Gan/Kass
Physics 416

Problem Set 4
Due May 14, 2012

1) The decay of an unstable particle is described by the following probability density function in terms of the decay time (t) and the particle's lifetime (λ).

$$p(t,\lambda) = \frac{e^{-\frac{t}{\lambda}}}{\lambda}$$

Three measurements of t ($t_1 = 7$ sec, $t_2 = 3$ sec, $t_3 = 4$ sec) are made.

a) Write down the likelihood function for this problem.
b) Use the Maximum Likelihood Method to calculate the value of λ for this data set.

2) A theory states that the angular distribution of electrons from the decay of an unstable particle should have a probability distribution function of the form (both N and α are constants):

$$p(\cos\theta) = N(1 + \alpha\cos^2\theta)$$

An experiment measures ten examples of the decay of this unstable particle and finds the following values of $\cos\theta$: (-0.05, -0.15, -0.25, -0.35, 0.45, 0.55, 0.65, 0.75, 0.85, 0.95). For this problem the limits on $\cos\theta$ are [-1, 1]. We wish to determine the value of α using the Maximum Likelihood Method.

a) Use the normalization condition for a probability distribution function to show that:

$$N = \frac{1}{2(1+\alpha/3)}$$

b) Write down the Likelihood Function for this problem.
c) Make a plot of the Likelihood Function vs. α for $-1.5 < \alpha < 1.5$. Use this plot to find the value of α that maximizes the Likelihood Function.

3) We wish to determine the acceleration due to gravity (g) using the following data and $\mathbf{h = 0.5gt^2}$.

a) Use the least squares technique to find the best value of g. Assume the error in each h (height) measurement is 0.01 m and the time is measured exactly. (See Taylor Problem 8.5)

h (m)	t (s)
0.05	0.1
0.44	0.3
1.23	0.5
2.40	0.7

b) What is the value of the chi-square (χ^2) for this problem?
c) How many degrees of freedom are there in this problem? (See Taylor Problem 12.14, part b))
d) Estimate the probability to get a χ^2 per degree of freedom ≥ what you obtain using parts b) and c).

4) Taylor P8.4, page 200.

5) Taylor P8.10, page 201. Just do the first part of the problem (weighted LSQ estimate of A and B). Skip everything after "Compare…"