

LAB 4

The propagation of errors is a statistical tool used for estimating the uncertainty of a quantity that is not directly measured. For a quantity represented by a function dependent on variables with known uncertainties, the uncertainties can be propagated to calculate the total uncertainty in the function.

Consider an example with a function, Q , dependent two variables, x and y ,

$$Q = Q(x, y).$$

The uncertainty of Q is given by

$$\sigma_Q^2 = \sigma_x^2 \left(\frac{dQ}{dx} \right)^2 + \sigma_y^2 \left(\frac{dQ}{dy} \right)^2 + 2\sigma_{xy} \left(\frac{dQ}{dx} \right) \left(\frac{dQ}{dy} \right).$$

If x and y are uncorrelated, the formula is simplified to

$$\sigma_Q^2 = \sigma_x^2 \left(\frac{dQ}{dx} \right)^2 + \sigma_y^2 \left(\frac{dQ}{dy} \right)^2.$$

In this experiment, we demonstrate the concept of propagation of errors by measuring the resistance of large number of resistor pairs. For each pair, three quantities are measured, R_1 and R_2 and the total resistance,

$$R_{12} = R_1 + R_2.$$

By propagation of errors, the uncertainty of R_{12} is related to the uncertainties of the individual resistances by

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2.$$

You will collect data on 200 pairs of resistors from five printed circuit boards (PCB) with each PCB containing 40 pairs. Please keep track of the PCB number written on each PCB used to avoid using the same PCB twice. Each pair of resistors is connected to a 4-pin connector. By inserting a 2-pin connector into the appropriate pair of pins, you can readily measure the three resistances. The LabView program, Error_Propagation, will record each measurement.

The experimental procedure is as follow:

1. Double click Error_Propagation to open the program. Click the right arrow on the upper left corner to start the program.
2. Plug the connector on one end of a banana cable into the red and black inputs on the upper right-hand corner of a BK Precision 2831E multimeter (Fig. 1). The red input has five

labels, including a “ Ω ” symbol for resistance measurement. The black input is labelled as “COM” for common or ground.

3. Insert the other end into the red and black connectors on a PCB (Fig. 2).
4. Turn on the power and push the white button labelled “ Ω ” to set the multimeter to measure resistance (Fig. 1).
5. Insert the connector of a 2-pin cable into the 2-pin connector next to the red and black connectors on the PCB.
6. Insert the other end of the 2-pin cable into the first two pins of the 4-pin connector to measure R_1 . Start with the resistor pair labelled R_1 and R_2 on the upper left-hand corner. If the measured resistance is $\sim 1.3 \text{ k}\Omega$, the measured value will be displayed and the button under “Measurement Good?” will turn from red to green (Fig. 3), indicating that you are ready to measure R_2 .
7. Move the 2-pin connector over by one pin to measure R_2 . The button will turn green if the resistance is $\sim 1.1 \text{ k}\Omega$.
8. Move the 2-pin connector over by one more pin to measure R_{12} . The button will turn green if the resistance is $\sim 2.4 \text{ k}\Omega$.
9. A dialog box to enter the file name will appear once you have completed 40 measurements.

Once you have completed the measurement of five PCB's, histogram R_1 , R_2 , and R_{12} and fit each distribution to a Gaussian to extract σ_1 , σ_2 , and σ_{12} . What is the χ^2 and number of degrees of freedom of each fit? Does the χ^2 indicate that the distribution is consistent with a Gaussian? Superimpose the fitted curves on the resistance distributions. Does the measured σ_{12} consistent with the expectation based on the propagation of errors?



Fig. 1: BK Precision 2831E multimeter. The connector on one end of a banana cable should be inserted into the two connectors inside the red box on the upper right-hand corner. Push the white button labelled “ Ω ” inside the red box to set the multimeter to measure resistance.

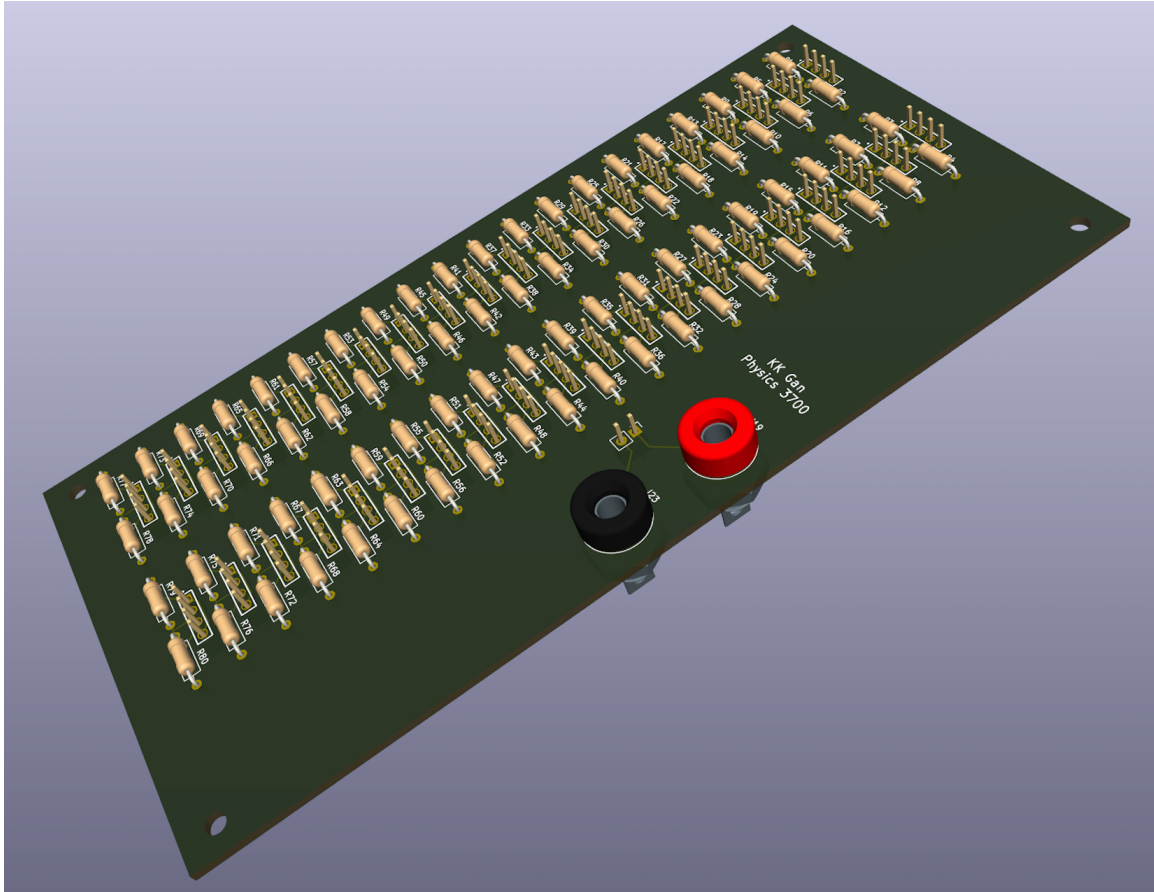


Fig. 2: A rendition of the PCB showing two columns of 20 resistor pairs. For each pair, the resistor on the left corresponds to R_1 (labelled in odd numbers, R1 to R79).

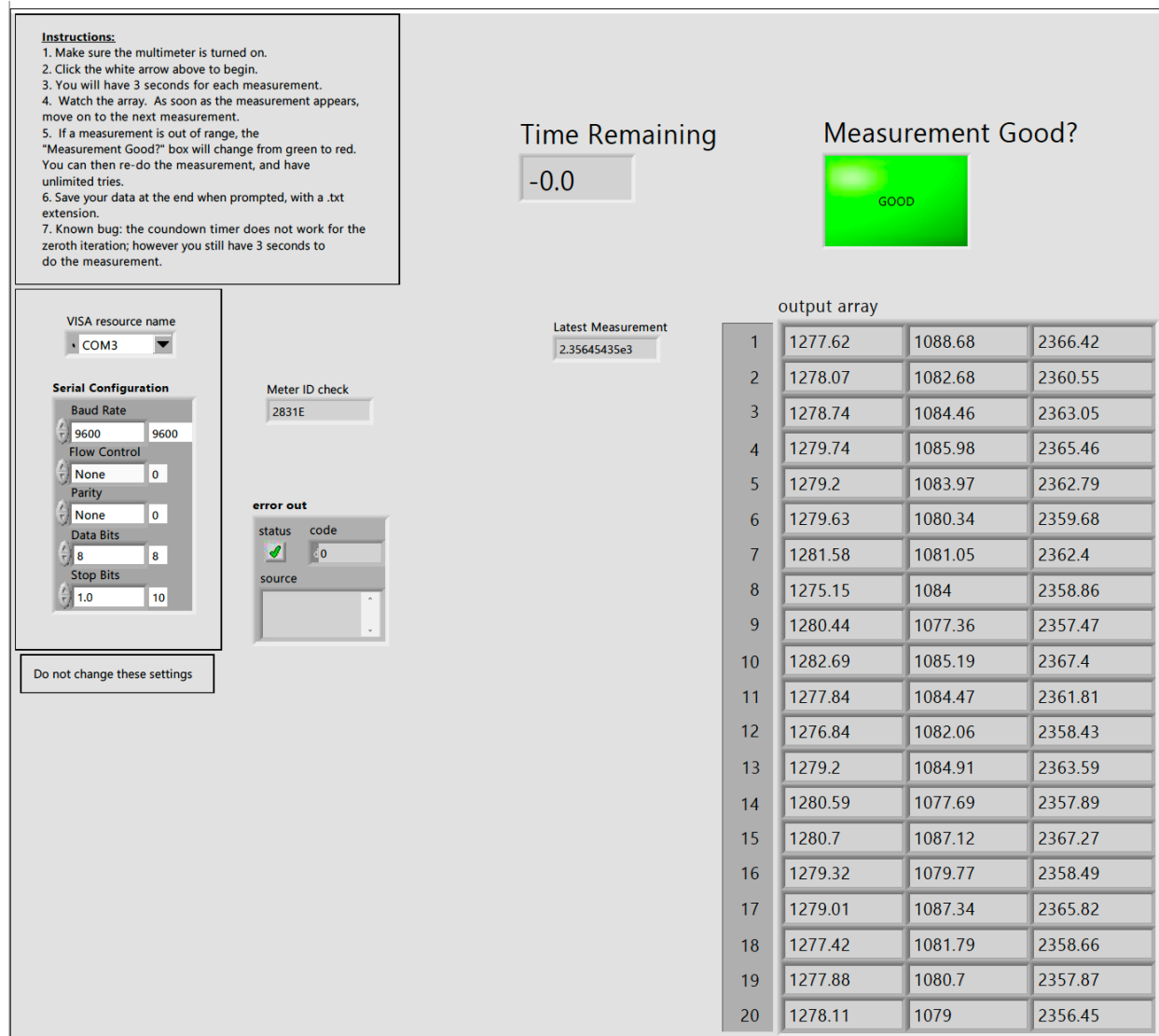


Fig. 3: A screen capture of the LabView program Error_Propagation.