

## LAB 2

### 1. Measurement of $\pi$

In this exercise we will determine a value for  $\pi$  by throwing darts:

- a) Determine  $\pi$  by throwing a dart 100 or more times. To do this first make a dart board target from an 8x11 inch sheet of paper. On this piece of paper inscribe (using pen or pencil) a circle inside of a square. How is the value of  $\pi$  related to the ratio of the circle's area to the square's area? Unless you are really an expert dart thrower, your dart tosses should be uniformly distributed throughout the square. Thus  $\pi$  can be determined from the *probability* of a dart falling within the circle. Calculate the value of your  $\pi$  and its uncertainty (please show the formula used).
- b) Determine  $\pi$  by writing a computer program (e.g. FORTRAN). One method of determining  $\pi$  using the random number generator is as follows:
  - i) Generate 2 random numbers each between [0,1]. Call one random number  $x$ , the other  $y$ .
  - ii) Assume that  $(x,y)$  represents the coordinates of a point inside a square with sides = 1. Note: the units used to measure the size of the sides of the square are arbitrary.
  - iii) Assume that there is a circle inscribed in this square with radius =  $1/2$  and calculate if  $(x,y)$  falls inside the circle.
  - iv) Repeat the above procedure keeping track of the number of  $(x,y)$  points generated (= dart tosses) and the number of times the point lies within the circle.
  - v) Once all the points have been generated calculate  $\pi$ .

Determine  $\pi$  using runs of  $4 \times 10^2$ ,  $10^3$ ,  $4 \times 10^3$ ,  $10^4$ ,  $4 \times 10^4$ ,  $10^5$ ,  $4 \times 10^5$ , and  $10^6$  points. For each case, calculate the measured  $\pi$  and its uncertainty. Plot the measured values and their uncertainties vs. the number of points (use semilog scales). Also display the result from part a) in the same plot. Draw a horizontal line indicating the true value of  $\pi$ . Comment on the precision of the measurements vs. sample size. How many measurements have their error bars touching the horizontal line? Is this consistent with the expectation that 68% of the measurements should be within one standard deviation of the expectation?

Determine  $\pi$  using 25 trials of 100 points. For each trial calculate  $\pi$  and make a histogram of your 25 trials. Calculate the average value of  $\pi$  and its standard deviation using your 25 trials. How is the uncertainty of  $\pi$  estimated from part a) compare with the standard deviation? Indicate the result you obtained from tossing the darts on the histogram. Using the histogram as a probability distribution for obtaining  $\pi$  from 100 dart tosses, comment on your expertise as a dart player.

### 2. Binomial Distribution

Take a dozen six sided dice and toss them 50 times. For each toss record the number of times (e.g.) a two comes up.

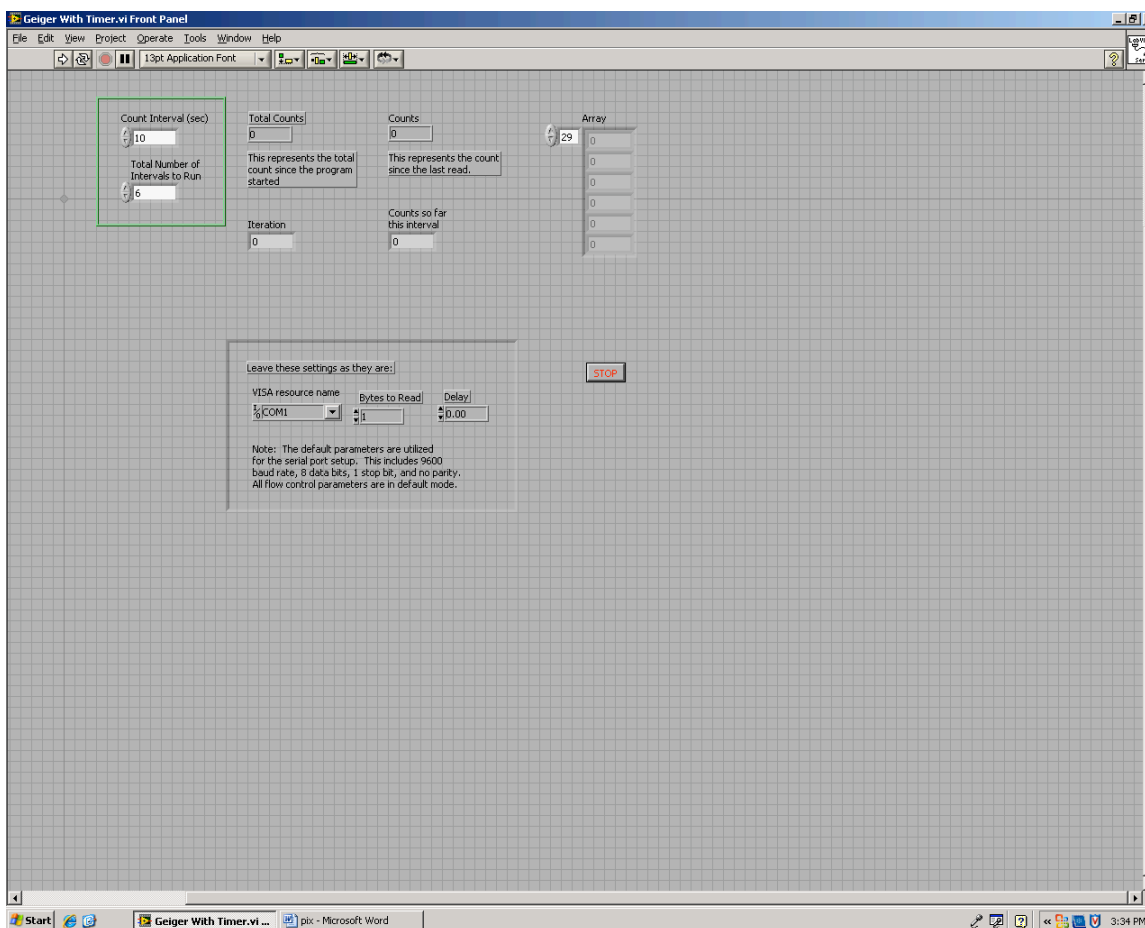
- a) Calculate the average and variance.
- b) Make a histogram (frequency distribution) of your results (i.e. the number of times there were no two's in a toss, one two, etc.).
- c) For each data point in the histogram, estimate its uncertainty, e.g. what is the uncertainty on the number of times there were one two in a toss? Display the uncertainty on the histogram using the "Error Bar" option under "Plot".

- d) Compare your results with that expected from a binomial distribution. Plot the theoretical expectations along with your experimental results. How does your measured average and variance compare with theoretical expectations?
- e) Compare your results from above with a Poisson distribution assuming the same average number of two's as above. Plot the Poisson distribution along with the binomial expectations. How does your measured average and variance compare with theoretical expectations? How would you change the dice and the experiment if you wanted to generate a Poisson distribution from the toss of dice?

### 3. Poisson Distribution

The rate of cosmic-ray particles passing through a Geiger counter is governed by Poisson statistics. The counter has a wire strung inside a chamber filled with gas. The wire (anode) is held at high voltage ( $\sim$ kV) with respect to the chamber (cathode). A traversing particle knocks electrons off a few gas molecules. The electrons accelerate toward the wire under the strong electric field, resulting in an avalanche of electrons. The electronic signal produces the well-known “click”. The signal is also being sent to an electronic device that uses the LabVIEW program, “Geiger with Timer”, to count the number of particles traversing the chamber. Turn the switch on the Geiger counter to the “Audio” position. You should hear occasional “click”, indicating that the counter is working.

- a) Double click the “Geiger with Timer” icon in C:\LabView to start the LabVIEW program. You should see the following:



b) Set the “Total Number of Intervals to Run” to 60 and the “Count Interval (sec)” to 15. This instructs the program to record the number of counts in 15-second intervals for the duration of 20 minutes. Click the “OFF” button to turn on the program and then click the “→” button at the top of the window to start the counting. After 20 minutes, the program asks you for the name of the file to store the count information. If necessary, you can stop the counting by clicking on the icon that looks like a stop sign.

c) Use the data stored in the file to histogram the number of counts per 15 seconds interval. Calculate the uncertainty of each data point and display the error on the histogram. Calculate the mean of the histogram and use it to calculate the expectations based on Poisson statistics. Superimpose the results on the histogram. Is the histogram consistent with the Poisson expectations? How many data points have zero counts and how many have 10 or more counts? Calculate the Poisson predictions. Is your observation consistent with the predictions?

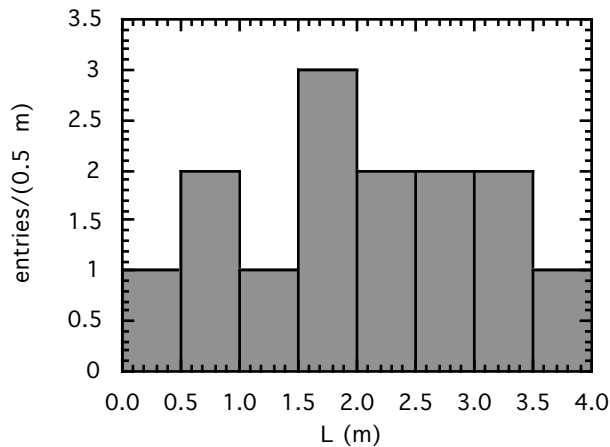
### *Note on Histogram:*

A Histogram is a very useful and convenient way of presenting data. To make a histogram, you first divide the variable you measured into some number of equal intervals (bins) and then count the number of entries in each interval. You then plot the number of entries vs. the central value of each bin.

For example, suppose we measured the length (L) of 14 snakes (in meters): 0.7, 1.3, 1.5, 0.3, 2.4, 3.1, 2.6, 1.6, 3.3, 2.2, 2.7, 3.5, 0.6, 1.7. Let's choose a bin width of 0.5 m, then you have the following number of snakes in each bin:

Length (m)	#snakes
0.0 - 0.5	1
0.5 - 1.0	2
1.0 - 1.5	1
1.5 - 2.0	3
2.0 - 2.5	2
2.5 - 3.0	2
3.0 - 3.5	2
3.5 - 4.0	1

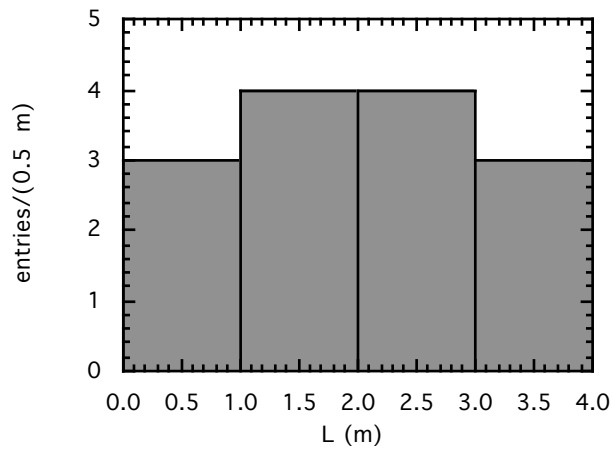
The histogram of the data looks like:



The bin width is somewhat arbitrary. For example, we could also use a bin width of 1 m:

length (m)	#snakes
0.0 - 1.0	3
1.0 - 2.0	4
2.0 - 3.0	4
3.0 - 4.0	3

Then the histogram looks like:



In general, we try to choose the bin width such that if possible each bin contains several entries. So, for this example, a bin width of 0.05m would be too small and a bin width of 5 m too large!

To make a histogram using Kaleidagraph, use the more versatile “Scatter” option under “Linear” from the “Gallery” menu, instead of the “Histogram” option.