Lecture 1: Introduction

Some Definitions:
- Current (I): Amount of electric charge (Q) moving past a point per unit time
    - $I = dQ/dt = $ Coulombs/sec
   - units = Amps (1 Coulomb = 6×10^{18} electrons)

- Voltage (V):
    - Work needed to move charge from point a to b
        Work = $V \bullet Q$
    - ☞ Volt = Work/Charge = Joules/Coulomb
    - Voltage is always measured with respect to something
    - "ground" is defined as zero Volts

- Direct Current (DC): In a DC circuit the current and voltage are constant as a function of time

- Power (P): Rate of doing work
    - $P = dW/dt$
    - units = Watts

- Ohms Law: Linear relationship between voltage and current
  - $V = I \bullet R$
  - R = Resistance (Ω)
  - units = Ohms
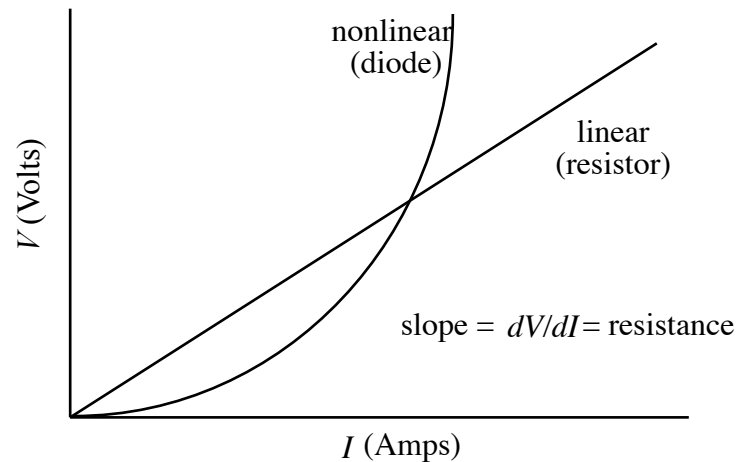
- Joules Law: When current flows through a resistor energy is dissipated

  $W = QV$

  $P = dW/dt = VdQ/dt + QdV/dt$

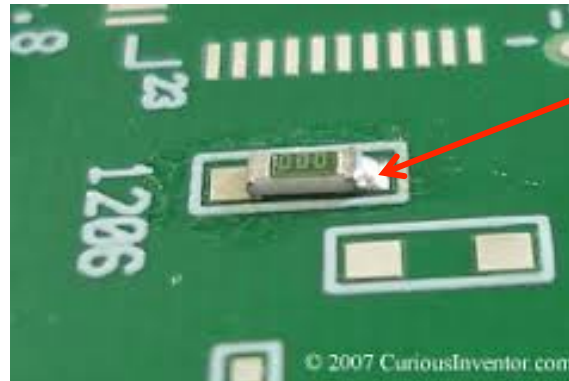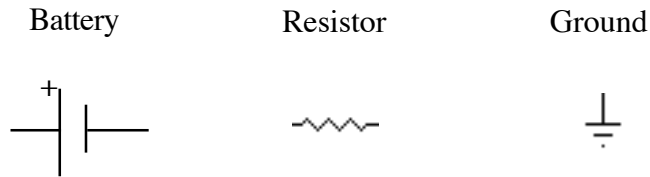 - $dV/dt = 0$ for DC circuit and averages to 0 for AC
  - ☞ Power $= VdQ/dt = V \bullet I$
  - Using Ohms law
    - ☞ $P = I^2 R = V^2 / R$
    - ■ 100 Watts = 10 V and 10 Amps or 10 V through 1 Ω

Simple Circuits

- Symbols:

| Battery | Resistor | Ground |

Solder paste

- ◆ 4700 Lab resistors
- ◆ Stick the leads into "bread board" to make connections

- ◆ Use in computers/cell phones
- ◆ Place with "pick & place" machine
- ◆ Surface tension automatically aligns the component on their pads!

- Dimension of surface mount components (e.g. 1206):
  - ◆ length: 12 x 250 μm = 3 mm
  - ◆ width: 6 x 250 μm = 1.5 mm
  - ■ smallest available: 01005 (0.4 mm × 0.2 mm, power rating = 0.03 W)
  - ■ slightly less insane version: 0201 (0.6 mm × 0.3 mm, power rating = 0.05 W)

- Simple(st) Circuit:
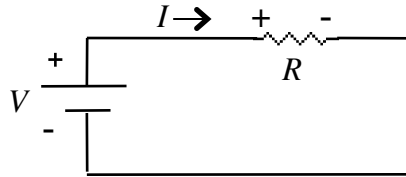
  - ◆ Convention: Current flow is in the direction of <u>positive</u> charge flow
    - ■ When we go across a battery in direction of current (- ➔ +)
      - ☞ $+V$
    - ■ Voltage drop across a resistor in direction of current (+ ➔ -)
      - ☞ $-IR$
      - ❑ **Conservation of Energy: sum of potential drops around the circuit should be zero**
        - ☞ $V - IR = 0$ or $V = IR$!!

- Next simple(st) circuit: two resistors in series
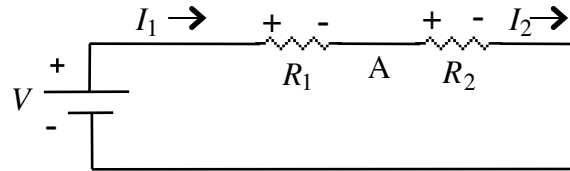
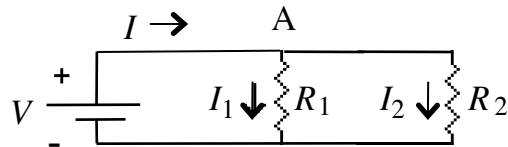- ◆ **Conservation of charge: $I_1 = I_2 = I$ at point A**
  - ☞ $V = I(R_1 + R_2) = IR$
  - ☞ $R = R_1 + R_2$
    - ★ ***Resistors in Series Add: $R = R_1 + R_2 + R_3$...***
- ◆ What's voltage across R_2?
  - ☞ $V_2 = I_2R_2 = VR_2/(R_1 + R_2)$ "**Voltage Divider Equation**"
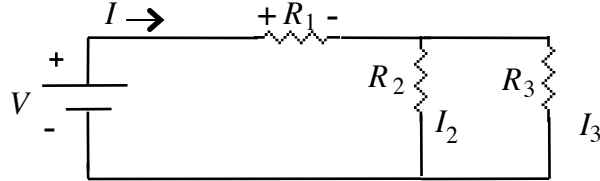
- Two resistors in parallel

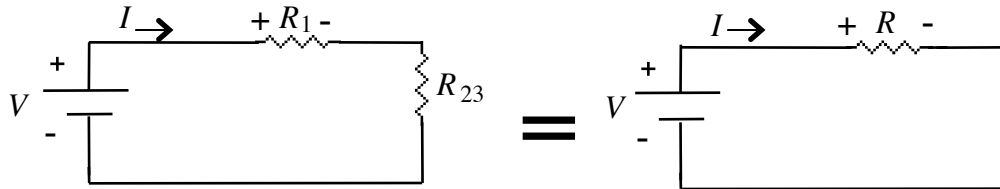- ◆ $I = I_1 + I_2 = V/R_1 + V/R_2 = V/R$
  - ☞ $1/R = 1/R_1 + 1/R_2$

  $$\therefore R = \frac{R_1R_2}{R_1 + R_2}$$

  - ★ ***Parallel Resistors add like:*** $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

- In a circuit with 3 resistors (series and parallel), what's $I_2 = V_2/R_2$?

- reduce to a simpler circuit:

- $I = V/R = V/(R_1 + R_{23})$

$$R_{23} = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_2 = I R_{23}$$

$$= \frac{V}{R_1 + \dfrac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2 R_3}{R_2 + R_3}$$

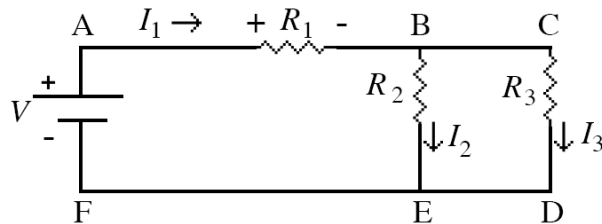$$= \frac{V R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_2 = \frac{V_2}{R_2}$$

$$= \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

If $R_3 \rightarrow \infty$ then $I_2 = I = V/(R_1 + R_2)$ as expected!

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Kirchoff's Laws

- We can formalize and generalize the previous examples using Kirchoff's Laws:
  1. $\Sigma I = 0$ at a node: conservation of charge
  2. $\Sigma V = 0$ around a closed loop: conservation of energy

  - example

  - node B: $I_1 = I_2 + I_3$ ➔ $I_1 - I_2 - I_3 = 0$
  - loop ABEF: $V - I_1R_1 - I_2R_2 = 0$
  - loop ACDF: $V - I_1R_1 - I_3R_3 = 0$
    - ☞ 3 linear equations with 3 unknowns: I_1, I_2, I_3
    - ☞ always wind up with as many linear equations as unknowns!
  - use matrix methods to solve these equations:
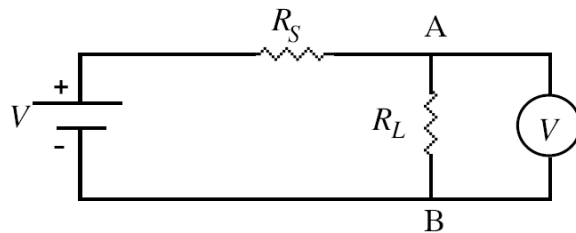
    $$V = RI$$

    $$\begin{bmatrix} V \\ V \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

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$$I_2 = \frac{\det \begin{bmatrix} R_1 & V & 0 \\ R_1 & V & R_3 \\ 1 & 0 & -1 \end{bmatrix}}{\det \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{VR_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

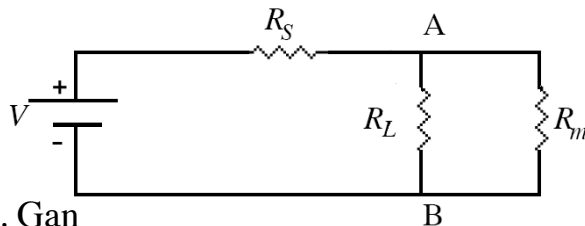☞ the *same* solution as in page 5!

Measuring Things

● Voltmeter: **Always** put in **parallel** with what you want to measure

◆ If no voltmeter we would have:

$$V_{AB} = \left[\frac{R_L}{R_S + R_L} \right] V$$

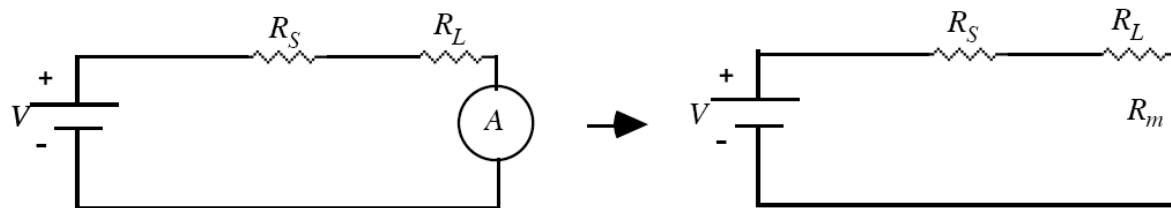◆ If the voltmeter has a finite resistance R_m then circuit looks like:

- ■ From previous pages we have:

$$V_{AB}^{*} = \left[\frac{R_m \| R_L}{R_S + R_m \| R_L} \right] V$$

$$= \frac{V R_m R_L}{R_S R_L + R_m R_L + R_S R_m}$$

$$= \frac{V R_L}{R_L + R_S + \dfrac{R_S R_L}{R_m}}$$

$$\cong V_{AB} \qquad \text{if } R_L << R_m$$

- ☞ good voltmeter has high resistance ($> 10^6$ Ω)

- ● Ammeter: measures current
  - ■ **Always** put in **series** with what you want to measure

- ◆ Without meter: $I = V/(R_S + R_L)$
- ◆ With meter: $I^* = V/(R_S + R_L + R_m)$
  - ☞ good ammeter has $R_m << (R_S + R_L)$, i.e. low resistance (0.1-1 Ω)

Thevenin's Equivalent Circuit Theorem

● Any network of resistors and batteries having 2 output terminals may be replaced by a series combination of resistor and battery

◆ Useful when solving complicated (!?) networks

◆ Solve problems by finding V_{eq} and R_{eq} for circuit without load, then add load to circuit.

◆ Use basic voltage divider equation:
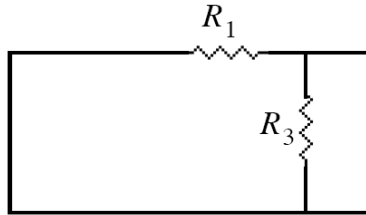
$$V_L = \frac{V_{eq}R_L}{R_L + R_{eq}}$$
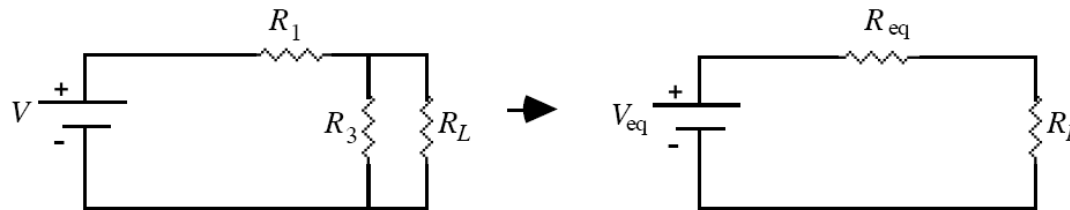
● Two rules for using Thevenin's Thereom:

1. Take the load out of the circuit to find V_{eq}:
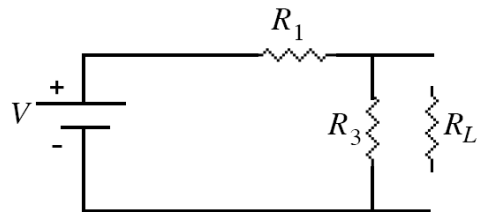
$$V_{eq} = \frac{VR_3}{R_1 + R_3}$$

2. Short circuit all power supplies (batteries) to find R_{eq}:

$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3}$$

■ Can now solve for I_L as in previous examples:

$$I_L = \frac{V_{eq}}{R_{eq} + R_L}$$

$$= \left[\frac{V R_3}{R_1 + R_3} \right] \times \frac{1}{\dfrac{R_1 R_3}{R_1 + R_3} + R_L}$$

$$= \frac{V R_3}{R_1 R_L + R_1 R_3 + R_L R_3}$$

☞ Same answer as previous examples!