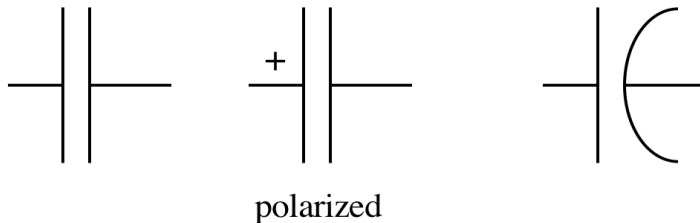


Lecture 2: Resistors and Capacitors

Capacitance:

- Capacitance (C) is defined as the ratio of charge (Q) to voltage (V) on an object:
 - ◆ $C = Q/V = \text{Coulombs/Volt} = \text{Farad}$
 - ☞ Capacitance of an object depends on geometry and its dielectric constant.
 - ◆ Symbol(s) for capacitors:

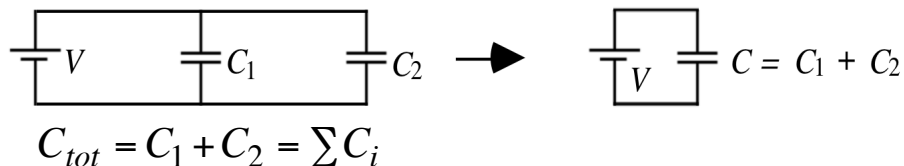


- ◆ A capacitor is a device that stores electric charge (memory devices).
- ◆ A capacitor is a device that stores energy

$$E = \frac{Q^2}{2C} = \frac{CV^2}{2}$$

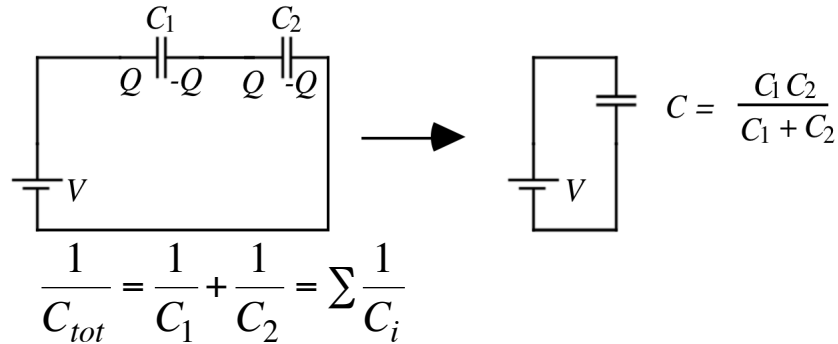
- ◆ Capacitors are easy to fabricate in small sizes (μm), use in chips

- How to combine capacitance:
 - ◆ capacitors in parallel adds like resistors in series:



Total capacitance is more than individual capacitance!

- ♦ capacitors in series add like resistors in parallel:



Total capacitance is less than individual capacitance!

- Energy and Power in Capacitors

- ♦ How much energy can a "typical" capacitor store?

- Pick a 4 μF Cap (it would read 4 mF) rated at 3 kV

$$E = \frac{1}{2} CV^2 = \frac{1}{2} 4 \times 10^{-6} \cdot 3000^2 = 18 \text{ J}$$

- This is the same as dropping a 2 kg weight (about 4 pounds) 1 meter

- ♦ How much power is dissipated in a capacitor?

$$\text{Power} = \frac{dE}{dt}$$

$$= \frac{d}{dt} \left(\frac{CV^2}{2} \right)$$

$$P = CV \frac{dV}{dt}$$

- dV/dt must be finite otherwise we source (or sink) an infinite amount of power!

THIS WOULD BE UNPHYSICAL.

- the voltage across a capacitor cannot change instantaneously
 - ☞ a useful fact when trying to guess the transient (short term) behavior of a circuit
- the voltage across a resistor can change instantaneously
 - ☐ the power dissipated in a resistor does not depend on dV/dt :

$$P = I^2 R \text{ or } V^2 / R$$

◆ Why do capacitors come in such small values?

- Example: Calculate the size of a 1 Farad parallel capacitor with air between the plates.

$$C = \frac{k\epsilon_0 A}{d}$$

k = dielectric constant (= 1 for air)

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ m}^{-2}$$

d = distance between plates (assumed 1 mm)

$$A = \text{area of plates} = 1.1 \times 10^8 \text{ m}^2 !!!$$

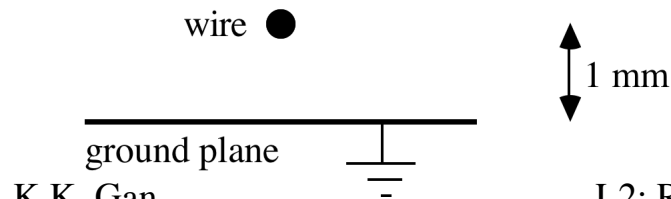
☞ square plate of 6.5 miles per side

☐ breakthroughs in capacitor technologies (driven by laptop/cell phone industries)

☞ 10 μF capacitor in 0402 package (1.0 mm \times 0.5 mm \times 0.5 mm) costs only 3 pennies

◆ How small can we make capacitors?

- A wire near a ground plane has $C \sim 0.1 \text{ pf} = 10^{-13} \text{ F}$.



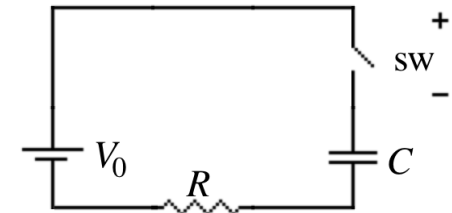
“Stray capacitance” slow down signal or induces cross talk by capacitive coupling!

- ◆ Some words to the wise on capacitors and their labeling.
 - Typical capacitors are multiples of micro Farads (10^{-6} F) or picoFarads (10^{-12} F).
 - ☞ Whenever you see mF it almost always is micro, *not* milli F and *never* mega F.
 - ☞ picoFarad (10^{-12} F) is sometimes written as pf and pronounced *puff*.
 - no *single* convention for labeling capacitors
 - ☐ Many manufacturers have their own labeling scheme (See Horowitz and Hill lab manual).

● Resistors and Capacitors

- ◆ Examine voltage and current vs. time for a circuit with one R and one C .
 - Assume that at $t < 0$ all voltages are zero, $V_R = V_C = 0$.
 - At $t \geq 0$ the switch is closed and the battery (V_0) is connected.
 - Apply Kirchhoff's voltage rule:

$$\begin{aligned}
 V_0 &= V_R + V_C \\
 &= IR + \frac{Q}{C} \\
 &= R \frac{dQ}{dt} + \frac{Q}{C}
 \end{aligned}$$



- Solve the differential equation by differentiating both sides of above equation:

$$\begin{aligned}
 \frac{dV_0}{dt} &= \frac{1}{C} \frac{dQ}{dt} + R \frac{d^2Q}{dt^2} \\
 0 &= \frac{I}{C} + R \frac{dI}{dt} \\
 \frac{dI}{dt} &= -\frac{I}{RC}
 \end{aligned}$$

- This is just an exponential decay equation with time constant RC (sec).
- The current as a function of time through the resistor and capacitor is:

$$I(t) = I_0 e^{-t/RC}$$

◆ What's $V_R(t)$?

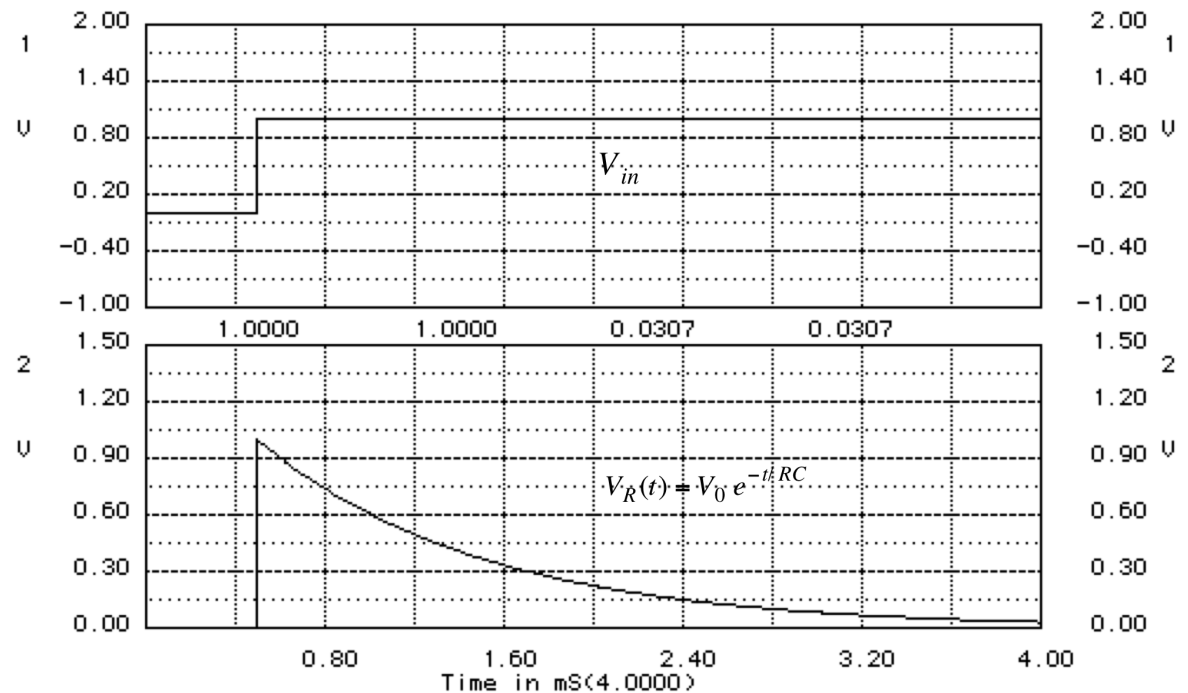
- By Ohm's law:

$$V_R(t) = I_R \cdot R$$

$$= I_0 R e^{-t/RC}$$

$$= V_0 e^{-t/RC}$$

- At $t = 0$ all the voltage appears across the resistor, $V_R(0) = V_0$.
- At $t = \infty$, $V_R(\infty) = 0$.



- ◆ What's $V_C(t)$?
 - Easiest way to answer is to use the fact that $V_0 = V_R + V_C$ is valid for all t .

$$V_C = V_0 - V_R$$

$$V_C = V_0(1 - e^{-t/RC})$$
 - ☞ At $t = 0$ all the voltage appears across the resistor so $V_C(0) = 0$.
 - ☞ At $t = \infty$, $V_C(\infty) = V_0$.

- ◆ Suppose we wait until $I = 0$ and then short out the battery.

$$0 = V_R + V_C$$

$$V_R = -V_C$$

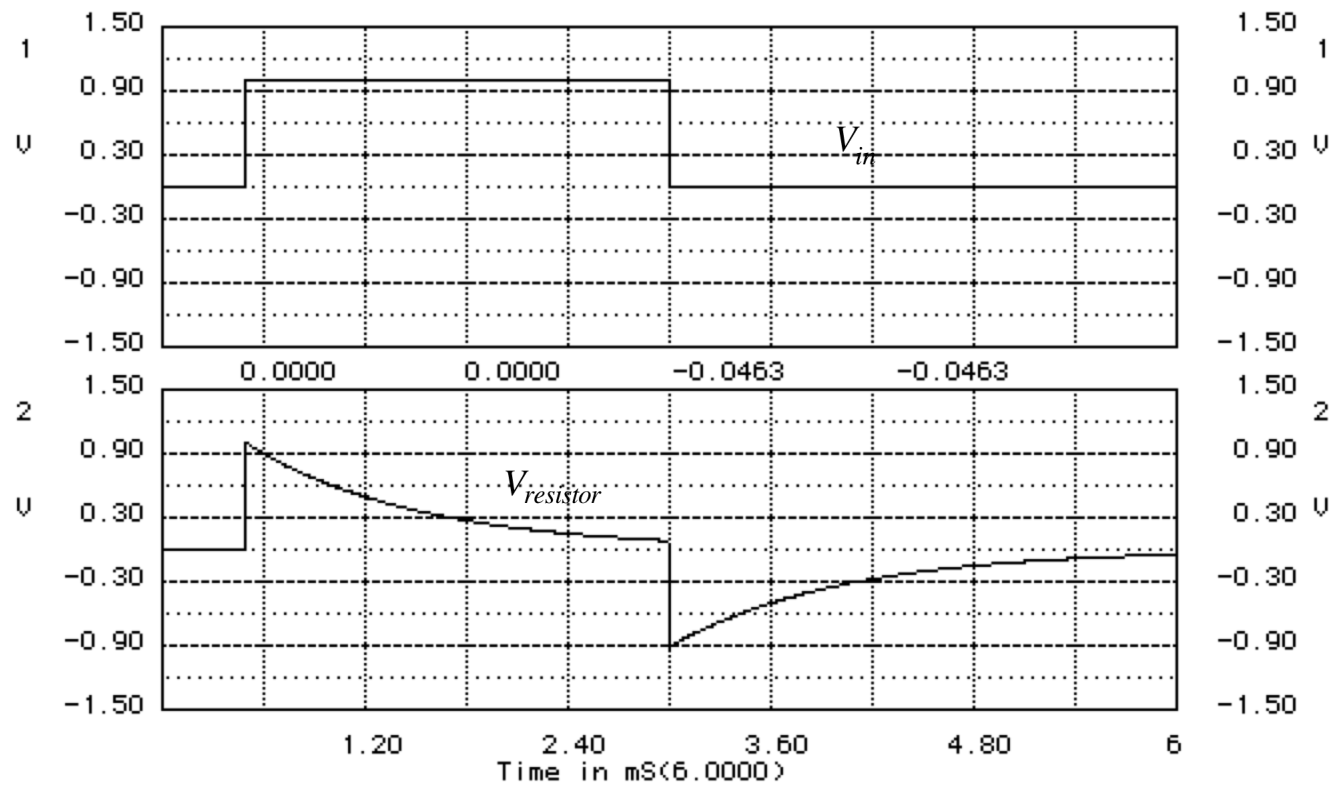
$$R \frac{dQ}{dt} = -\frac{Q}{C}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$
 - Solving the exponential equation yields,

$$Q(t) = Q_0 e^{-t/RC}$$
 - We can find V_C using $V = Q/C$,

$$V_C(t) = V_0 e^{-t/RC}$$
 - Finally we can find the voltage across the resistor using $V_R = -V_C$,

$$V_R(t) = -V_0 e^{-t/RC}$$



- ◆ Suppose $V_C(t) = V_0 \sin \omega t$ instead of DC

👉 What happens to V_C and I_C ?

$$Q(t) = CV(t)$$

$$= CV_0 \sin \omega t$$

$$I_C = dQ/dt$$

$$= \omega CV_0 \cos \omega t$$

$$= \omega CV_0 \sin(\omega t + \pi/2)$$

👉 **current in capacitor varies like a sine wave too, but 90° out of phase with voltage.**

- We can write an equation that looks like Ohm's law by defining V^* :

$$V^* = V_0 \sin(\omega t + \pi/2)$$
- ☞ the relationship between the voltage and current in C looks like:


$$V^* = I_C / \omega C$$

$$= I_C R^*$$
- ☞ $1/\omega C$ can be identified as a kind of resistance, capacitive reactance:

$$X_C \equiv 1/\omega C \text{ (Ohms)}$$
- $X_C = 0$ for $\omega = \infty$
 - ☞ high frequencies: a capacitor looks like a short circuit
 - ☞ “bypass capacitor” on power supply “short circuit” ripple (noise) to ground
- $X_C = \infty$ for $\omega = 0$
 - ☞ low frequencies: a capacitor looks like an open circuit (high resistance).
 - ☞ look at the AC output of a circuit via a capacitor

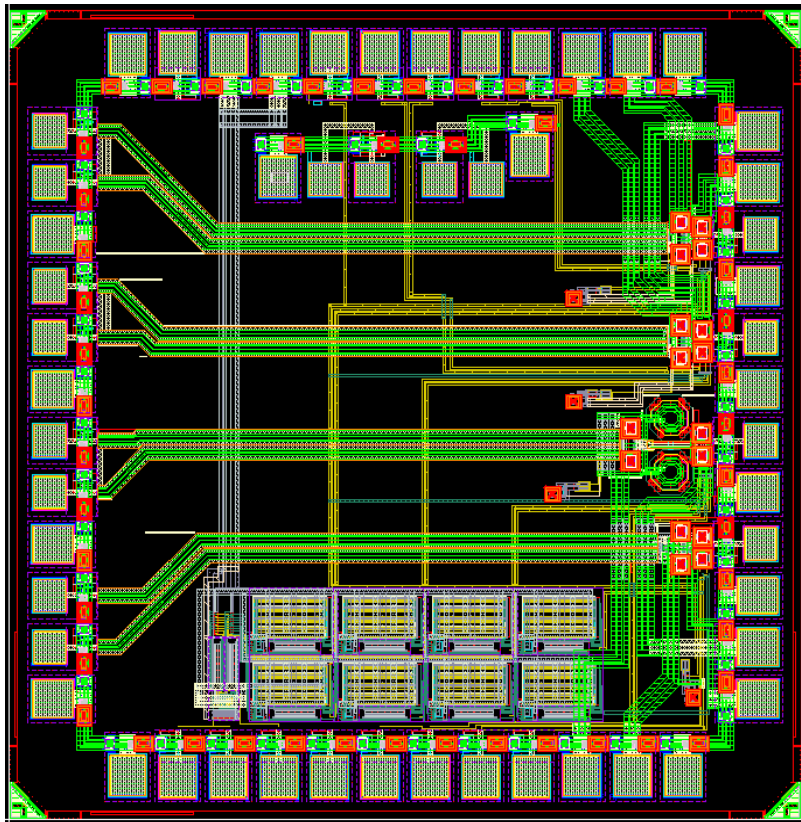
Inductance:

- Define inductance by: $V = L di/dt$
 - ◆ Unit: Henry
 - ◆ Symbol:

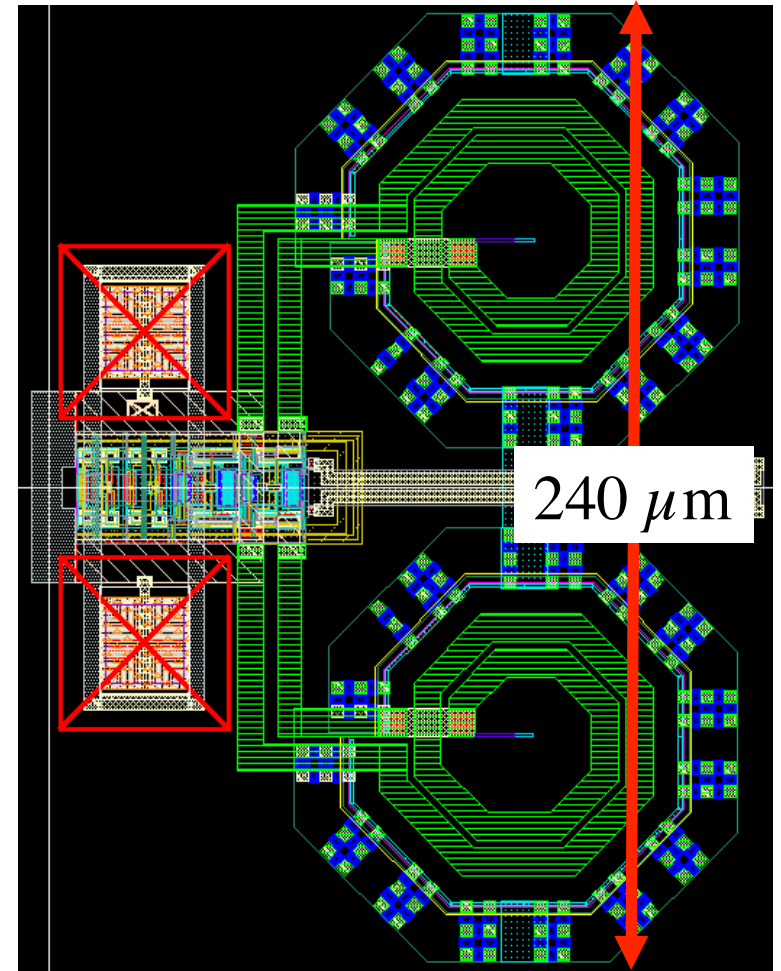

 - ◆ Inductors are usually made from a coil of wire
 - tend to be bulky and are hard to fabricate in small sizes (μm), seldom used in chips.
 - ◆ Two inductors next to each other (transformer) can step up or down a voltage
 - no change in the frequency of the voltage
 - provide isolation from the rest of the circuit

10 Gb/s VCSEL (Laser) Array Driver

- 4-channel test chip (65 nm CMOS)



2 mm



- Capacitance and inductance on long transmission lines degrade signal

L2: Resistors and Capacitors

- How much energy is stored in an inductor?

$$dE = VdQ$$

$$I = \frac{dQ}{dt}$$

$$dE = VI dt$$

$$V = L \frac{dI}{dt}$$

$$dE = LI dI$$

$$E = L \int_0^I IdI$$

$$E = \frac{1}{2} LI^2$$

- How much power is dissipated in an inductor?

$$Power = \frac{dE}{dt}$$

$$= \frac{d}{dt} \left(\frac{LI^2}{2} \right)$$

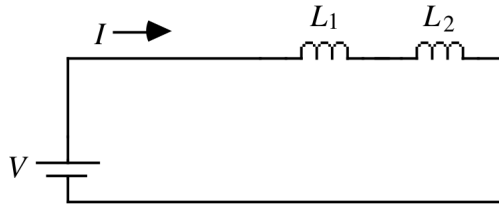
$$P = LI \frac{dI}{dt}$$

- dI/dt must be finite as we can't source (or sink) an infinite amount of power in an inductor!

THIS WOULD BE UNPHYSICAL.

- the *current* across an inductor cannot change *instantaneously*.

- ◆ Two inductors in series:



- Apply Kirchhoff's Laws,

$$V = V_1 + V_2$$

$$= L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$

$$\equiv L_{tot} \frac{dI}{dt}$$

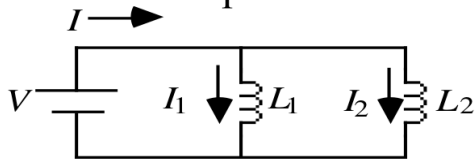
$$L_{tot} = L_1 + L_2$$

$$= \sum L_i$$

- 👉 Inductors in series add like resistors in series.

The *total* inductance is *greater* than the individual inductances.

- Two inductors in parallel:



- Since the inductors are in parallel,

$$V_1 = V_2 = V$$

- The total current in the circuit is

$$I = I_1 + I_2$$

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$= \frac{V}{L_1} + \frac{V}{L_2}$$

$$\equiv \frac{V}{L_{tot}}$$

$$\frac{1}{L_{tot}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$L_{tot} = \frac{L_1 L_2}{L_1 + L_2}$$

- If we have more than 2 inductors in parallel, they combine like:

$$\frac{1}{L_{tot}} = \sum \frac{1}{L_i}$$

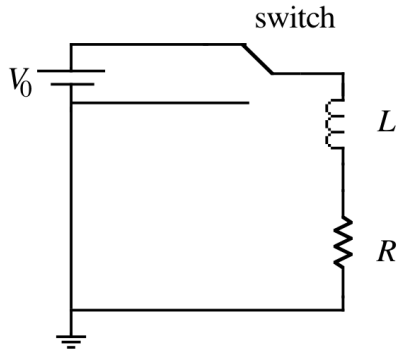
- Inductors in parallel add like resistors in parallel.

The *total* inductance is *less* than the individual inductances.

- Resistors and Inductors

- ◆ Examine voltage and current versus time for a circuit with one R and one L .

- Assume that at $t < 0$ all voltages are zero, $V_R = V_L = 0$.
- At $t \geq 0$ the switch is closed and the battery (V_0) is connected.



- Like the capacitor case, apply Kirchhoff's voltage rule:

$$\begin{aligned} V_0 &= V_R + V_L \\ &= IR + L \frac{dI}{dt} \end{aligned}$$

- Solving the differential equation, assuming at $t = 0$, $I = 0$:

$$I(t) = \frac{V_0}{R} \left(1 - e^{-tR/L} \right)$$

☞ This is just an exponential decay equation with time constant L/R (seconds).

- ◆ What's $V_R(t)$?

- By Ohm's law $V_R = I_R R$ at any time:

$$V_R = I(t)R = V_0 \left(1 - e^{-tR/L} \right)$$

- At $t = 0$, none of the voltage appears across the resistor, $V_R(0) = 0$.
- At $t = \infty$, $V_R(\infty) = V_0$.

◆ What's $V_L(t)$?

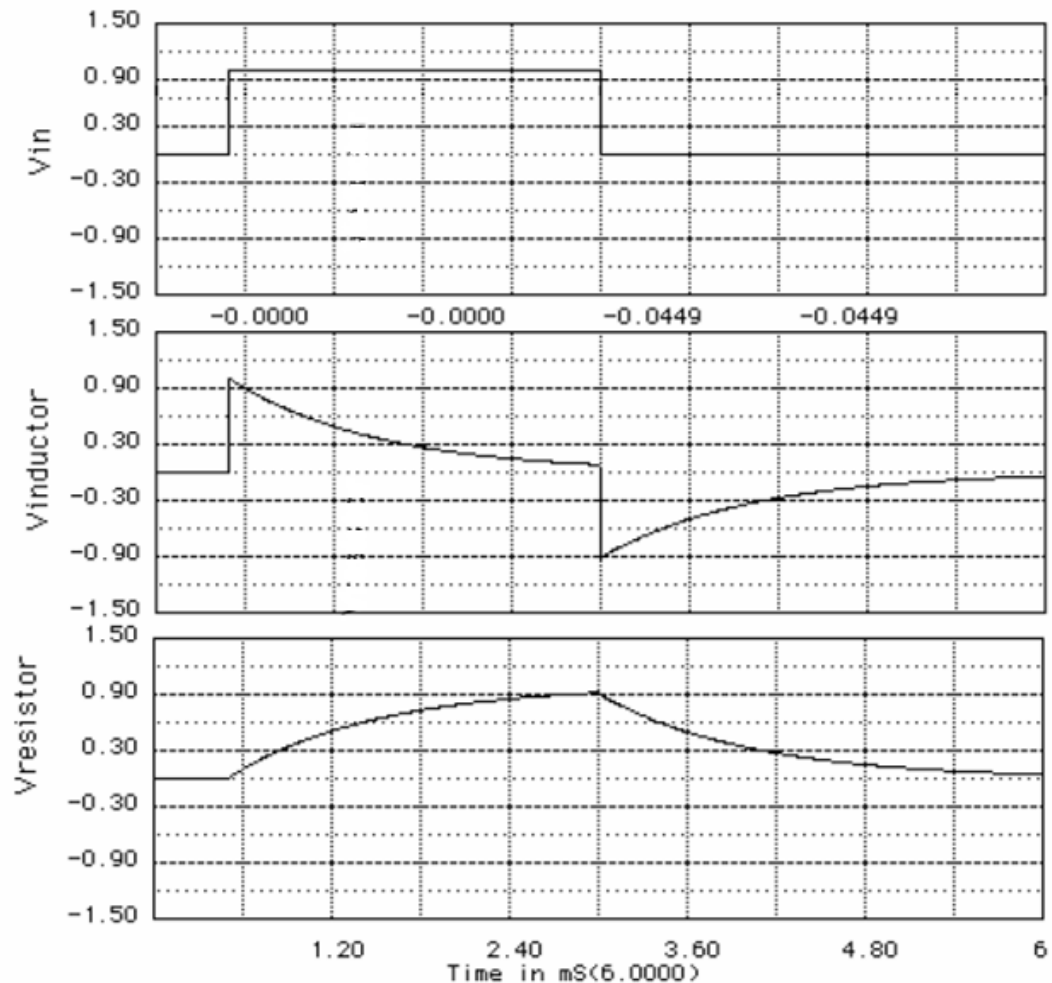
- Easiest way to answer is to use the fact that $V_0 = V_R + V_L$ is valid for all t .

$$V_L = V_0 - V_R$$

$$V_L(t) = V_0 e^{-tR/L}$$

- At $t = 0$, all the voltage appears across the inductor so $V_L(0) = V_0$.
- At $t = \infty$, $V_L(\infty) = 0$.

Pick $L/R = 1$ ms



- ◆ Suppose $V_C(t) = V_0 \sin \omega t$ instead of DC, what happens to V_L and I_L ?

$$V = L \frac{dI_L}{dt}$$

$$I_L = \frac{1}{L} \int_0^t V dt$$

$$= -\frac{V_0}{\omega L} \cos \omega t$$

$$I_L(t) = \frac{V_0}{\omega L} \sin(\omega t - \pi/2)$$

☞ The current in an inductor varies like a sine wave too, but 90° out of phase with the voltage.

- We can write an equation that looks like Ohm's law by defining V^* :

$$V^* = V_0 \sin(\omega t - \pi/2)$$

☞ $V^* = I_L \omega L = I_L R^*$

- ωL can be identified as a kind of resistance, inductive reactance:

$$X_L \equiv \omega L \text{ (Ohms)}$$

- $X_L = 0$ if $\omega = 0$

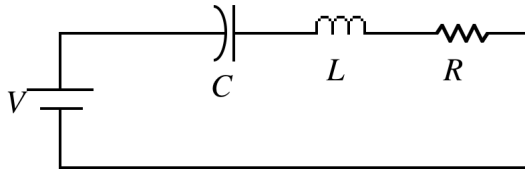
☞ low frequencies: an inductor looks like a short circuit (low resistance).

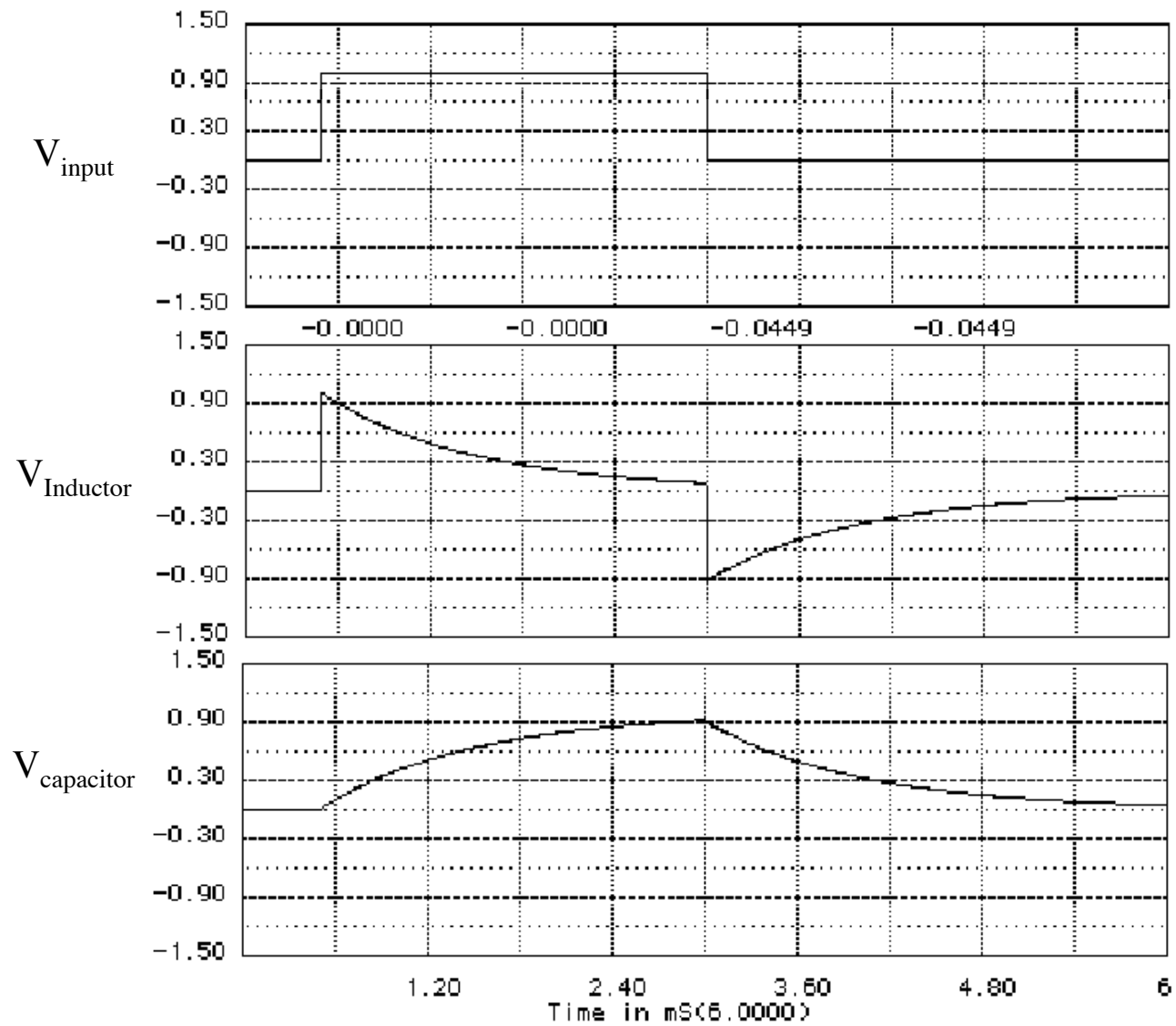
- $X_L = \infty$ if $\omega = \infty$

☞ high frequencies: an inductor looks like an open circuit.

☞ connect a circuit to the power supply via an inductor to filter out ripple (noise).

- Some things to remember about R , L , and C 's.
 - ◆ For DC circuits, after many time constants (L/R or RC):
 - ☞ Inductor acts like a wire ($0\ \Omega$).
 - ☞ Capacitor acts like an open circuit ($\infty\ \Omega$).
 - ◆ For circuits where the voltage changes very rapidly or transient behavior:
 - ☞ Capacitor acts like a wire ($0\ \Omega$).
 - ☞ Inductor acts like an open circuit ($\infty\ \Omega$).
 - ◆ Example, RLC circuit with DC supply:
 - At $t = 0$, voltages on R , C are zero and $V_L = V_0$.
 - At $t = \infty$, voltages on R , L are zero and $V_C = V_0$.





L2: Resistors and Capacitors