Lecture 1: Introduction

Some Definitions:

- Current (I): Amount of electric charge (Q) moving past a point per unit time
  - $I = dQ/dt$ = Coulombs/sec
  - units = Amps (1 Coulomb = 6×10^{18} electrons)

- Voltage (V):
  - Work needed to move charge from point a to b

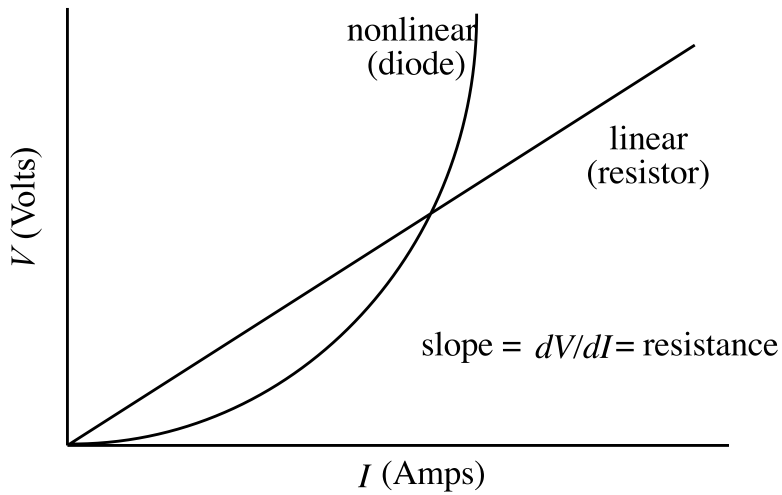
    Work = $V \bullet Q$

    ⇨ Volt = Work/Charge = Joules/Coulomb
  - Voltage is always measured with respect to something
  - "ground" is defined as zero Volts

- Direct Current (DC): In a DC circuit the current and voltage are constant as a function of time

- Power (P): Rate of doing work
  - $P = dW/dt$
  - units = Watts

- Ohms Law: Linear relationship between voltage and current
  - $V = I \bullet R$
  - R = Resistance (Ω)
  - units = Ohms



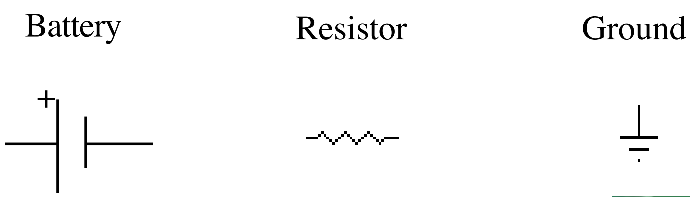
- Joules Law: When current flows through a resistor energy is dissipated

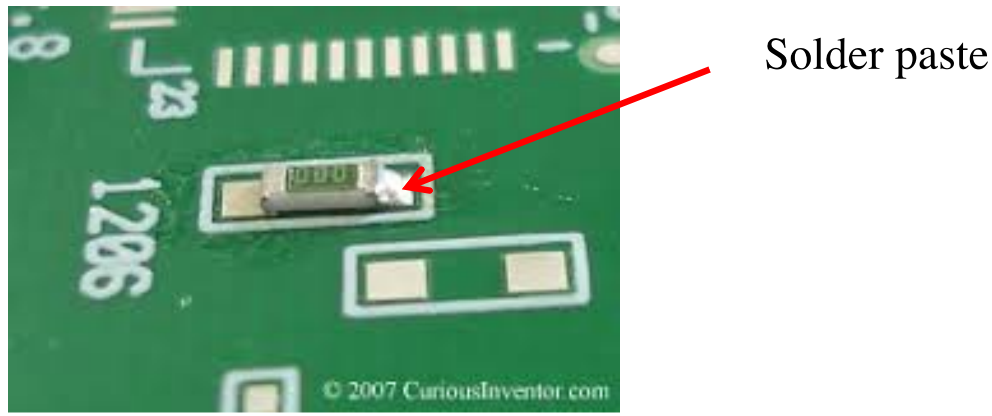
  $W = QV$

  $P = dW/dt = VdQ/dt + QdV/dt$
  - $dV/dt = 0$ for DC circuit and averages to 0 for AC
  - ⇨ Power = $VdQ/dt = V \bullet I$
  - Using Ohms law
    - ⇨ $P = I^2 R = V^2 / R$
      - 100 Watts = 10 V and 10 Amps or 10 V through 1 Ω

Simple Circuits

- Symbols:



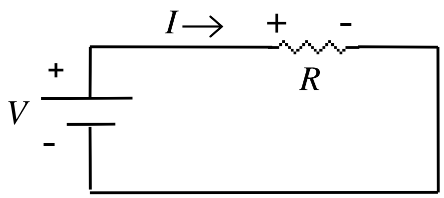


  - 4700 Lab resistors
  - Stick the leads into "bread board" to make connections

  - Use in computers/cell phones
  - Place with "pick & place" machine
  - Surface tension automatically aligns the component on their pads!

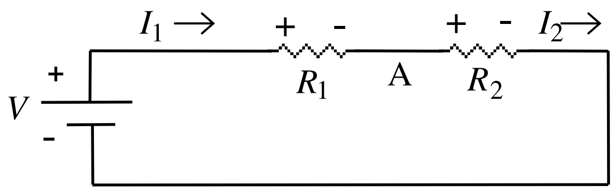
- Dimension of surface mount components (e.g. 1206):
  - length: 12 x 250 μm = 3 mm
  - width: 6 x 250 μm = 1.5 mm
  - smallest available: 01005 (0.4 mm × 0.2 mm, power rating = 0.03 W)
  - slightly less insane version: 0201 (0.6 mm × 0.3 mm, power rating = 0.05 W)

- Simple(st) Circuit:



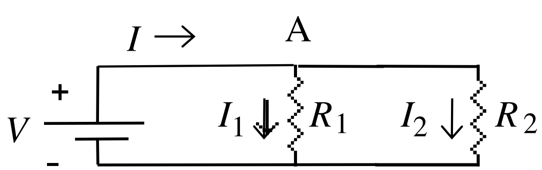
  - Convention: Current flow is in the direction of <u>positive</u> charge flow
    - When we go across a battery in direction of current (- ➔ +)
      - ⇨ $+V$
    - Voltage drop across a resistor in direction of current (+ ➔ -)
      - ⇨ $-IR$
      - **Conservation of Energy: sum of potential drops around the circuit should be zero**
        - ⇨ $V - IR = 0$ or $V = IR$!!

- Next simple(st) circuit: two resistors in series



  - **Conservation of charge:** $I_1 = I_2 = I$ **at point A**
    - ⇨ $V = I(R_1+R_2) = IR$
    - ⇨ $R = R_1 + R_2$
      - ⇨ ***Resistors in Series Add:*** $R = R_1 + R_2 + R_3 \ldots$
  - What's voltage across R_2?
    - ⇨ $V_2 = I_2R_2 = VR_2/(R_1 + R_2)$ "**Voltage Divider Equation**"

- Two resistors in parallel

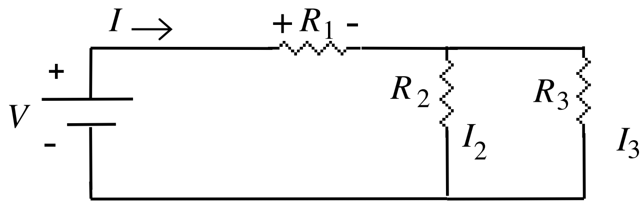


  - $I = I_1 + I_2 = V/R_1 + V/R_2 = V/R$
    - ⇨ $1/R = 1/R_1 + 1/R_2$

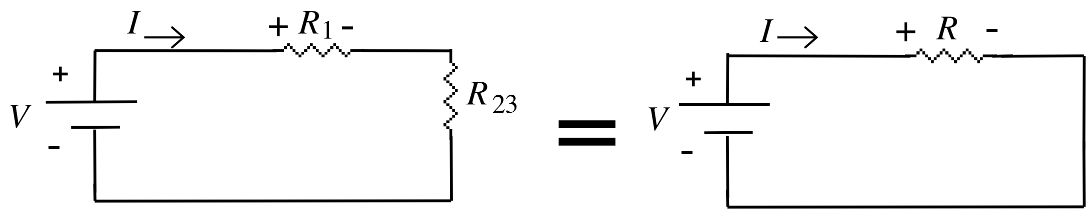
$$\therefore \; R = \frac{R_1 R_2}{R_1 + R_2}$$

  - ⇨ ***Parallel Resistors add like:*** $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

- In a circuit with 3 resistors (series and parallel), what's $I_2 = V_2/R_2$?



  - reduce to a simpler circuit:



  - $I = V/R = V/(R_1 + R_{23})$

$$R_{23} = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_2 = IR_{23}$$

$$= \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2 R_3}{R_2 + R_3}$$

$$= \frac{V R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

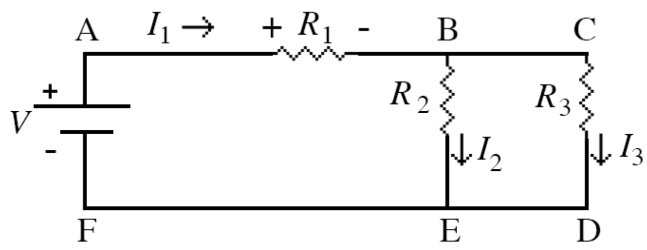
$$I_2 = \frac{V_2}{R_2}$$

$$= \frac{V R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

If $R_3 \to \infty$ then $I_2 = I = V/(R_1 + R_2)$ as expected!

Kirchoff's Laws

- We can formalize and generalize the previous examples using Kirchoff's Laws:
  1. $\Sigma I = 0$ at a node: conservation of charge
  2. $\Sigma V = 0$ around a closed loop: conservation of energy
  - example



- node B: $I_1 = I_2 + I_3$ ➔ $I_1 - I_2 - I_3 = 0$
- loop ABEF: $V - I_1R_1 - I_2R_2 = 0$
- loop ACDF: $V - I_1R_1 - I_3R_3 = 0$
  - ⇨ 3 linear equations with 3 unknowns: I_1, I_2, I_3
  - ⇨ always wind up with as many linear equations as unknowns!
- use matrix methods to solve these equations:

$$\boldsymbol{V} = \boldsymbol{RI}$$

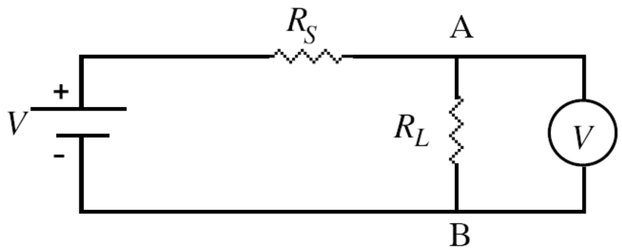
$$\begin{bmatrix} V \\ V \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_2 = \frac{\det\begin{bmatrix} R_1 & V & 0 \\ R_1 & V & R_3 \\ 1 & 0 & -1 \end{bmatrix}}{\det\begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

⇨ the *same* solution as in page 6!

Measuring Things

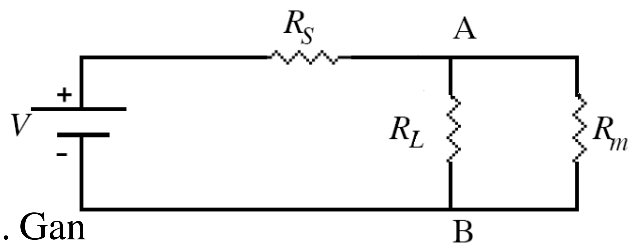
- Voltmeter: **Always** put in **parallel** with what you want to measure



- ◆ If no voltmeter we would have:

$$V_{AB} = \left[\frac{R_L}{R_S + R_L}\right]V$$

- ◆ If the voltmeter has a finite resistance R_m then circuit looks like:

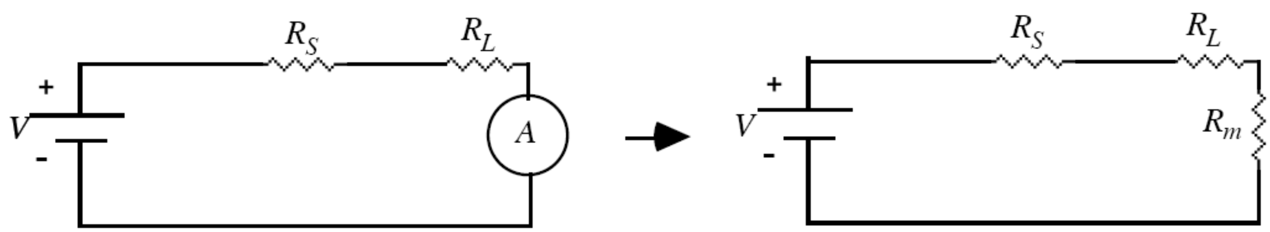


- From previous pages we have:

$$V_{AB}^* = \left[\frac{R_m \| R_L}{R_S + R_m \| R_L}\right] V$$
$$= \frac{V R_m R_L}{R_S R_L + R_m R_L + R_S R_m}$$
$$= \frac{V R_L}{R_L + R_S + \frac{R_S R_L}{R_m}}$$
$$\cong V_{AB} \qquad \text{if } R_L << R_m$$

⇨ good voltmeter has high resistance (> 10^6 Ω)

- Ammeter: measures current
  - **Always** put in **series** with what you want to measure



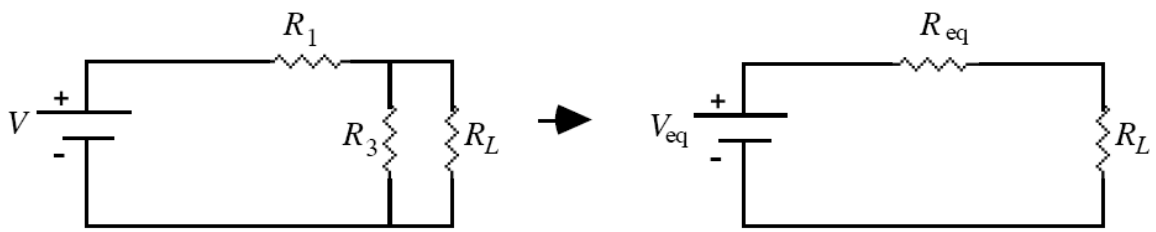
- Without meter: $I = V/(R_S + R_L)$
- With meter: $I^* = V/(R_S + R_L + R_m)$

⇨ good ammeter has $R_m << (R_S + R_L)$, i.e. low resistance (0.1-1 Ω)

Thevenin's Equivalent Circuit Theorem

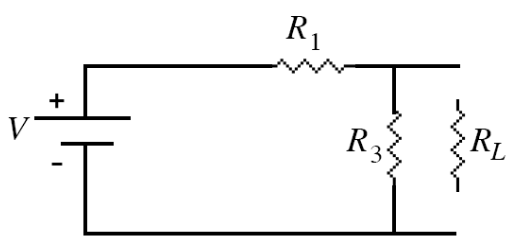
- Any network of resistors and batteries having 2 output terminals may be replaced by a series combination of resistor and battery
  - Useful when solving complicated (!?) networks
  - Solve problems by finding V_{eq} and R_{eq} for circuit without load, then add load to circuit.
  - Use basic voltage divider equation:

$$V_L = \frac{V_{eq} R_L}{R_L + R_{eq}}$$



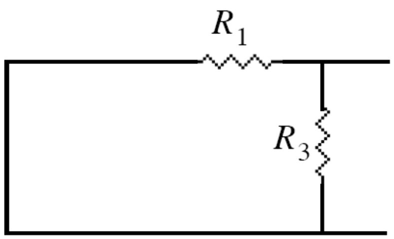
- Two rules for using Thevenin's Thereom:

1. Take the load out of the circuit to find V_{eq}:



$$V_{eq} = \frac{VR_3}{R_1 + R_3}$$

2. Short circuit all power supplies (batteries) to find R_{eq}:



$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3}$$

- Can now solve for I_L as in previous examples:

$$I_L = \frac{V_{eq}}{R_{eq} + R_L}$$

$$= \left[\frac{VR_3}{R_1 + R_3}\right] \times \frac{1}{\frac{R_1 R_3}{R_1 + R_3} + R_L}$$

$$= \frac{VR_3}{R_1 R_L + R_1 R_3 + R_L R_3}$$

⇨ Same answer as previous examples!