Gan/Kass
Physics 416

Problem Set 4
Due May 5, 2009

1) The probability density function (*pdf*) for an electron in the lowest energy level (n = 1) state of a hydrogen atom, as a function of radial distance (r) from the nucleus, is given by:

$p(r) = \frac{4}{a^3} r^2 e^{-2r/a}$ with a = constant (know which one?)

a) Show that this is a properly normalized *pdf*.
b) What is the most probable radial distance (in terms of a) of the electron?
c) What is the average radial distance (in terms of a) of the electron?

2) We wish to determine the acceleration due to gravity (g) using the following data and $\mathbf{h = 0.5}\boldsymbol{g}\mathbf{t^2}$.
a) Use the least squares technique to find the best value of g. Assume the error in each h (height) measurement is 0.01 m and the time is measured exactly. (See Taylor Problem 8.5)

h (m)	t (s)
0.05	0.1
0.44	0.3
1.23	0.5
2.40	0.7

b) What is the value of the chi-square (χ^2) for this problem?
c) How many degrees of freedom are there in this problem? (See Taylor Problem 12.14, part b))
d) Estimate the probability to get a χ^2 per degree of freedom ≥ what you obtain using parts b) and c).

3) Taylor, Problem 8.14, page 202.

4) Two different experiments have measured the mass of the Ohio boson. Experiment #1 measured 1.00 ± 0.01 gm while experiment 2 measured 1.04 ± 0.02 gm.
a) What is the best estimate of the mass of the Ohio boson if we combine the two experiments?
b) Calculate the χ^2 for the two measurements in this problem using:

$$\chi^2 = \sum_{i=1}^{2} \frac{(m_i - m)^2}{\sigma_i^2}$$

with m_i the measurement from experiment i and σ_i the standard deviation of the measurement, and m the best estimate of the mass obtained by combining the two experiments.
c) How many degrees of freedom are there for this χ^2?
d) What's the probability of getting a value of χ^2 per degree of freedom ≥ to the one in this problem?

5) Taylor, Problem 12.7, page 280. Give the value of the constraint for problems 12.2,12.3, 12.4.

6) Taylor, Problem 12.8, page 280.

7) A theory states that the angular distribution of electrons from the decay of an unstable particle should have a probability distribution function of the form (both N and α are constants):

$$p(\cos\theta) = N(1 + \alpha \cos^2 \theta)$$

An experiment measures ten examples of the decay of this unstable particle and finds the following values of $\cos\theta$: (-0.05, -0.15, -0.25, -0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95). For this problem the limits on $\cos\theta$ are [-1, 1]. We wish to determine the value of α using the Maximum Likelihood Method.
a) Use the normalization condition for a probability distribution function to show that:

$$N = \frac{1}{2(1 + \alpha / 3)}$$

b) Write down the Likelihood Function for this problem.
c) Make a plot of the Likelihood Function vs. α for $-1.5 < \alpha < 1.5$. Use this plot to find the value of α that maximizes the Likelihood Function.