Physics 3700
Problem Set 5
Due November 13, 2023

1) Suppose our variables x and y are related by:

$$y = \alpha x + \beta x^3$$

Assume we have n measurement pairs: $(x_i, y_i \pm \sigma)$ (all y's have the same uncertainty, σ).
Use the method of Least Squares to derive formulas for the best estimate of α and β.

2) Taylor, Problem 8.14, page 202.

3) Two different experiments have measured the mass of the Ohio boson. Experiment #1 measured 1.00 ± 0.01 gm while experiment 2 measured 1.04 ± 0.02 gm.
a) What is the best estimate of the mass of the Ohio boson if we combine the two experiments?
b) Calculate the χ^2 for the two measurements in this problem using:

$$\chi^2 = \sum_{i=1}^{2} \frac{(m_i - m)^2}{\sigma_i^2}$$

with m_i the measurement from experiment i and σ_i the standard deviation of the measurement, and m the best estimate of the mass obtained by combining the two experiments.
c) How many degrees of freedom are there for this χ^2?
d) What's the probability of getting a value of χ^2 per degree of freedom $\geq$ to the one in this problem?

4) A set of n data points $(x_i, y_i \pm \sigma_i)$ are related by: $y = A + 5x$.
a) Use the method of Least Squares to show that the best estimate of the intercept, A, is given by:

$$A = \frac{\sum_{i=1}^{n} y_i / \sigma_i^2 - 5 \sum_{i=1}^{n} x_i / \sigma_i^2}{\sum_{i=1}^{n} 1/\sigma_i^2}$$

b) Use propagation of errors to show that the variance of A is given by:

$$\sigma_A^2 = \frac{1}{\sum_{i=1}^{n} 1/\sigma_i^2}$$

5) Taylor, Problem 8.24, page 205.

6) Taylor, Problem 12.7, page 280. Give the value of the constraint for problems 12.2, 12.3, 12.4.

7) Taylor, Problem 12.8, page 280.