LAB 5

The goal of this lab is to become familiar with the concept of energy resolution and to measure the energy resolution of your NaI detector. In Lab 4, you measured the response of the NaI detector to gamma rays and saw that a gamma ray shows up as a "peak" or "bump" in the MCA (Multi Channel Analyzer) spectrum. The shape of the peak is to a very good approximation well described by a Gaussian function. The mean of this Gaussian corresponds to the average energy of the gamma ray while the width of the Gaussian tells us something about how precisely the NaI detector measures the energy of the gamma ray.

The standard definition of energy resolution (R) is:

$$R = \frac{Full\ Width\ Half\ Maximum}{Position\ of\ Peak}$$

For a Gaussian distribution the position of the peak = mean, and the full width at half maximum is related to the standard deviation by FWHM = 2.35σ.

(I) To get a feeling for the spread in gamma ray energy from a relatively long-lived isotope estimate the natural line width (ΔE) of Cs137 using the Heisenberg Uncertainty Principle:

$$\Delta E \Delta t \geq \hbar$$

with $\hbar = 6.58 \times 10^{-16} eV \cdot \sec$, Δt taken to be the half-life of Cs137, 30 years. What is your ΔE?

Using your energy calibration curve from Lab 4 roughly how many voltage bins does this energy spread cover? Could you possibly ever see this energy spread using the equipment that we have in this lab?

(II) Measure the energy resolution of your NaI detector using Co60, Na22, and Cs137.
(a) *Crude calculation of energy resolution*:
Calculate the energy resolution using the spectrum measured in Lab 5. For each of the five gamma rays estimate the peak voltage and FWHM in terms of the voltage and calculate the energy resolution (R). Make a plot of your energy resolution (y-axis) vs. the true energy (x-axis) (no error bars).

(b) *A better estimate of the energy resolution*:
Calculate the energy resolution by first converting the voltage into energy using your calibration curve from Lab 4. For each gamma ray estimate the FWHM in terms of the energy spread and using the known energy of each gamma ray calculate the energy resolution for the five gamma rays. Why is this a better way of estimating the energy resolution compared to (a), i.e. what is the potential problem in using (a)? Compare with the results of part (a) by plotting your new energy resolution on the same plot.

(c) *The ultimate calculation of the energy resolution*:

Fit your data to a function that represents the measured gamma ray spectrum. For the voltage bins near the gamma ray the spectrum can be thought to consist of two components, the gamma ray itself (our signal), which we can model with a Gaussian, and background. If there were no background then the estimated number of counts in bin i would be N_i^S:

$$N_i^S = \frac{A}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

Here x_i is either the voltage or energy. We would like to determine the constants A, μ, and σ that best describe our data. Unfortunately, we also have background in our spectrum. Let's assume that N_i^B, the number of background count in bin i, in the area of interest can be modeled by a straight line (is this reasonable?):

$$N_i^B = C + Dx_i$$

Again, the constants C and D are to be determined from our data. Now, since our data really consists of background and signal the best representation of our data is given by the sum of the signal and background functions:

$$N_i^{S+B} = C + Dx_i + \frac{A}{\sqrt{2\pi}\sigma} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

The problem now is to determine the 5 constants (A, C, D, μ, σ) from our data points. In principle this is simple, we use a technique such as Maximum Likelihood or Least Squares to find the 5 parameters. In practice this is difficult since the above function is non-linear in some of the variables, thus we would have to solve a set of 5 coupled non-linear equations! Rather than write a computer program to do this (you can if you want), you can use your plotting such as SciDAVis to fit the data with the complicated functions. You should weigh each data point using its statistical error and make sure that each statistical error is included (and visible) in the plot. Produce a table that summarizes the χ^2 and number of degree of freedom of the fits. Are the χ^2 reasonable?

To help the program to perform the fit in the right region, you must provide your best guesses of A, σ, and μ as the starting values (seeds) for the fit. A is the area of the Gaussian, i.e. the total number of events in the Gaussian. It is more difficult to guess what C and D should be but the fitting program will work with arbitrary guesses for these provided you supply a good guess for the other parameters.

Note: We only want to fit a small part of the spectrum (e.g. 50 voltage bins out of 250) to the function. Therefore it is best to copy the data in the region of interest to a new file and fit.

For each gamma ray calculate the energy resolution using the mean (μ) and standard deviation (σ) obtained from your fit. Don't forget to convert standard deviation into FWHM (you could fit for the FWHM instead of σ)! Plot the resolutions on the same plot from parts (a) and (b). Is the energy resolution constant as a function of energy?

III) Simulate, by writing a computer program, the performance of a NaI detector. In this part of the lab we would like to get a feeling for what a Co60 gamma ray spectrum would look like if the NaI detector had a percent energy resolution (R) of 3, 6, or 25%. To do this you will write a program that simulates the response of a detector with a known energy resolution. In this exercise assume that the response of the detector is given by a Gaussian function with known mean (μ) and

standard deviation (σ). When Co60 decays it gives off two distinct gamma rays (not always, but 99% of the time), a lower energy gamma with $E_l = 1.172$ MeV and a higher energy gamma ray with $E_u = 1.333$ MeV. Let's represent the response of the NaI detector as a Gaussian with $\mu = E_l$ or E_u depending on which gamma ray we are detecting. Using the definition of energy resolution (R), and defining E as the energy of the gamma ray the σ of the Gaussian is given by:

$$\sigma = \frac{E \times R}{2.35}$$

From a previous lab we know how to generate a Gaussian distribution using random numbers. Recall that the Gaussian you generated had $\mu = 0$ and $\sigma = 1$. However for this lab we want a Gaussian with $\mu = E_l$ or E_u and σ given by the energy resolution. To transform (apart from a normalization constant) from a set of numbers $\{g_1...g_n\}$ that are distributed according to a Gaussian distribution with $\mu = 0$ and $\sigma = 1$ to a set of numbers $\{G_1...G_n\}$ also Gaussian distributed but with $\mu = m$ and $\sigma = s$ use:

$$G_i = s \cdot g_i + m$$

Using the definition of a Gaussian distribution show that the above transformation really does what it claims to do! This proof is similar to the example in Lecture 4.

You can provide the proof using the equation editor in MS Word or write it on a piece of paper for attachment to the lab report. Simply showing your work on a mathematical program is not readable and hence unacceptable.

Use the following prescription to simulate on the computer the energy response of the detector to Co60. The goal here is to make a histogram similar to the one displayed by the LABVIEW program MCA.

   a) Choose an energy resolution (first do 3%, then 6%, then 25%).
   b) Use your Gaussian number generator to obtain a set of g_i's.
   c) Transform each g_i to the new Gaussian variable G_i. Remember s is the standard deviation corresponding to the chosen energy resolution and m is the true energy of the gamma ray. The G_i's now represent the measured energy of the gamma rays. For each G_i you generate you need to decide if represents the low energy or high energy gamma ray. For the low energy gamma $m = 1.172$ MeV, for the high energy gamma $m = 1.333$ MeV.
   d) Keep track of the number of measured gamma rays with energies in the interval $[E, E+\Delta E]$. The LABVIEW program uses a bin size of $\Delta E \sim 15$ KeV. I suggest using 25 bins with this bin size, the first bin starting at $E = 1.1$ MeV.

For each energy resolution generate 10^4 low energy gamma rays and 10^4 high energy gamma rays (this represents 10^4 decays of Co60). Make a histogram of your energy spectrum for each of the three energy resolutions (Superimpose all three histograms on the same plot with no error bars.) Which of the three histograms looks most like your actual Co60 spectrum? Comment on how easy/difficult it is to tell that Co60 decays to two gamma rays if the energy resolution of your detector is ~25%.