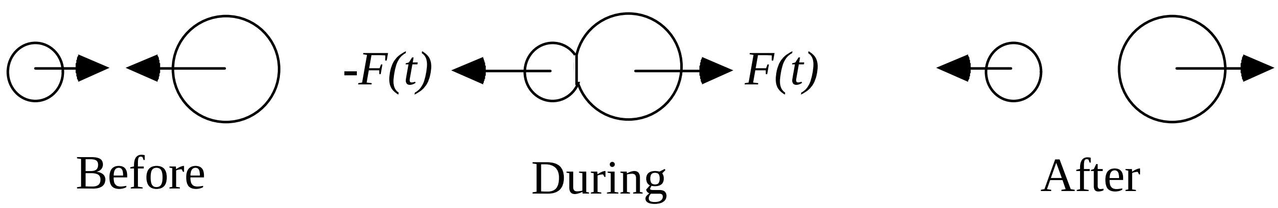
10. Collisions

- Use conservation of momentum and energy and the center of mass to understand collisions between two objects.
- During a collision, two or more objects exert a force on one another for a short time:

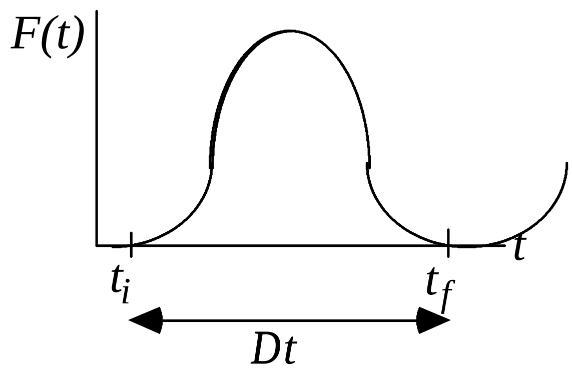


- It is not necessary for the objects to touch during a collision, e.g. an asteroid flied by the earth is considered a collision because its path is changed due to the gravitational attraction of the earth. One can still use conservation of momentum and energy to analyze the collision.

Impulse:

During a collision, the objects exert a force on one another. This force may be complicated and change with time. However, from Newton's 3rd

Law, the two objects must exert an equal and opposite force on one another.



From Newton's 2nd Law:

$$\frac{d\vec{p}}{dt} = \vec{F}(t)$$

$$d\vec{p} = \vec{F}(t)dt$$

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

The change in the momentum is defined as the **impulse** of the collision.

- Impulse is a vector quantity.

Impulse-Linear Momentum Theorem:

In a collision, the impulse on an object is equal to the change in momentum:

$$\vec{J} = \Delta\vec{p}$$

Conservation of Linear Momentum:

In a system of two or more particles that are colliding, the forces that these objects exert on one another are internal forces. These internal forces cannot change the momentum of the system. Only an external force can change the momentum.

The linear momentum of a closed isolated system is conserved during a collision of objects within the system. This follows directly from Newton's 2nd Law:

$$\frac{d\vec{p}}{dt} = \sum F_{ext} = 0$$

$$\Rightarrow \vec{p} = \text{constant}$$

For a two-body collision, the change in momentum of one object is equal and opposite to the change in the momentum of the other object:

$$\vec{p}_i = \vec{p}_f$$

$$\vec{p}_{1_i} + \vec{p}_{2_i} = \vec{p}_{1_f} + \vec{p}_{2_f}$$

$$\vec{p}_{1_f} - \vec{p}_{1_i} = -(\vec{p}_{2_f} - \vec{p}_{2_i})$$

$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

This can also be understood from the impulses delivered by the two equal but opposite internal forces:

$$\Delta\vec{p}_1 = \int \vec{F}_{\text{int}}dt = \Delta\vec{J}$$

$$\Delta\vec{p}_2 = \int(-\vec{F}_{\text{int}})dt = -\Delta\vec{J}$$

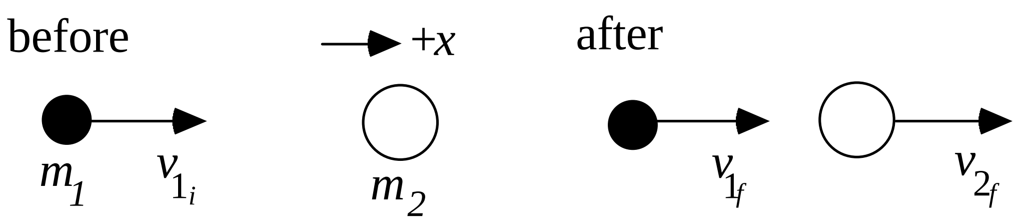
$$\Rightarrow \Delta\vec{p}_1 = -\Delta\vec{p}_2$$

Elastic Collision:

In an elastic collision, the kinetic energy of the **system** is conserved during the collision.

- The kinetic energy of each object will change

Consider a collision in which one of the objects is stationary before the collision:



- Conservation of momentum for an isolated system:

$$m_1 v_{1_i} = m_1 v_{1_f} + m_2 v_{2_f}$$

$$m_1(v_{1_i} - v_{1_f}) = m_2 v_{2_f} \qquad (1)$$

- Conservation of energy for an elastic collision:

$$\tfrac{1}{2}m_1v_{1_i}^2 = \tfrac{1}{2}m_1v_{1_f}^2 + \tfrac{1}{2}m_2v_{2_f}^2$$

$$m_1v_{1_i}^2 - m_1v_{1_f}^2 = m_2v_{2_f}^2$$

$$m_1(v_{1_i} - v_{1_f})(v_{1_i} + v_{1_f}) = m_2v_{2_f}^2 \qquad (2)$$

$(2)\div(1)$: $$v_{1_i} + v_{1_f} = v_{2_f} \qquad (3)$$

Substitute into (1):

$$m_1(v_{1_i} - v_{1_f}) = m_2(v_{1_i} + v_{1_f})$$

$$v_{1_f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1_i}$$

Substitute into (3):

$$v_{2_f} = v_{1_i} + \frac{m_1 - m_2}{m_1 + m_2}v_{1_i}$$

$$= \frac{2m_1}{m_1 + m_2}v_{1_i}$$

- Particle #2 always moves in the positive direction. Particle #1 will move in the positive direction if $m_1 > m_2$ else it will recoil and move in the negative direction.

- If $m_1 = m_2$,

$$v_{1_f} = 0$$

$$v_{2_f} = v_{1_i}$$

i.e. particle #1 comes to rest and particle #2 moves off with original speed of particle #1.

- If $m_2 >> m_1$,

$$v_{1_f} \approx -v_{1_i}$$

$$v_{2_f} \approx \frac{2m_1}{m_2} v_{1_i} << v_{1_i}$$

i.e. particle #1 bounces back in the opposite direction with almost the same speed it had originally. The massive particle #2 moves slowly in the original direction of particle #1.

- If $m_1 >> m_2$,

$$v_{1_f} \approx v_{1_i}$$

$$v_{2_f} \approx 2v_{1_i}$$

i.e. the massive particle #1 continues to move almost as if it did not hit anything, whereas particle #2 flies off with twice the initial speed of the massive particle.

We can solve for the case where both particles are moving before the collision using conservation of momentum and energy:

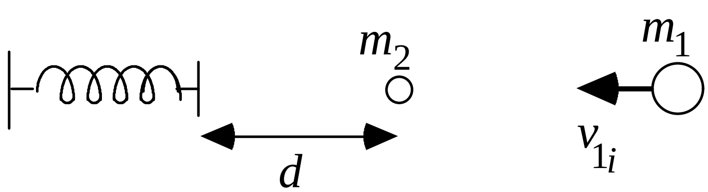
$$v_{1_f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1_i} + \frac{2m_2}{m_1 + m_2} v_{2_i}$$

$$v_{2_f} = \frac{2m_1}{m_1 + m_2} v_{1_i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2_i}$$

- The labeling "1" and "2" is completely arbitrary. We can change 1⁄2 everywhere and get back the same answer.
- Setting $v_{2_i} = 0$ yields the previous set of equations.

Example:

A 350-g target glider is at rest on a track, a distance d = 53 cm from the end of the track. A 590-g projectile glider approaches the target glider with a velocity $v_{1_i} = -75\ \text{cm/s}$ and collides elastically. The target glider rebounds elastically from a short spring at the end of the track and meets the projectile glider for a second time. How far from the end of the track does this second collision occur?

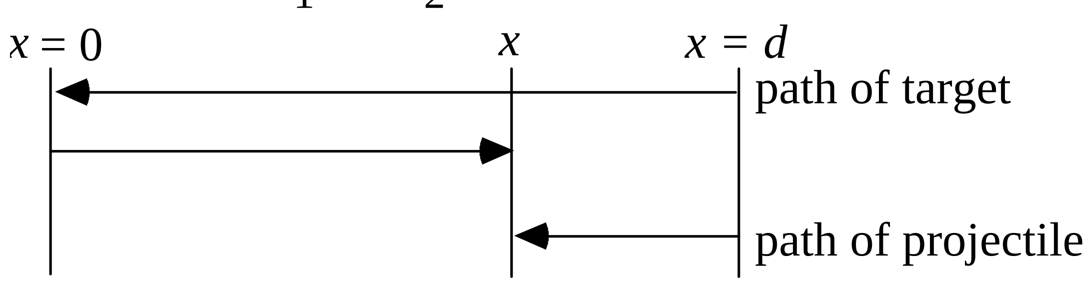


- The projectile glider will move with some speed after the 1st collision toward the spring.
- The 1st collision gives the target some velocity to travel to the spring.
- The target rebounds elastically from the spring with the same speed after 1st collision.

After 1st collision:

$$v_{2_f} = \frac{2m_1}{m_1 + m_2} v_{1_i} = -94 \text{ cm / s}$$

$$v_{1_f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1_i} = -19 \text{ cm / s}$$



$$t = \frac{d+x}{v_{2_f}} = \frac{d-x}{v_{1_f}}$$

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$$x = d\,\frac{v_{2_f} - v_{1_f}}{v_{2_f} + v_{1_f}} = 34 \text{ cm}$$

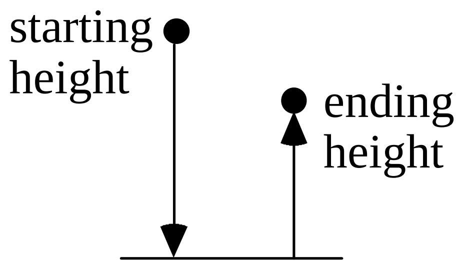
Conceptual Question:

A popular swinging-balls apparatus consists of an aligned row of identical elastic balls that are suspended by strings so that they barely touch each other. When two balls are lifted from one end and released, they strike the row and two balls pop out from the other end, If instead one ball popped out with twice the velocity of the other two. This would violate the conservation of:

1. momentum
2. energy
3. both of these
4. none - it is possible for one ball to fly out

Inelastic Collision:

In an inelastic collision between two objects in an isolated system, kinetic energy is not conserved, but the linear momentum is conserved. Most collision we observe everyday are inelastic with some loss of kinetic energy.

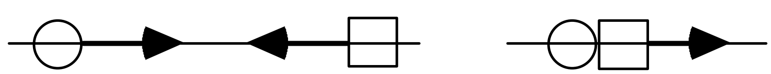


If we drop a ball from a height *h*, it will collide with the earth (ground) and bounce back up. However, it will not return to the original height because some kinetic energy is lost during the collision.

When two cars collide with one another, parts of the cars crumple and bend. Some of the kinetic energy of the system goes into this deformation, so kinetic energy is lost during the collision. This is part of the design by automatic engineers by having a "crumple zone" to take energy out of the collision to protect the driver.

Completely Inelastic Collision:

- Two objects stick together after the collision.
- After the collision, the combined objects have a mass equal to the sum of the two masses and move with the velocity of the center of mass.
- Linear momentum is **conserved**.
- Maximum amount of kinetic energy is lost.



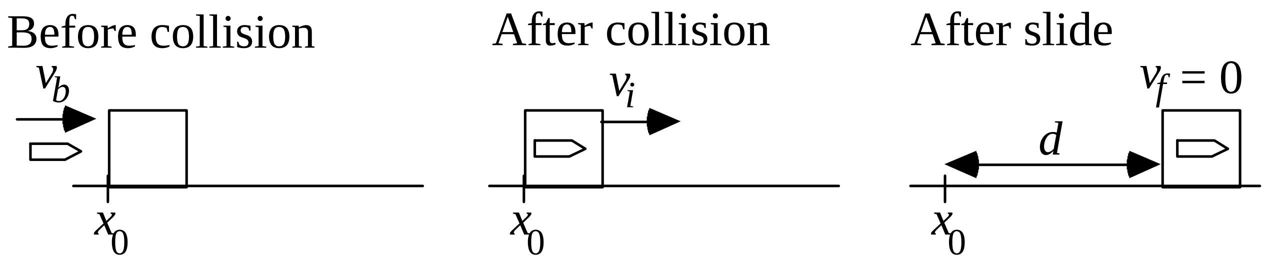
$$p_i^{sys} = p_f^{sys}$$

$$m_1 v_{1_i} + m_2 v_{2_i} = (m_1 + m_2) v$$

$$v = v_{cm} = \frac{m_1 v_{1_i} + m_2 v_{2_i}}{m_1 + m_2}$$

Example:

A bullet of mass 4.5 g is fired horizontally into a 2.4-kg wooden block at rest on a horizontal surface. The coefficient of kinetic friction between the block and the surface is 0.20. The bullet comes to rest in the block, which moves 1.8 m. (a) What is the speed of the block immediately after the bullet comes to rest within it, and (b) at what speed is the bullet fired?



(a)
$$f = mN$$
$$= m(m_b + m_w)g$$
$$(m_b + m_w)a = -m(m_b + m_w)g$$
$$a = -mg$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$0 = v_i^2 - 2mgd$$
$$v_i = \sqrt{2mgd}$$
$$= 2.65 \text{ m / s}$$

(b) Conservation of momentum:
$$m_b v_b + 0 = (m_b + m_w)v_i$$
$$v_b = \frac{m_b + m_w}{m_b} v_i$$
$$= 1400 \text{ m / s}$$

Conceptual Question:

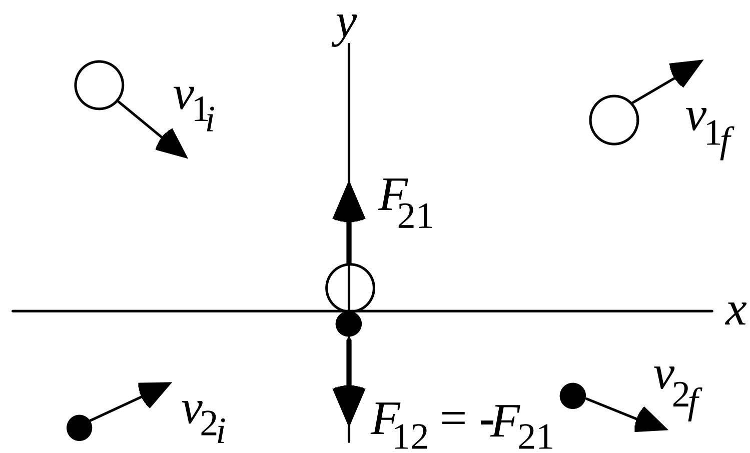
A piece of taffy slams into and sticks to another identical piece at rest. The momentum of the two pieces stuck together after the collision is the same as it was before the collision, but this is not true of the kinetic energy which is partly turned into heat. What percentage of the kinetic energy is turned into heat?

1. 0% 2. 25%
3. 50% 4. 75%
5. more information must be given

Collisions in 2-Dimensions:

★ When we studied the motion of objects, we started by studying motion in one dimension and then we found that every thing we learned could be easily applied to 2 or 3 dimensions. The same is true for collisions in 2 or 3 dimensions:

- Linear momentum is each direction is conserved.
- In elastic collisions, the kinetic energy of the system is conserved.
- In totally inelastic collisions, the two objects stick together and move with a common velocity, the velocity of the center of mass.



$$p_{i_x}^{sys} = p_{f_x}^{sys}$$

$$m_1 v_{1_{i_x}} + m_2 v_{2_{i_x}} = m_1 v_{1_{f_x}} + m_2 v_{2_{f_x}}$$

$$p_{i_y}^{sys} = p_{f_y}^{sys}$$

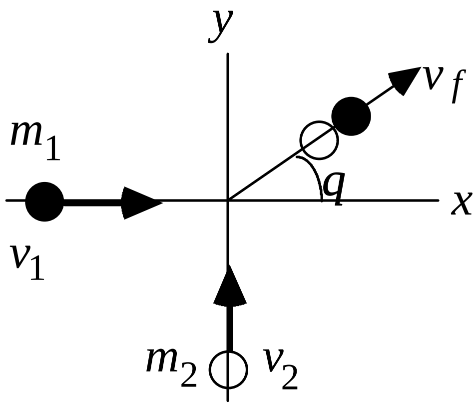
$$m_1 v_{1_{i_y}} + m_2 v_{2_{i_y}} = m_1 v_{1_{f_y}} + m_2 v_{2_{f_y}}$$

For elastic collisions:

$$\tfrac{1}{2} m_1 v_{1_i}^2 + \tfrac{1}{2} m_2 v_{2_i}^2 = \tfrac{1}{2} m_1 v_{1_f}^2 + \tfrac{1}{2} m_2 v_{2_f}^2$$

Example:

One ice skater (m_1) is skating due east along a frozen lake with a speed v_1. Another skater (m_2) is skating due north with speed v_2. They collide into a big heap and slide across the ice together. What is their velocity after the collision?



$$p_{i_x}^{sys} = p_{f_x}^{sys}$$

$$m_1 v_1 = (m_1 + m_2) v_{f_x}$$

$$v_{f_x} = \frac{m_1 v_1}{m_1 + m_2}$$

$$p_{i_y}^{sys} = p_{f_y}^{sys}$$

$$m_2 v_2 = (m_1 + m_2) v_{f_y}$$

$$v_{f_y} = \frac{m_2 v_2}{m_1 + m_2}$$

$$q = \tan^{-1}\left(\frac{v_{f_y}}{v_{f_x}}\right) = \tan^{-1}\left(\frac{m_2 v_2}{m_1 v_1}\right)$$

$$v_f = \sqrt{v_{f_x}^2 + v_{f_y}^2}$$

$$= \frac{\sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}}{m_1 + m_2}$$

Check:

- If two skaters of equal momentum collided:

$$m_1 v_1 = m_2 v_2$$

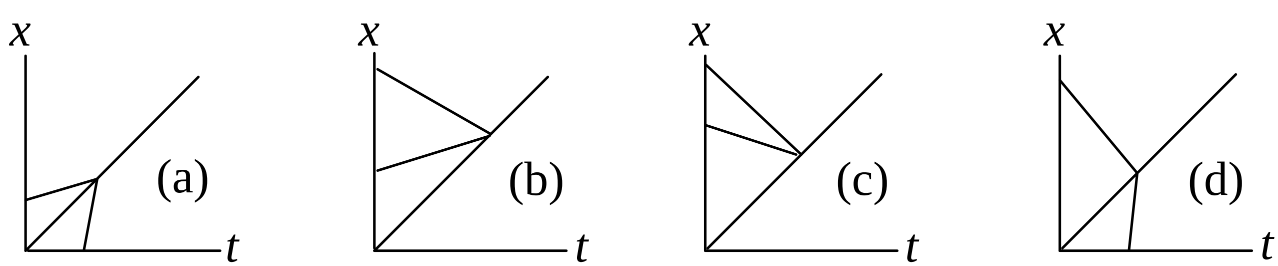
$$q = 45°$$

- If skater 2 was stationary:

$$q = \tan^{-1}(0) = 0^{\circ}$$

Conceptual Question:

(a) The figures below show the position vs. time plot for two bodies and their center of mass. The two bodies undergo a completely inelastic one-dimensional collision while moving along the x-axis. Which graph corresponds to a physically impossible situation?



(b) The figure below shows seven identical blocks on a frictionless floor. Initially, block *a* and *b* are moving rightward and block *g* leftward, each with the same speed. A series of elastic collisions occur. After the last collision, what are the speeds and direction of motion of each block?