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Physics 3700

Problem Set 1
Due Monday, January 29, 2024

Note: To receive credit for the homework problem you must show how you arrived at your answer, e.g. give the relevant formula and show your calculations.

1) The temperature (in degrees Kelvin) at which a certain material becomes a superconductor has been measured 24 times as follows:

18.9,18.7,19.3,19.2,18.9,19.0,20.2,19.9,18.6,19.4,19.3,18.8,19.3,19.2,18.7,18.5, 18.6,19.7,19.9,20.0,19.5,19.4,19.6,19.0

a) Calculate the mean temperature at which the material becomes a superconductor.
b) Calculate the standard deviation (use the n-1 form, Taylor eq. 4.9) of the temperature at which the material becomes a superconductor.
c) Histogram the temperature distribution using a suitable bin size.

2) Taylor, Problem 2.6, page 37.

3) Taylor, Problem 4.2, page 113. You can do part b) with Excel or a software program.

4) The probability distribution that describes the sum of the dots (x) showing on a pair of dice is:

$$p(x) = \frac{x-1}{36} \quad x = 2,3,4,5,6,7$$
$$p(x) = \frac{13-x}{36} \quad x = 8,9,10,11,12$$

Show that this probability distribution has the proper normalization and find the mean and variance of the distribution.

5) The probability density function describing the time (t) between the creation and decay of a certain unstable elementary particle is given by:

$$f(t) = 0 \quad t < 0$$
$$f(t) = ae^{-\lambda t} \; t \geq 0$$

with λ and a constant.
a) Using the normalization condition (eq. 5.13) on page 130 find the normalization constant a in terms of λ.
b) Find the average time it takes for a particle to decay in terms of λ.
c) What is the probability for a particle to "live" more than twice as long as the average time?
d) Find the variance of the probability density function in terms of λ.

6) According to quantum mechanics, the position (x) of a particle in a one dimensional box with dimensions $-L/2 \leq x \leq L/2$ (L constant) can be described by the following probability distribution function p(x):

$p(x) = A\cos^2[\pi x/L]$ for $-L/2 \leq x \leq L/2$, and 0 for all other x.

a) Find the normalization constant A in terms of L.
b) Find the mean, mode, and median position of the particle in the box.
c) Show that the variance (σ^2) of x is given by:

$$\sigma^2 = \left(\frac{L}{\pi}\right)^2 \frac{\pi^2 - 6}{12}$$

d) What is the probability of finding the particle in the region: $L/4 \leq x \leq L/2$?

7) If a constant c is added to each x_i in a sample ($i = 1, n$) such that $y_i = x_i + c$ how does the mean and variance of the y_i's relate to the mean and variance of the x_i's? Alternatively, supposed we multiplied each x_i by the constant, i.e. $y_i = cx_i$. What is the new mean and variance?

8) The probability density function (*pdf*) (often called a Maxwellian Distribution) that describes the speed v of molecules in an ideal gas is given by:

$$p(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}$$

Here m is the mass of the molecule, k is the Boltzmann's constant, and T is the temperature.

a) Show this is a properly normalized *pdf*.
b) Find the most probable speed.
c) Find the average speed.
d) Find the variance of the speed.

9) The probability density function (*pdf*) for an electron in the lowest energy level ($n = 1$) state of a hydrogen atom, as a function of radial distance (r) from the nucleus, is given by:

$p(r) = \frac{4}{a^3} r^2 e^{-2r/a}$ with a = constant (know which one?)

a) Show that this is a properly normalized *pdf*.
b) What is the most probable radial distance (in terms of a) of the electron?
c) What is the average radial distance (in terms of a) of the electron?

10) The following plot (histogram) shows the count vs. R. The count in each bin is as follow:

2, 10, 23, 36, 39, 41, 21, 12, 6, 6, 3.

Calculate the mean (μ) and variance (σ^2) of the distribution. Plot the data with error bars together with the expectation based on a Gaussian distribution:

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$