transistor has only 3 legs, one of the terminals is common between the input and output.
  - There are 4 variables in the problem, $I_i$, $V_i$, $I_o$, and $V_o$.
    - The subscript i refer to the input side while the subscript o refers to the output side.
    - We assume that we know $I_i$ and $V_o$. 

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Kirchhoff’s laws relate all the currents and voltages:

\[ V_i = V_i(I_i, V_o) \]
\[ I_o = I_o(I_i, V_o) \]

For a linear model of the transistor with a small changes in \( I_i \) and \( V_o \):

\[
dV_i = \left( \frac{\partial V_i}{\partial I_i} \right)_{V_o} dI_i + \left( \frac{\partial V_i}{\partial V_o} \right)_{I_i} dV_o
\]

\[
dI_o = \left( \frac{\partial I_o}{\partial I_i} \right)_{V_o} dI_i + \left( \frac{\partial I_o}{\partial V_o} \right)_{I_i} dV_o
\]

The partial derivatives are called the hybrid (or \( h \) parameters):

\[
dV_i = h_{ii} dI_i + h_{io} dV_o
\]
\[
dI_o = h_{oi} dI_i + h_{oo} dV_o
\]

- \( h_{oi} \) and \( h_{io} \) are unitless
- \( h_{oo} \) has units \( 1/\Omega \) (mhos)
- \( h_{ii} \) has units \( \Omega \)

The four \( h \) parameters are easily measured.

- e.g. to measure \( h_{ii} \) hold \( V_o \) (the output voltage) constant and measure \( V_{in}/I_{in} \).

Unfortunately the \( h \) parameters are not constant.

- e.g. Figs. 11-14 of the 2N3904 spec sheet show the variation of the parameters with \( I_C \).
There are 3 sets of the 4 hybrid parameters.

- One for each type of amp: common emitter, common base, common collector
- In order to differentiate one set of parameters from another the following notation is used:

<table>
<thead>
<tr>
<th>First subscript</th>
<th>Second subscript</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = input impedance</td>
<td>e = common emitter</td>
</tr>
<tr>
<td>o = output admittance</td>
<td>b = common base</td>
</tr>
<tr>
<td>r = reverse voltage ratio</td>
<td>c = common collector</td>
</tr>
<tr>
<td>f = forward current ratio</td>
<td></td>
</tr>
</tbody>
</table>

For a common emitter amplifier we would write:

\[
dV_i = h_{ie} dI_i + h_{re} dV_o \\
dI_o = h_{fe} dI_i + h_{oe} dV_o
\]

- Typical values for the \( h \) parameters for a 2N3904 transistor in the common emitter configuration:
  - \( h_{fe} = 120 \), \( h_{oe} = 8.7 \times 10^{-6} \, \Omega^{-1} \), \( h_{ie} = 3700 \, \Omega \), \( h_{re} = 1.3 \times 10^{-4} \) for \( I_C = 1 \, mA \)
- The equivalent circuit for a transistor in the common emitter configuration looks like:

  ![Equivalent circuit diagram](image)

  - Circle: voltage source
    - the voltage across this element is always equal to \( h_{re} V_o \) independent of the current through it.
  - Triangle: current source
    - the current through this element is always \( h_{fe} I_{in} \) independent of the voltage across the device.

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- We can use the model to calculate voltage/current gain and the input/output impedance of a CE amp.
- Equivalent circuit for a CE amp with a voltage source (with resistance $R_s$) and load resistor ($R_{load}$):

![Equivalent circuit for a CE amp](image)

- **Current gain**: $G_I = \frac{I_o}{I_{in}}$
  - Using Kirchhoff's current law at the output side we have:
    $$h_{fe} I_{in} + V_o h_{oc} = I_o$$
  - Using Kirchhoff's voltage rule at the output we have:
    $$V_o = -I_o R_{load}$$
    $$h_{fe} I_{in} = h_{oe} I_o R_{load} + I_o$$
    $$G_I = \frac{I_o}{I_{in}} = h_{fe} / (1 + h_{oe} R_{load})$$
  - For typical CE amps, $h_{oe} R_{load} \ll 1$ and the gain reduces to familiar form:
    $$G_I = h_{fe} = \beta$$

- **Voltage gain**: $G_V = \frac{V_o}{V_{in}}$
  - This gain can be derived in a similar fashion as the current gain:
    $$G_V = \frac{V_o}{V_{in}} = -h_{fe} R_{load} / (\Delta R_{load} + h_{ie})$$
    with $\Delta = h_{ie} h_{oc} - h_{fe} h_{re} \approx 10^{-2}$
  - This reduces to a familiar form for most cases where $\Delta R_{load} \ll h_{ie}$
    $$G_V = -h_{fe} \frac{R_{load}}{h_{ie}} = -R_{load} / r_{BE}$$

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**Input Impedance:** \( Z_i = \frac{V_{in}}{I_{in}} \)

\[ Z_i = \left( \Delta R_{load} + h_{ie} \right) / \left( 1 + h_{oe} R_{load} \right) \]

This reduces to a familiar form for most cases where \( \Delta R_{load} \ll h_{ie} \) and \( h_{oe} R_{load} \ll 1 \)

**Output Impedance:** \( Z_o = \frac{V_o}{I_o} \)

\[ Z_o = \frac{R_s + h_{ie}}{\Delta + h_{oe} R_s} \]

- \( Z_o \) does not reduce to a simple expression.
- As the denominator is small, \( Z_o \) is as advertised large.

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**Feedback and Amplifiers**

- Consider the common emitter amplifier shown.
- This amp differs slightly from the CE amp we saw before:
  - bias resistor \( R_2 \) is connected to collector resistor \( R_1 \) instead of directly to \( V_{cc} \).
- How does this affect \( V_{out} \)?
  - If \( V_{out} \) *decreases* (moves away from \( V_{cc} \))
    - \( I_2 \) increases
    - \( V_B \) decreases (gets closer to ground)
    - \( V_{out} \) will increase since \( \Delta V_{out} = -\Delta V_B R_1/R_E \)
  - If \( V_{out} \) *increases* (moves towards \( V_{cc} \))
    - \( I_2 \) decreases
    - \( V_B \) increases (moves away from ground).
    - \( V_{out} \) will decrease since \( \Delta V_{out} = -\Delta V_B R_1/R_E \)

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This is an example of NEGATIVE FEEDBACK
- Negative Feedback is **good**:  
  - Stabilizes amplifier against oscillation  
  - Increases the input impedance of the amplifier  
  - Decreases the output impedance of the amplifier  
- Positive Feedback is **bad**:  
  - Causes amplifiers to oscillate

**Feedback Fundamentals:**

- Without feedback the output and input are related by:
  \[ V_{\text{out}} = AV_{\text{in}} \]
- The feedback (box B) returns a portion of the output voltage to the amplifier through the "mixer".
  - The feedback network on the AM radio is the collector to base resistors (R₃, R₅)
- The input to the amplifier is:
  \[ V_x = V_{\text{in}} + BV_{\text{out}} \]
- The gain with feedback is:
  \[ V_{\text{out}} = AV_x = A(V_{\text{in}} + BV_{\text{out}}) \]
  \[ G = V_{\text{out}} / V_{\text{in}} = A / (1 - AB) \]

Oscillation is a large fluctuation of output signal with no input

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Positive and negative feedback:

- Let's define $A > 0$ (positive)
  
  $$ G = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A}{1-AB} $$

- Positive feedback, $AB > 0$:
  - As $AB \rightarrow 1$, $G \rightarrow \infty$.
    - Circuit is unstable
    - Oscillates if $AB = 1$

- Negative feedback, $AB < 0$:
  - As $A \rightarrow \infty$, an amazing thing happens:
    $$ |AB| \rightarrow \infty $$
    $$ |G| \rightarrow |1/B| $$
  - Example: $A = 10^5$ and $B = -0.01$ then $G = 100$.
  - The stability of the gain is determined by the feedback loop ($B$) and not the amplifier ($A$).
  - Example: $B$ is held fixed at 0.01 and $A$ varies:
    
    | $A$     | Gain |
    |---------|------|
    | $5 \times 10^3$ | 98.3 |
    | $1 \times 10^4$  | 99.0 |
    | $2 \times 10^4$  | 99.6 |

  - Circuits can be made stable with respect to variations in the transistor characteristics as long as $B$ is stable.
  - $B$ can be made from precision components such as resistors.
Operational Amplifiers (Op Amps)

- Op amps are very high gain \( A = 10^5 \) differential amplifiers.
  - Differential amp has two inputs \( (V_1, V_2) \) and output \( V_{out} = A \( V_1 - V_2 \) \) where \( A \) is the amplifier gain.

  ![Op Amp Diagram]

  **non inverting input** + 
  **inverting input** -

  (power connections not shown)

- If an op amp is used without feedback and \( V_1 \neq V_2 \)
  - \( V_{out} \) saturates at the power supply voltage (either positive or negative supply).

- Example: Assume the maximum output swing for an op amp is ±15 V.
  - If there is no feedback in the circuit:
    - \( V_{out} = 15 \) V if \( V_{non-invert} > V_{invert} \)
    - \( V_{out} = -15 \) V if \( V_{non-invert} < V_{invert} \)

- Op amps are almost always used with negative feedback.
  - The output is connected to the (inverting) input.

- Op amps come in “chip” form. They are made up of complex circuits with 20-100 transistors.

<table>
<thead>
<tr>
<th>Ideal Op Amp</th>
<th>Real Op Amp µA741</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage gain (open loop) ( \infty )</td>
<td>( 10^5 )</td>
</tr>
<tr>
<td>Input impedance ( \infty )</td>
<td>2 M( \Omega )</td>
</tr>
<tr>
<td>Output impedance 0</td>
<td>75 ( \Omega )</td>
</tr>
<tr>
<td>Slew rate ( \infty )</td>
<td>0.5 V/( \mu )sec</td>
</tr>
<tr>
<td>Power consumption 0</td>
<td>50 mW</td>
</tr>
<tr>
<td>( V_{out} ) with ( V_{in} = 0 ) 0</td>
<td>2 mV (unity gain)</td>
</tr>
<tr>
<td>Price 0$</td>
<td>$0.25</td>
</tr>
</tbody>
</table>

  **Slew rate is how fast output can change**
When working with op amps using negative feedback two simple rules (almost) always apply:

- **No current goes into the op amp.**
  - This reflects the high input impedance of the op amp.
- **Both input terminals of the op amp have the same voltage.**
  - This has to do with the actual circuitry making up the op amp.

Some examples of op amp circuits with negative feedback:

- **Voltage Follower:**
  
  ![Voltage Follower Diagram](image)

  - The feedback network is just a wire connecting the output to the input.
  - By rule #2, the inverting (-) input is also at \( V_{\text{in}} \).
  - \( V_{\text{out}} = V_{\text{in}} \).
  
  What good is this circuit?
  - Mainly as a buffer as it has high input impedance (M\(\Omega\)) and low output impedance (100 \(\Omega\)).
Inverting Amplifier:

- By rule #2, point A is at ground.
- By Rule #1, no current is going into the op amp.
- We can redraw the circuit as:

\[
\frac{V_{in}}{R_1} + \frac{V_{out}}{R_f} = 0
\]

\[
V_{out} / V_{in} = -\frac{R_f}{R_1}
\]

- The closed loop gain is \( R_f / R_1 \).
- The minus sign in the gain means that the output has the opposite polarity as the input.
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Non-Inverting Amplifier:

- By rule #2, point A is $V_{in}$.
- By Rule #1, no current is going into the op amp.
- We can redraw the circuit as:

$$V_{in} / R_1 + (V_{in} - V_{out}) / R_f = 0$$
$$V_{out} / V_{in} = (R_1 + R_f) / R_1$$

- The closed loop gain is $(R_1 + R_f) / R_1$.
- The output has the same polarity as the input.
Integrating Amplifier:

- Again, using the two rules for op amp circuits we redraw the circuit as:

  - The output voltage is related to the integral of the input voltage.
  - The negative sign in the gain means that $V_{in}$ and $V_{out}$ have opposite polarity.

\[
\begin{align*}
V_{in} + \frac{dQ}{dt} &= 0 \\
\frac{V_{in}}{R_1} + C \frac{dV_{out}}{dt} &= 0
\end{align*}
\]

\[V_{out} = -\frac{1}{CR_1} \int V_{in} dt\]
**Op Amps and Analog Calculations:**
- Op amps were invented before transistors to perform analog calculations.
- Their main function was to solve differential equations in real time.
- Example: Suppose we wanted to solve the following:
  \[
  \frac{d^2 x}{dt^2} = g
  \]
  - This describes a body under constant acceleration (gravity if \( g = 9.8 \text{ m/s}^2 \)).
- The following circuit gives an output which is the solution to the differential equation:
- The input voltage is a constant (= \( g \)).
- For convenience we pick \( RC = 1 \).
- At point A:
  \[
  V_A = -\int V_{in} \, dt = -\int \frac{d^2 x}{dt^2} \, dt = -\frac{dx}{dt}
  \]
  - The output voltage (\( V_{out} \)) is the integral of \( V_A \):
    \[
    V_{out} = -\int V_A \, dt = \int \frac{dx}{dt} \, dt = x(t)
    \]

If we want non-zero boundary conditions (e.g. \( V(t = 0) = 1 \text{ m/s} \)) we add a DC voltage at point A.