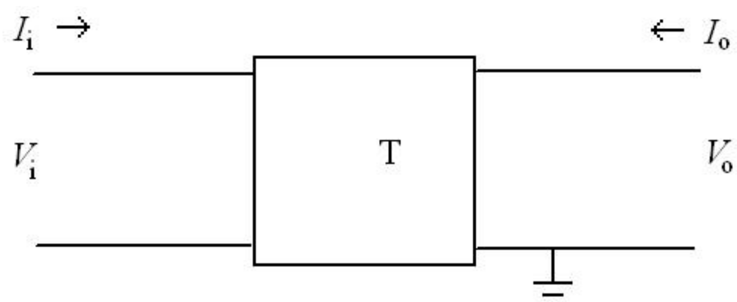
Lecture 7: Transistors and Amplifiers

Hybrid Transistor Model for small AC :

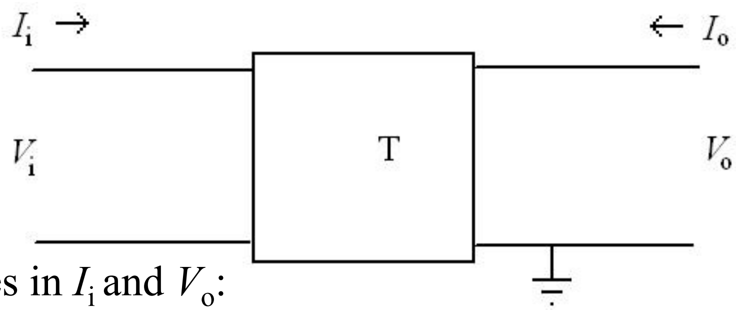
- The previous model for a transistor used one parameter (β, the current gain) to describe the transistor.
  - doesn't explain many features of three common forms of transistor amplifiers (common emitter etc.)
  - e.g. could not calculate the output impedance of the common emitter amp.

- Very often in electronics we describe complex circuits in terms of an equivalent circuit or model.
  - need a model that relates the input currents and voltages to the output currents and voltages.
  - the model needs to be linear in the currents and voltages.
    - For a transistor this condition of linearity is true for *small* signals.

- The most general linear model of the transistor is a 4-terminal "black box".



  - *In this model we assume the transistor is biased on properly and do not show the biasing circuit*.
  - Since a transistor has only 3 legs, one of the terminals is common between the input and output.
  - There are 4 variables in the problem, I_i, V_i, I_o, and V_o.
    - The subscript i refer to the input side while the subscript o refers to the output side.
    - We *assume* that we know I_i and V_o.



- Kirchhoff's laws relate all the currents and voltages:

$$V_i = V_i(I_i, V_o)$$

$$I_o = I_o(I_i, V_o)$$

- For a linear model of the transistor with a small changes in I_i and V_o:

$$dV_i = \left(\frac{\partial V_i}{\partial I_i}\right)_{V_o} dI_i + \left(\frac{\partial V_i}{\partial V_o}\right)_{I_i} dV_o$$

$$dI_o = \left(\frac{\partial I_o}{\partial I_i}\right)_{V_o} dI_i + \left(\frac{\partial I_o}{\partial V_o}\right)_{I_i} dV_o$$

  - The partial derivatives are called the hybrid (or *h*) parameters:

$$dV_i = h_{ii} dI_i + h_{io} dV_o$$

$$dI_o = h_{oi} dI_i + h_{oo} dV_o$$

    - h_{oi} and h_{io} are unitless
    - h_{oo} has units 1/Ω (mhos)
    - h_{ii} has units Ω
  - The four *h* parameters are easily measured.
    - e.g. to measure h_{ii} hold V_o (the output voltage) constant and measure V_{in}/I_{in}.
  - Unfortunately the *h* parameters are not constant.
    - e.g. Figs. 11-14 of the 2N3904 spec sheet show the variation of the parameters with I_C.

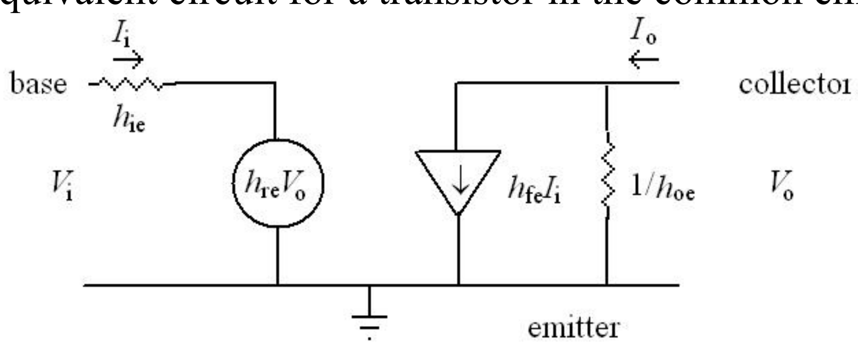
- There are 3 sets of the 4 hybrid parameters.
  - One for each type of amp: common emitter, common base, common collector
  - In order to differentiate one set of parameters from another the following notation is used:

First subscript	**Second subscript**
i = input impedance	e = common emitter
o = output admittance	b = common base
r = reverse voltage ratio	c = common collector
f = forward current ratio	

  ☞ For a common emitter amplifier we would write:

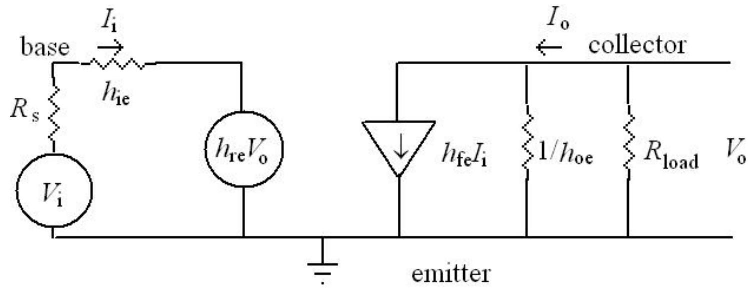
$$dV_i = h_{ie}dI_i + h_{re}dV_o$$
$$dI_o = h_{fe}dI_i + h_{oe}dV_o$$

  - Typical values for the h parameters for a 2N3904 transistor in the common emitter configuration: $h_{fe} = 120$, $h_{oe} = 8.7\times10^{-6}\ \Omega^{-1}$, $h_{ie} = 3700\ \Omega$, $h_{re} = 1.3\times10^{-4}$ for $I_C = 1$ mA
  - The equivalent circuit for a transistor in the common emitter configuration looks like:



  - Circle: voltage source
    ☞ the voltage across this element is always equal to $h_{re}V_o$ independent of the current through it.
  - Triangle: current source
    ☞ the current through this element is always $h_{fe}I_{in}$ independent of the voltage across the device.

- We can use the model to calculate voltage/current gain and the input/output impedance of a CE amp.
- Equivalent circuit for a CE amp with a voltage source (with resistance R_s) and load resistor (R_{load}):



biasing network not shown

- **Current gain**: $G_I = I_o/I_{in}$
  - Using Kirchhoff's current law at the output side we have:

$$h_{fe}I_{in} + V_o h_{oe} = I_o$$

  - Using Kirchhoff's voltage rule at the output we have:

$$V_o = -I_o R_{load}$$

$$h_{fe}I_{in} = h_{oe}I_o R_{load} + I_o$$

$$G_I = I_o / I_{in} = h_{fe} / (1 + h_{oe} R_{load})$$

  - For typical CE amps, $h_{oe}R_{load} << 1$ and the gain reduces to familiar form:

$$G_I \approx h_{fe} = \beta$$

- **Voltage gain**: $G_v = V_o/V_{in}$
  - This gain can be derived in a similar fashion as the current gain:

$$G_V = V_o / V_{in} = -h_{fe} R_{load} / (\Delta R_{load} + h_{ie})$$

with $\Delta = h_{ie}h_{oe} - h_{fe}h_{re} \approx 10^{-2}$

  - This reduces to a familiar form for most cases where $\Delta R_{load} << h_{ie}$

$$G_V = -h_{fe} R_{load} / h_{ie} = -R_{load} / r_{BE}$$

- **Input Impedance**: $Z_i = V_{in}/I_{in}$

  $$Z_i = (\Delta R_{load} + h_{ie})/(1 + h_{oe} R_{load})$$

  - This reduces to a familiar form for most cases where $\Delta R_{load} << h_{ie}$ and $h_{oe} R_{load} << 1$

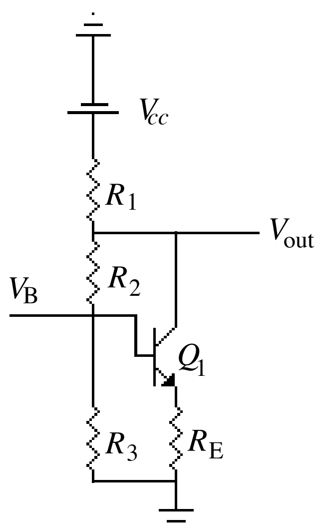
    $Z_i = h_{ie} = h_{fe} r_{BE}$
- **Output Impedance**: $Z_o = V_o/I_o$

  $$Z_o = (R_s + h_{ie})/(\Delta + h_{oe} R_s)$$

  - Z_o does not reduce to a simple expression.
  - As the denominator is small, Z_o is as advertised large.

Feedback and Amplifiers

- Consider the common emitter amplifier shown.
  - This amp differs slightly from the CE amp we saw before:
    - bias resistor R_2 is connected to collector resistor R_1 instead of directly to V_{cc}.
  - How does this effect V_{out}?
    - If V_{out} *decreases* (moves away from V_{cc})
      - I_2 increases
      - V_B decreases (gets closer to ground)
      - V_{out} will increase since $\Delta V_{out} = -\Delta V_B R_1/R_E$
    - If V_{out} *increases* (moves towards V_{cc})
      - I_2 decreases
      - V_B increases (moves away from ground).
      - V_{out} will decrease since $\Delta V_{out} = -\Delta V_B R_1/R_E$



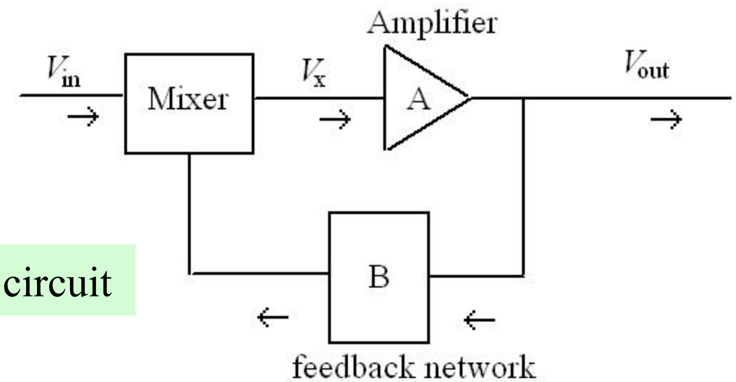
This is an example of NEGATIVE FEEDBACK

- Negative Feedback is good:
  - Stabilizes amplifier against oscillation
  - Increases the input impedance of the amplifier
  - Decreases the output impedance of the amplifier
- Positive Feedback is bad:
  - Causes amplifiers to oscillate

Oscillation is a large fluctuation of output signal with no input

- **Feedback Fundamentals**:

general feedback circuit



- Without feedback the output and input are related by:

$$V_{out} = AV_{in}$$

- The feedback (box B) returns a portion of the output voltage to the amplifier through the "mixer".
  - The feedback network on the AM radio is the collector to base resistors (R_3, R_5)
- The input to the amplifier is:

$$V_x = V_{in} + BV_{out}$$

- The gain with feedback is:

$$V_{out} = AV_x = A(V_{in} + BV_{out})$$

$$G = V_{out} / V_{in} = A/(1 - AB)$$

A: open loop gain
AB: loop gain
G: closed loop gain

- Positive and negative feedback:
  - Lets define $A > 0$ (positive)

    $$G = V_{out}/V_{in} = A/(1 - AB)$$
  - Positive feedback, $AB > 0$:
    - As $AB \to 1$, $G \to \infty$.
      - circuit is unstable
      - oscillates if $AB = 1$
  - Negative feedback, $AB < 0$:
    - As $A \to \infty$, an amazing thing happens:

      $|AB| \to \infty$

      $|G| \to |1/B|$

For large amplifier gain (A) the circuit properties are determined by the feedback loop.

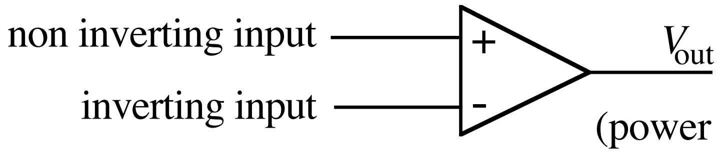
    - Example: $A = 10^5$ and $B = -0.01$ then $G = 100$.
    - The stability of the gain is determined by the feedback loop (B) and not the amplifier (A).
    - Example: B is held fixed at 0.01 and A varies:

A	Gain
5×10^3	98.3
1×10^4	99.0
2×10^4	99.6

☞ circuits can be made stable with respect to variations in the transistor characteristics as long as B is stable.
- B can be made from precision components such as resistors.

Operational Amplifiers (Op Amps)

- Op amps are very high gain ($A = 10^5$) differential amplifiers.
  - Differential amp has two inputs (V_1, V_2) and output $V_{out} = A\,(V_1 - V_2)$ where A is the amplifier gain.



(power connections not shown)

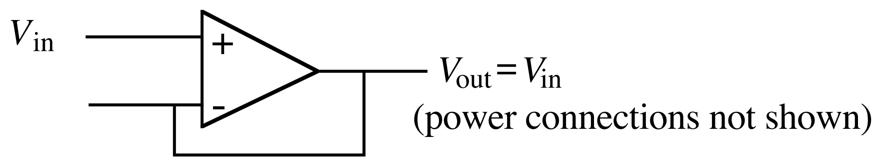
  - If an op amp is used without feedback and $V_1 \neq V_2$
    - V_{out} saturates at the power supply voltage (either positive or negative supply).
  - Example: Assume the maximum output swing for an op amp is ±15 V.
    - If there is no feedback in the circuit:
      - $V_{out} = 15$ V if $V_{non\text{-}invert} > V_{invert}$
      - $V_{out} = -15$ V if $V_{non\text{-}invert} < V_{invert}$
- Op amps are almost always used with negative feedback.
  - The output is connected to the (inverting) input.
- Op amps come in "chip" form. They are made up of complex circuits with 20-100 transistors.

	Ideal Op Amp	**Real Op Amp** μA741
Voltage gain (open loop)	∞	10^5
Input impedance	∞	2 MΩ
Output impedance	0	75 Ω
Slew rate	∞	0.5 V/μsec
Power consumption	0	50 mW
V_{out} with $V_{in} = 0$	0	2 mV (unity gain)
Price	0$	$0.25

Slew rate is how fast output can change

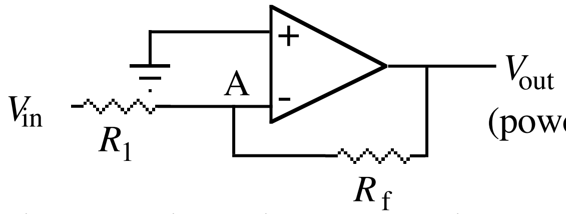
- When working with op amps using negative feedback two simple rules (almost) always apply:
  - No current goes into the op amp.
    - This reflects the high input impedance of the op amp.
  - Both input terminals of the op amp have the same voltage.
    - This has to do with the actual circuitry making up the op amp.

- Some examples of op amp circuits with negative feedback:
  - **Voltage Follower:**



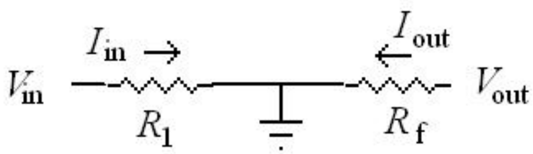
    - The feedback network is just a wire connecting the output to the input.
    - By rule #2, the inverting (-) input is also at V_{in}.
      - ☞ $V_{out} = V_{in}$.
    - What good is this circuit?
      - Mainly as a buffer as it has high input impedance (MΩ) and low output impedance (100 Ω).

- **Inverting Amplifier:**



(power connections not shown)

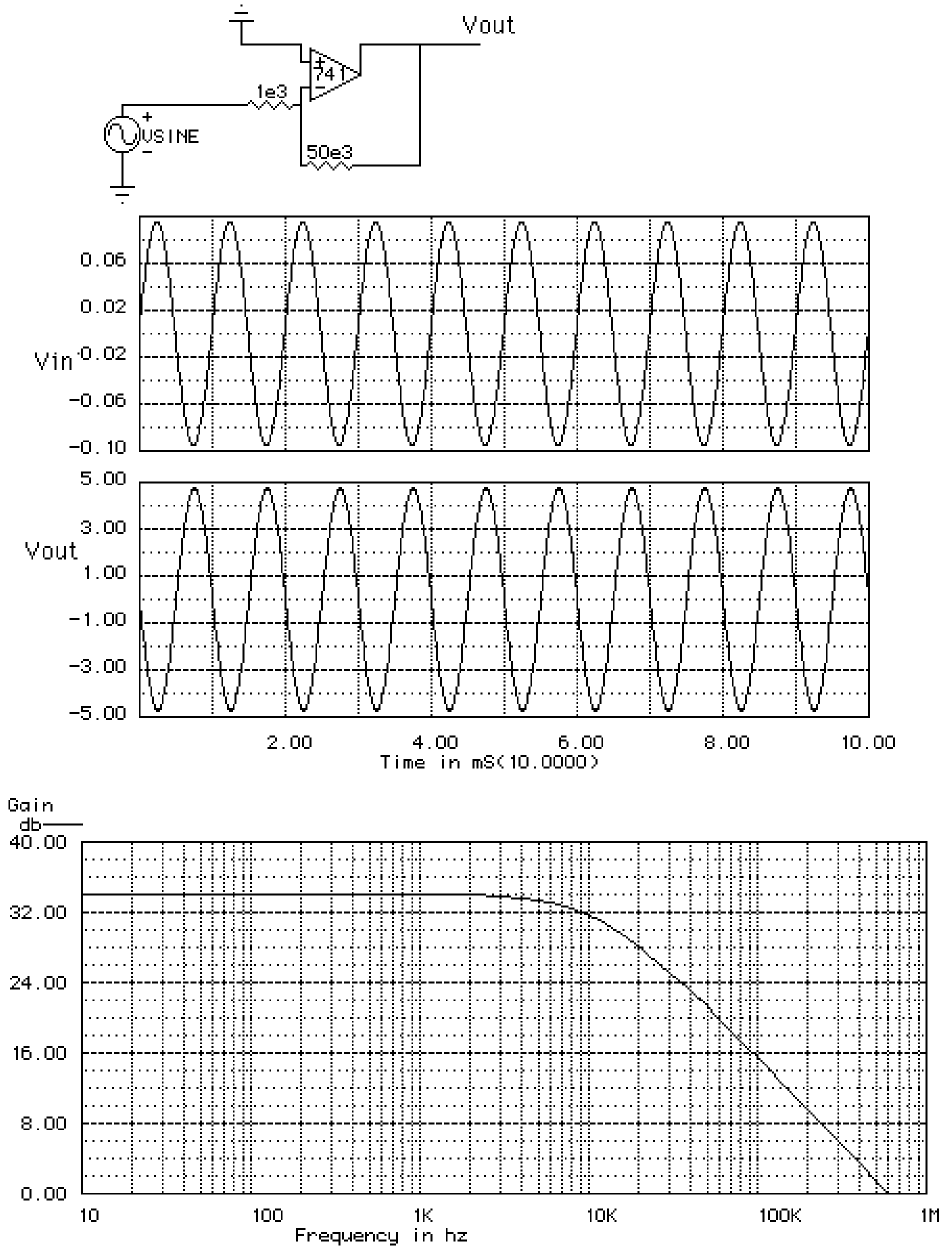
- By rule #2, point A is at ground.
- By Rule #1, no current is going into the op amp.
- We can redraw the circuit as:



$$V_{in} / R_1 + V_{out} / R_f = 0$$

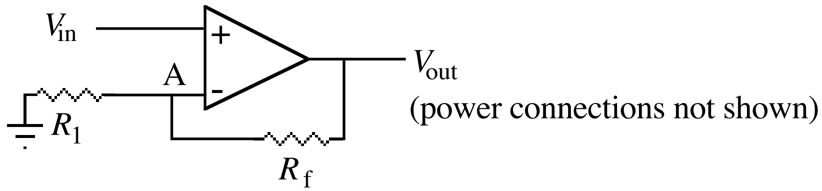
$$V_{out} / V_{in} = -R_f / R_1$$

- ☞ The closed loop gain is R_f/R_1.
- ❑ The minus sign in the gain means that the output has the opposite polarity as the input.

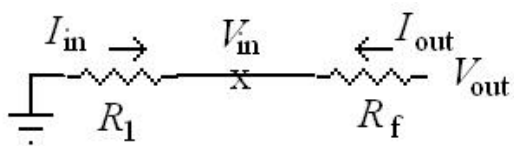


Vout
741
1e3
50e3
VSINE
Vin
0.06
0.02
-0.02
-0.06
-0.10
Vout
5.00
3.00
1.00
-1.00
-3.00
-5.00
2.00
4.00
6.00
8.00
10.00
Time in mS(10.0000)
Gain
db
40.00
32.00
24.00
16.00
8.00
0.00
10
100
1K
10K
100K
1M
Frequency in hz

- **Non-Inverting Amplifier:**



  - By rule #2, point A is V_{in}.
  - By Rule #1, no current is going into the op amp.
  - We can redraw the circuit as:

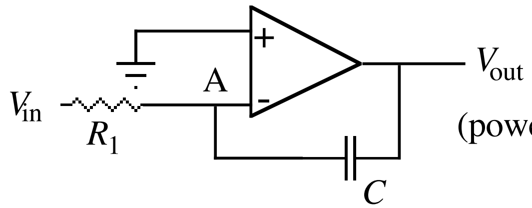


$$V_{in}/R_1 + (V_{in} - V_{out})/R_f = 0$$

$$V_{out}/V_{in} = (R_1 + R_f)/R_1$$

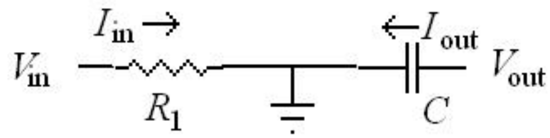
- ☞ The closed loop gain is $(R_1 + R_f) / R_1$.
- ❑ The output has the same polarity as the input.

- **Integrating Amplifier:**



(power connections not shown)

- Again, using the two rules for op amp circuits we redraw the circuit as:



$$\frac{V_{in}}{R_1} + \frac{dQ}{dt} = 0$$

$$\frac{V_{in}}{R_1} + C\frac{dV_{out}}{dt} = 0$$

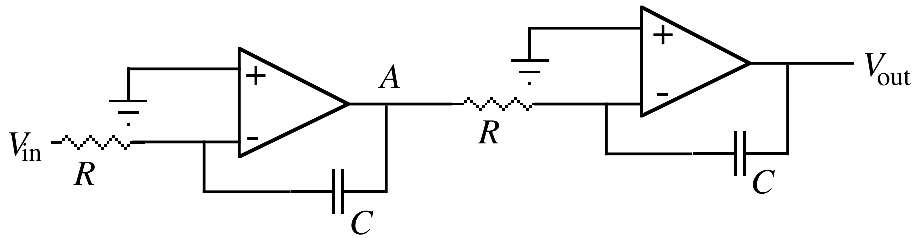
$$V_{out} = \frac{-1}{CR_1}\int V_{in}dt$$

- ☞ The output voltage is related to the integral of the input voltage.
- ❑ The negative sign in the gain means that V_{in} and V_{out} have opposite polarity.

- **Op Amps and Analog Calculations:**
  - Op amps were invented before transistors to perform analog calculations.
  - Their main function was to solve differential equations in real time.
  - Example: Suppose we wanted to solve the following:

$$\frac{d^2x}{dt^2} = g$$

    - This describes a body under constant acceleration (gravity if $g = 9.8$ m/s^2).
  - The following circuit gives an output which is the solution to the differential equation:



  - The input voltage is a constant (= g).
  - For convenience we pick $RC = 1$.
  - At point A:

$$V_A = -\int V_{in}\,dt = -\int \frac{d^2x}{dt^2}dt = -\frac{dx}{dt}$$

  - The output voltage (V_{out}) is the integral of V_A:

$$V_{out} = -\int V_A\,dt = \int \frac{dx}{dt}dt = x(t)$$

If we want non-zero boundary conditions (e.g. $V(t = 0) = 1$ m/s) we add a DC voltage at point A.