Lecture 9: Digital Electronics

Introduction:

- We can classify the building blocks of a circuit or system as being either analog or digital in nature.
  - If we focus on voltage as the circuit parameter of interest:
    - Analog: The voltage can take on a range of voltages, e.g. any value between 0.1 and 2 Volts.
    - Digital: The voltage can have only two values, e.g. 0 or 5 Volts
      - We say the voltage is either on or off
      - ☞ Digital circuits are useful when we don't need a continuous range of voltage or current.
      - Examples: Representing numbers, binary logic, counting circuits.
      - Example: Represent base 10 numbers using the binary system:

        $2|_{10} = 10|_2 = 1 \times 2^1 + 0 \times 2^0$

        $10|_{10} = 1010|_2 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

- Digital circuits use standard voltages (or currents) to denote <u>ON</u> (high, 1) or <u>OFF</u> (low, 0).
  - These standards are called "Logic Families" and there are several families.
  - Two of the most popular families are:
    - TTL (**T**ransistor-**T**ransistor-**L**ogic): ON = 5 Volts, OFF = 0 Volts
    - ECL (**E**mitter-**C**oupled-**L**ogic): ON = -1 Volt, OFF = -1.6 Volts
  - ☞ For practical reasons both ON and OFF are given by a range of voltages or currents.
  - ON for an input to a circuit might have slightly different voltage than ON for an output to a circuit.

- A description of several logic families is given in the table below:

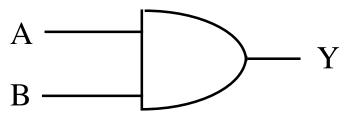
	Delay (ns)	Max. FF Rate (MHz)	Power/Gate (mW)	High (V)	Low (V)
Standard TTL (7400)	10	35	15	3.5	0.2
Low-power Schottky (74LS00)	9.0	33	2	3.5	0.2
Fast TTL (74F00)	3.5	125	5.5	2.7	0.5
CMOS (74C00)	25 @ 10 V 50 @ 25 V	10 @ 10 V 3.5 @ 5 V	0.01	5-15	0
High-speed CMOS (74HC00)	8.0	40	0.01	2-6	0.1
ECL	2	250	25	-0.9	-1.8
100k ECL	0.75	500	40	-1.0	-1.7

- Advantages of Digital:
  - only deal with two voltage levels (either ON or OFF)
  - voltages (or currents) are standardized
  - do not deal with individual transistors…
- Disadvantages of Digital:
  - too many "black" boxes
  - need good power supplies, clocks etc. for circuits to work properly

Logic Gates:

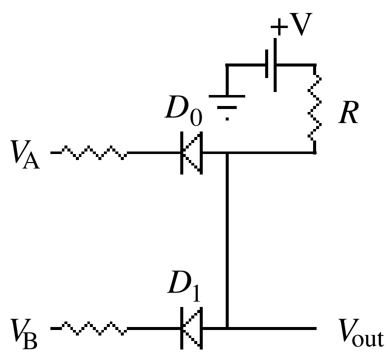
- We want to make decisions based on digital information.
  - For now consider the basic building blocks with one or two inputs and one output.
- The basic logic units (gates) are: AND, OR, NOT.
  - These functions are defined by their truth tables.

AND



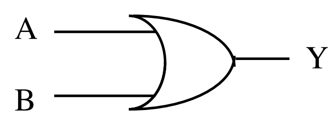
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

$A \bullet B = Y$



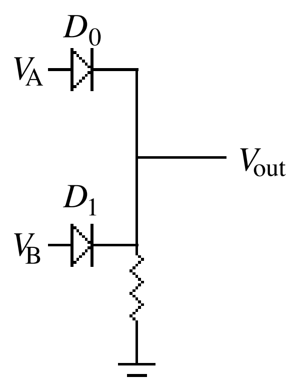
$V_{out} = +V$ if both diodes off
$= 0$ if one or more diodes conduct

OR



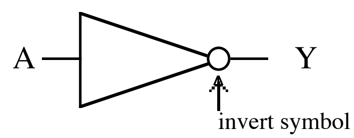
A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

$A + B = Y$



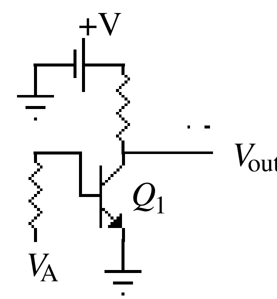
$V_{out} = V_A$ or V_B if either diodes conduct
$= 0$ if both diodes are off

NOT (inverter)



A	Y
0	1
1	0

$\overline{A} = Y$



$V_A > 0.6$ V, Q_1 is on, $V_{out} \sim 0$
$V_A < 0.6$ V, Q_1 is off, $V_{out} = +V$

A and B stand for the inputs
Y stands for the output
0: low input or output
1: high input or output

Boolean Algebra or the Algebra of 1's and 0's

- Circuits consisting of logic gates are described by Boolean algebra.
  - Use of this algebra can greatly simplify circuit design, e.g. minimize the number of components.
- The following theorems can be proved using a truth table and the definition of OR, AND, and NOT.

1) $A + A = A,\ A + 1 = 1,\ A + 0 = A$

2) $AA = A$

3) $AB = BA$

4) $ABC = (AB)C = A(BC)$

5) $A(B + C) = AB + AC$

6) $\overline{1} = 0,\ \overline{0} = 1$

7) $A + \overline{A} = 1,\ A\overline{A} = 0,\ A \cdot 1 = A$

8) $\overline{\overline{A}} = A$

9) $\overline{A + B} = \overline{A} \cdot \overline{B}$

10) $\overline{AB} = \overline{A} + \overline{B}$

(9 and 10: DeMorgan's Theorem)

See Simpson page 540 for more theorems.

For clarity "·" (AND) is not shown in some theorems.

- Example using Boolean algebra:
  - **Prove:** $X + YZ = (X + Y)(X + Z)$

$(X + Y)(X + Z) = XX + XZ + YX + YZ$ by 5)

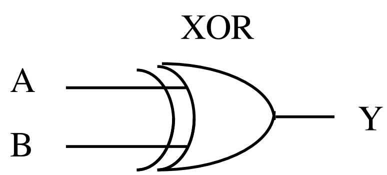
$= X + X(Z + Y) + YZ$ by 2) and 5)

$= X(1 + Z + Y) + YZ$ by 5)

$= X + YZ$ by 1)

- We could also have proven the above using a truth table.
  - There are 8 (2^3) possible combinations of X, Y, Z.
  - For a large number of inputs using a truth table becomes unwieldy.
  - Example, if there are 10 inputs
    - ☞ $2^{10} = 1024$ possible combinations!

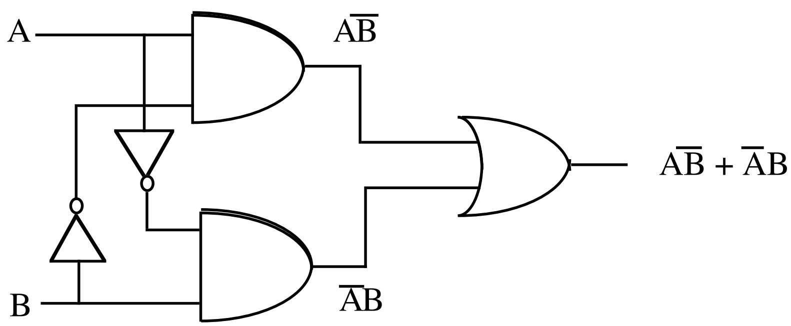
- Example: Exclusive OR = XOR = $A \oplus B$.
  - Output is high if inputs are different.



A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

$A\bar{B} + \bar{A}B = Y$

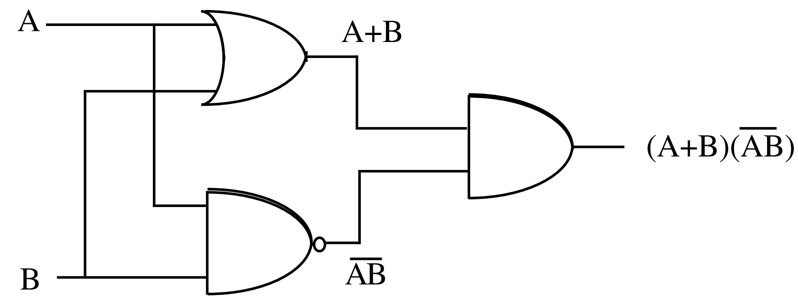
  - How do we make an exclusive OR with AND, OR, and NOT gates?



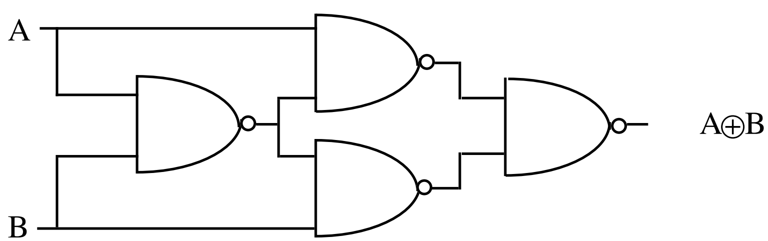
Brute force method

- Can we simplify this circuit with the use of less parts?
  - Use logical theorems:

$$A \oplus B = A\bar{B} + \bar{A}B$$
$$= A\bar{A} + A\bar{B} + \bar{A}B + B\bar{B} \quad \text{7) and 1)}$$
$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B}) \quad \text{5)}$$
$$= A(\overline{AB}) + B(\overline{AB}) \quad \text{10)}$$
$$= (A + B)(\overline{AB}) \quad \text{5)}$$

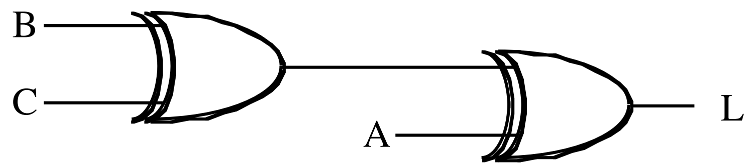


  - The circuit uses only 3 parts (OR, NAND, AND), but each of them is different!
  - Usually there are many ways to synthesize the same function (circuit).
  - Must decide if you want to minimize:
    - number of components
    - types of components
    - number of connections
    - power consumption
  - For example we can make an XOR using only 4 NAND gates:



- Final example: Suppose you have a light controlled by 3 switches.
  - You want the light to be on if any one of the 3 switches is on or if all 3 switches are on.

A	B	C	Light
1	1	1	1
1	0	0	1
1	1	0	0
0	1	1	0
0	1	0	1
0	0	1	1
1	0	1	0
0	0	0	0



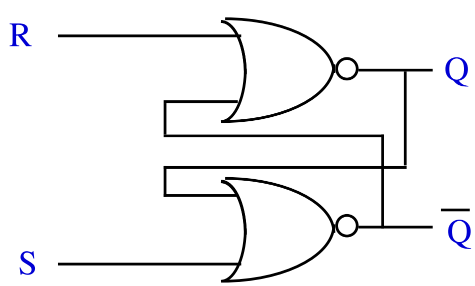
1 = ON
0 = OFF

$$
\begin{aligned}
L &= ABC + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C \\
&= A(BC + \bar{B}\bar{C}) + \bar{A}(B\bar{C} + \bar{B}C) \\
&= A(B + \bar{C})(\bar{B} + C) + \bar{A}(B \oplus C) \\
&= A\overline{\bar{B}C}\,\overline{B\bar{C}} + \bar{A}(B \oplus C) \\
&= A\overline{\bar{B}C + B\bar{C}} + \bar{A}(B \oplus C) \\
&= A\overline{B \oplus C} + \bar{A}(B \oplus C) \\
&= A \oplus (B \oplus C)
\end{aligned}
$$

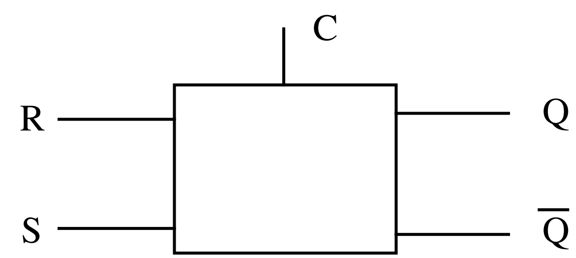
Flip-Flops:

- Basic counting unit in computer:
  - counters
  - shift registers
  - memory
- Circuit whose output depends on the history of its inputs.
- Can make a flip-flop with just 2 transistors (or 2 vacuum tubes 1919!).
- Lots of different types of flip-flops (e.g. RS, JK, T, D).

- Example: RS flip-flop or Reset-Set flip-flop
  - Flip-flops, like logic gates are defined by their truth table.
  - Flip-flops are controlled by an external clock pulse.
  - All inputs and outputs are logic levels (e.g. TTL, ECL).
  - Can make an RSFF out of NOR gates:

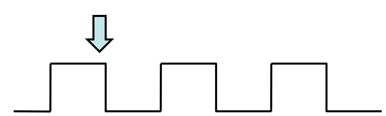


R	S	Q_{n+1}
0	0	Q_n
1	0	0
0	1	1
1	1	undefined

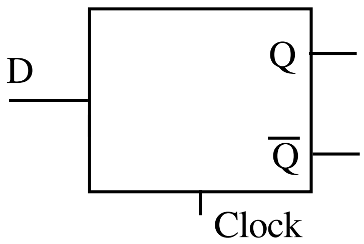


- Q_n is the present state of the FF.
- Q_{n+1} will be the output after the clock enables the FF to look at its inputs (R and S).
- Many FF change state ($Q_n \rightarrow Q_{n+1}$) on the trailing edge of the clock.

The state with R = S =1 *is undefined. The output is not predictable!*



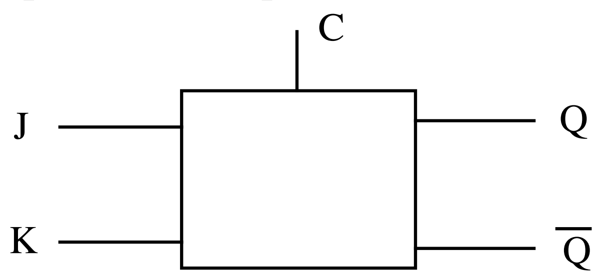
- Example: D flip-flop (Like RS but only one input)



D	Q	Q_{next}
0	0	0
0	1	0
1	0	1
1	1	1

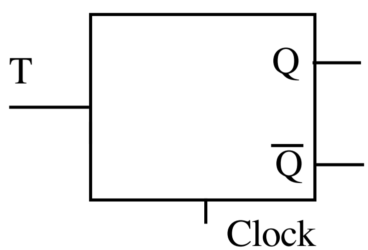
- Example: JK flip-flop
  - JKFF is like the RSFF except that both inputs (J and K) can be high (1).

J	K	Q_{n+1}
0	0	Q_n
1	0	1
0	1	0
1	1	$\overline{Q}_n$



  - Most JKFF' s have a connection for forcing Q = 0 (clear) or forcing Q = 1 (preset).

- Example: T (*Toggle*) flip-flop
  - T flip-flop is like the JKFF with both inputs (J and K) tied to each other.



T	Q	Q_{next}
0	0	0
0	1	1
1	0	1
1	1	0

- Flip-Flops are a class of circuits called "multivibrators"
  - Multivibrators are circuits with one or more stable states.
    - Monostable multivibrators (one shot) have one stable state.
      - If the circuit is forced out of its stable state (e.g. by an input pulse)
        - it eventually returns back to the stable state by itself.
    - Bistable multivibrators have two stable states.
      - Transitions between states occur only by an external action (e.g. voltage pulse for flip-flops).
      - Transition voltages can be different for the two states (e.g. Schmitt trigger).
    - Astable multivibrators are two-state devices which switch on their own accord.
      - Commonly used as oscillators.

- Example: Bistable Multivibrator.
  - This circuit has two stable states.
  - When either transistor conducts there is 4 mA flowing (by design) in the collector (I_{C1} or I_{C2}).
  - State 1: transistor 1 off, transistor 2 on
    - $V_{C1} \approx 11$ V, $V_{B1} \approx 2$ V, $V_{E1} \approx 4$ V
    - $V_{C2} \approx 4$ V, $V_{B2} \approx 5.5$ V, $V_{E2} \approx 4$ V

      T_2 is saturated ($V_{CE} \approx 0$)
  - Transition from state 1 to state 2:
    - Input pulse forces T_1 to conduct, T_1 conducting means that V_{C1} drops.
    - V_{C1} causes V_{B2} to drop to the point where T_2 is not conducting.
  - State 2: transistor 1 on, transistor 2 off
    - $V_{C1} \approx 4$ V, $V_{B1} \approx 5.5$ V, $V_{E1} \approx 4$ V

      T_1 is saturated ($V_{CE} \approx 0$)
    - $V_{C2} \approx 11$ V, $V_{B2} \approx 2$ V, $V_{E2} \approx 4$ V