

Lecture 1: Introduction

Some Definitions:

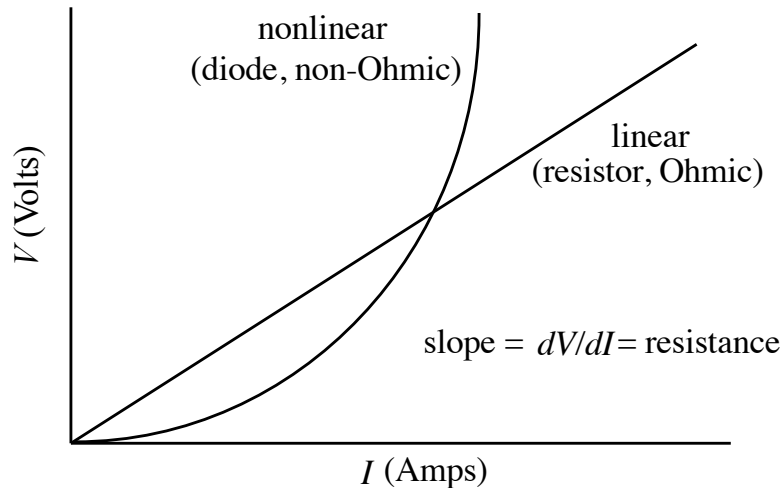
- Current (I): Amount of electric charge (Q) moving past a point per unit time
 - ◆ $I = dQ/dt = \text{Coulombs/sec}$
 - ◆ units = Amps (1 Coulomb = 6×10^{18} electrons)
 - ⇒ 1 A can do a lot of damage given the large number of electrons flowing through a wire
 - Voltage (V):
 - ◆ Work needed to move charge from point a to b
$$\text{Work} = V \cdot Q$$
 - ⇒ Volt = Work/Charge = Joules/Coulomb
 - ◆ Voltage is always measured with respect to something
 - ◆ "ground" is defined as zero Volts
 - ◆ always capitalize the "V", e.g. 110 V, 2 kV, 13 eV
 - in honor of Alessandro Volta (1745 – 1827) that invented the battery in 1799
 - Direct Current (DC): In a DC circuit the current and voltage are constant as a function of time
 - Power (P): Rate of doing work
 - ◆ $P = dW/dt$
 - ◆ units = Watts
- K.K. Gan



Tempio Voltiano, Como

- Ohms Law: Linear relationship between voltage and current

- ◆ $V = I \bullet R$
- ◆ $R = \text{Resistance}$
- ◆ units = Ohms (Ω)



- Joules Law: When current flows through a resistor energy is dissipated

$$W = QV$$

$$P = dW/dt = VdQ/dt + QdV/dt$$

- ◆ $dV/dt = 0$ for DC circuit and averages to 0 for AC

⇒ Power = $VdQ/dt = V \bullet I$

- ◆ Using Ohms law

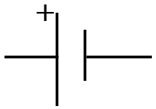
⇒ $P = I^2 R = V^2 / R$

- 100 Watts = 10 V and 10 Amps or 10 V through 1 Ω

Simple Circuits

- Symbols:

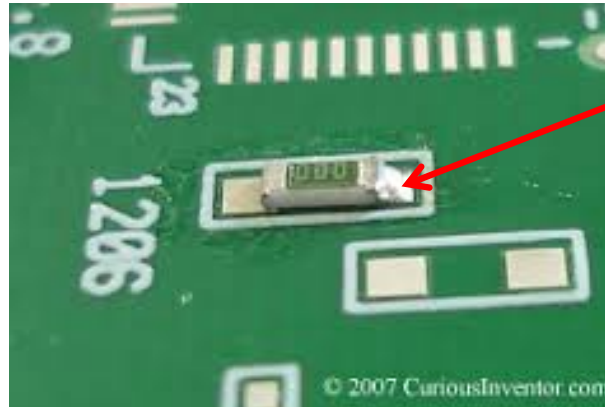
Battery



Resistor



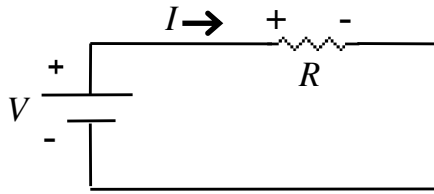
Ground



Solder paste

- ◆ 4700 Lab resistors
 - ◆ Stick the leads into “breadboard” to make connections
 - ◆ Use in computers/cell phones
 - ◆ Place with “pick & place” machine
 - ◆ Surface tension automatically aligns the component on their pads!
- Dimension of surface mount components (e.g. 1206):
 - ◆ length: 12 mils = $12 \times 250 \mu\text{m} = 3 \text{ mm}$
 - ◆ width: 6 mils = $6 \times 250 \mu\text{m} = 1.5 \text{ mm}$
 - smallest available: 01005 ($0.4 \text{ mm} \times 0.2 \text{ mm}$, power rating = 0.03 W)
 - slightly less insane version: 0201 ($0.6 \text{ mm} \times 0.3 \text{ mm}$, power rating = 0.05 W)

- Simple(st) Circuit:



- ◆ Convention: Current flow is in the direction of positive charge flow

- When we go across a battery in direction of current ($- \rightarrow +$)

⇒ $+V$

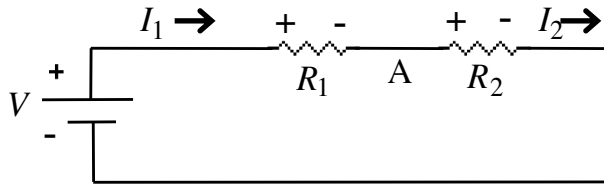
- Voltage drop across a resistor in direction of current ($+ \rightarrow -$)

⇒ $-IR$

- **Conservation of Energy: sum of potential drops around the circuit should be zero**

⇒ $V - IR = 0$ or $V = IR!!$

- Next simple(st) circuit: two resistors in series



◆ **Conservation of charge: $I_1 = I_2 = I$ at point A**

⇒ $V = I(R_1 + R_2) = IR$

⇒ $R = R_1 + R_2$

⇒ **Resistors in Series Add:** $R = R_1 + R_2 + R_3 \dots$

◆ What's voltage across R_2 ?

⇒ $V_2 = I_2 R_2 = VR_2 / (R_1 + R_2)$ "**Voltage Divider Equation**"

- Use of voltage divider

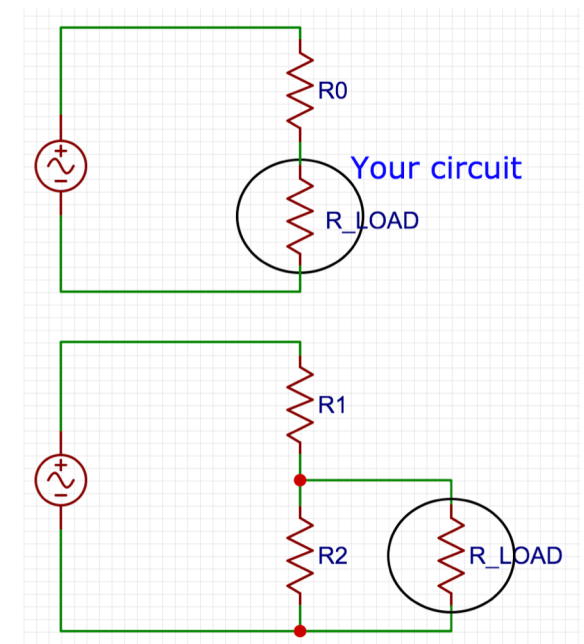
◆ When the signal from a signal generator is too large

⇒ don't use a large resistor (R_0) to attenuate it

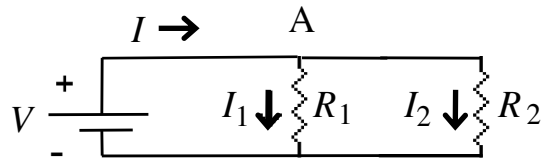
⇒ use a voltage divider to control the signal size

⇒ choose R_2 to be small compared to R_{LOAD}

⇒ the ratio $R_2 / (R_1 + R_2)$ controls the size of the signal



- Two resistors in parallel



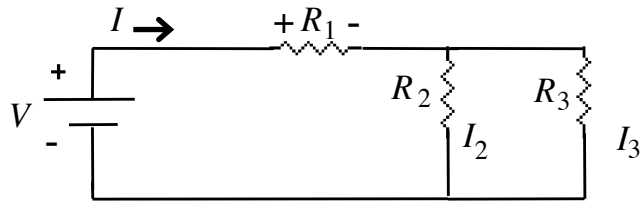
◆ $I = I_1 + I_2 = V/R_1 + V/R_2 = V/R$

⇒ $1/R = 1/R_1 + 1/R_2$

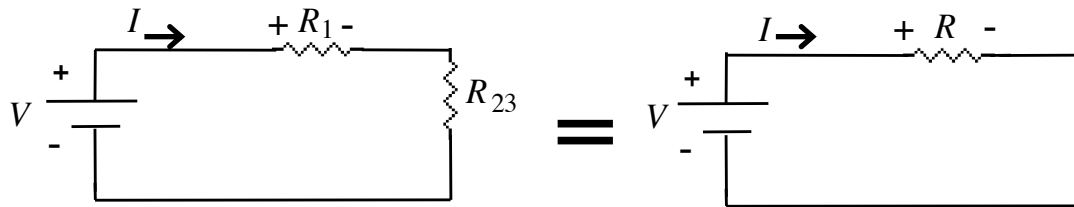
∴ $R = \frac{R_1 R_2}{R_1 + R_2}$

⇒ **Parallel Resistors add like:** $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \dots$

- In a circuit with 3 resistors (series and parallel), what's $I_2 = V_2/R_2$?



- ◆ reduce to a simpler circuit:



- ◆ $I = V/R = V/(R_1 + R_{23})$

$$R_{23} = R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_2 = IR_{23}$$

$$= \frac{V}{R_1 + \frac{R_2 R_3}{R_2 + R_3}} \times \frac{R_2 R_3}{R_2 + R_3}$$

$$= \frac{VR_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$I_2 = \frac{V_2}{R_2}$$

$$= \frac{VR_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

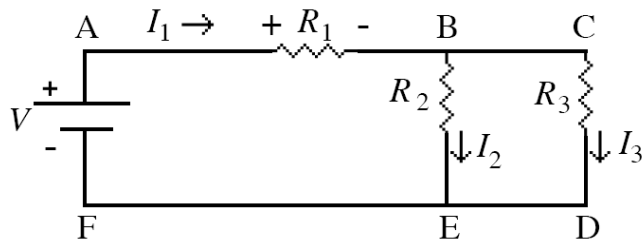
If $R_3 \rightarrow \infty$ then $I_2 = I = V/(R_1 + R_2)$ as expected!

Kirchoff's Laws

- We can formalize and generalize the previous examples using Kirchoff's Laws:

1. $\Sigma I = 0$ at a node: conservation of charge
2. $\Sigma V = 0$ around a closed loop: conservation of energy

- ◆ example



- node B: $I_1 = I_2 + I_3 \rightarrow I_1 - I_2 - I_3 = 0$
- loop ABEF: $V - I_1 R_1 - I_2 R_2 = 0$
- loop ACDF: $V - I_1 R_1 - I_3 R_3 = 0$
- ⇒ 3 linear equations with 3 unknowns: I_1, I_2, I_3
- ⇒ always wind up with as many linear equations as unknowns!
- use matrix methods to solve these equations:

$$\mathbf{V} = \mathbf{R}\mathbf{I}$$

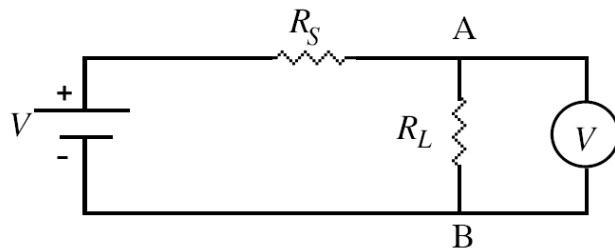
$$\begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$I_2 = \frac{\det \begin{bmatrix} R_1 & V & 0 \\ R_1 & V & R_3 \\ 1 & 0 & -1 \end{bmatrix}}{\det \begin{bmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{bmatrix}} = \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

⇒ the *same* solution as in page 7!

Measuring Things

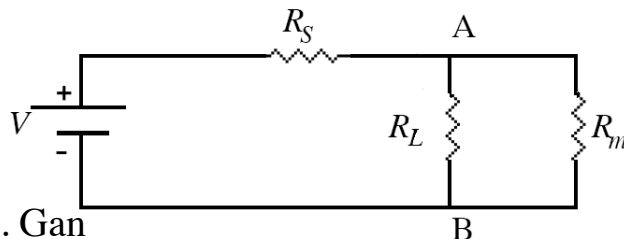
- Voltmeter: **Always** put in **parallel** with what you want to measure



- ◆ If no voltmeter we would have:

$$V_{AB} = \left[\frac{R_L}{R_S + R_L} \right] V$$

- ◆ If the voltmeter has a finite resistance R_m then circuit looks like:



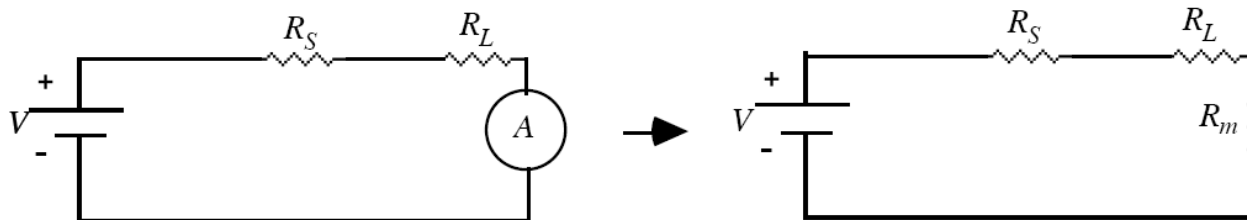
- From previous pages we have:

$$\begin{aligned}
 V_{AB}^* &= \left[\frac{R_m \parallel R_L}{R_S + R_m \parallel R_L} \right] V \\
 &= \frac{VR_m R_L}{R_S R_L + R_m R_L + R_S R_m} \\
 &= \frac{VR_L}{R_L + R_S + \frac{R_S R_L}{R_m}} \\
 &\cong V_{AB} \quad \text{if } R_L \ll R_m
 \end{aligned}$$

⇒ good voltmeter has high resistance ($> 10^6 \Omega$)

- Ammeter: measures current

- **Always** put in **series** with what you want to measure



◆ Without meter: $I = V/(R_S + R_L)$

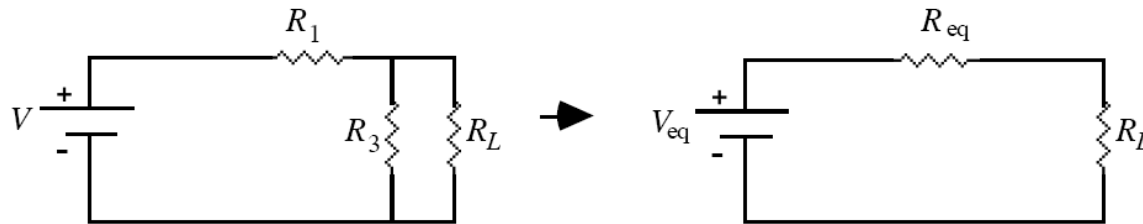
◆ With meter: $I^* = V/(R_S + R_L + R_m)$

⇒ good ammeter has $R_m \ll (R_S + R_L)$, i.e. low resistance (0.1-1 Ω)

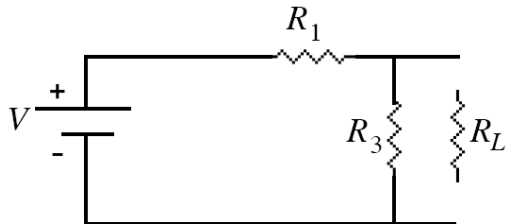
Thevenin's Equivalent Circuit Theorem

- Any network of resistors and batteries having 2 output terminals may be replaced by a series combination of resistor and battery
 - Useful when solving complicated (!?) networks
 - Solve problems by finding V_{eq} and R_{eq} for circuit without load, then add load to circuit.
 - Use basic voltage divider equation:

$$V_L = \frac{V_{eq} R_L}{R_L + R_{eq}}$$

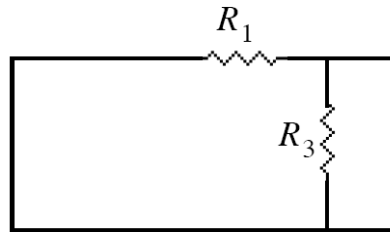


- Two rules for using Thevenin's Theorem:
 - Take the load out of the circuit to find V_{eq} :



$$V_{eq} = \frac{V R_3}{R_1 + R_3}$$

2. Short circuit all power supplies (batteries) to find R_{eq} :



$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3}$$

■ Can now solve for I_L as in previous examples:

$$\begin{aligned} I_L &= \frac{V_{eq}}{R_{eq} + R_L} \\ &= \left[\frac{VR_3}{R_1 + R_3} \right] \times \frac{1}{\frac{R_1 R_3}{R_1 + R_3} + R_L} \\ &= \frac{VR_3}{R_1 R_L + R_1 R_3 + R_L R_3} \end{aligned}$$

⇒ Same answer as previous examples!