13. Equilibrium

Static Equilibrium:

An object is in static equilibrium if its center of mass is stationary.

There is no net force or torque acting on it:

$$\sum F_x = 0 \qquad \sum t_x = 0$$

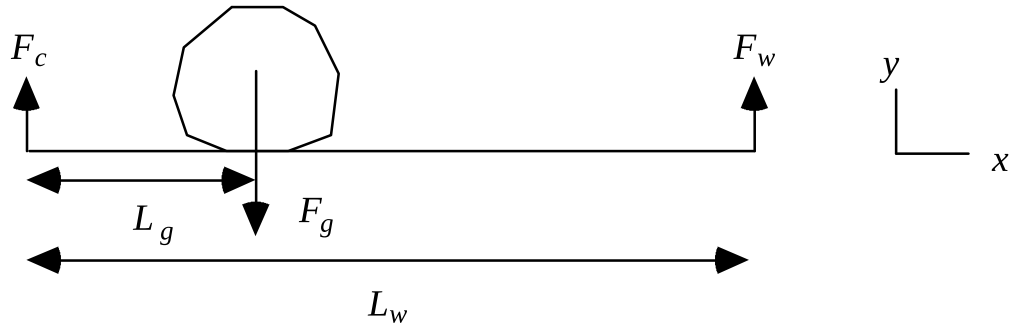
$$\sum F_y = 0 \qquad \sum t_y = 0$$

$$\sum F_z = 0 \qquad \sum t_z = 0$$

- The torque can be measured about any point.
- The study of static equilibrium is essential in structural engineering, e.g. buildings, bridges.

Example:

A 71.0 N boulder is placed 1.4 m from one end of a 2.00 m long board. Cliff and Will support the board at each end so that it is horizontal. Cliff is nearest the boulder. Neglecting the weight of the board, what forces does each apply to the board?



$$\sum F_y = F_c + F_w - F_g = 0$$

$$\sum t_z = 0$$

- The torque is along the z direction because the force is in the y direction.
- Pick a point where some of the forces have zero torques to simplify problem.

$$\sum t_z = 0 + F_w L_w - F_g L_g = 0$$

$$F_w = \frac{F_g L_g}{L_w}$$

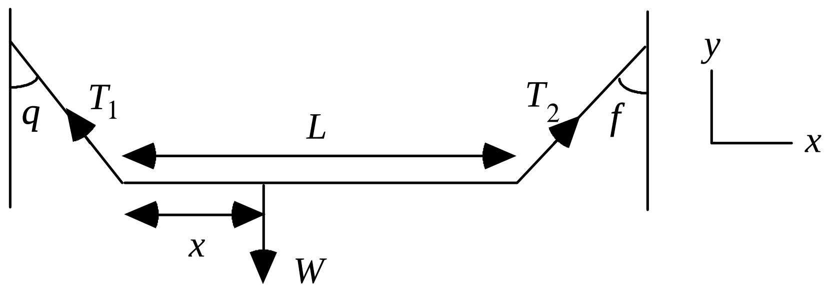
$$= 21.3 \text{ N}$$

$$F_c = F_g - F_w$$

$$= 49.7 \text{ N}$$

Example:

A nonuniform bar of weight W is suspended at rest in a horizontal position by two massless cords. One cord makes the angle q = 36.9° with the vertical; the other makes the angle f = 53.1° with the vertical. If the length L of the bar is 6.10 m, compute the distance x from the left-hand end of the bar to its center of mass.



$$\sum F_x = T_2 \sin f - T_1 \sin q = 0 \qquad (1)$$

$$\sum F_y = T_2 \cos f + T_1 \cos q - W = 0 \qquad (2)$$

Take the torque around the left end of the bar:

$$\sum t_z = -Wx + (T_2 \cos f)L = 0$$

$$x = \frac{(T_2 \cos f)L}{W} \qquad (3)$$

(1): $$T_1 = T_2 \frac{\sin f}{\sin q}$$

(2): $$0 = T_2 \cos f + T_2 \frac{\sin f}{\sin q} \cos q - W$$

$$W = T_2 \left(\cos f + \sin f \cot q \right)$$

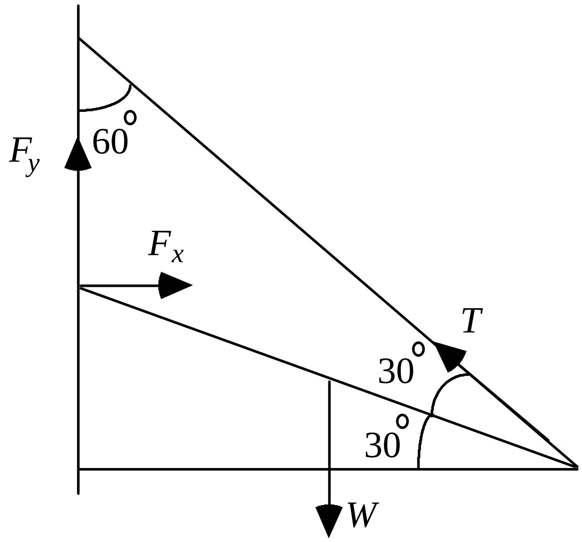
$$\frac{T_2}{W} = \frac{1}{\cos f + \sin f \cot q}$$

(3): $$x = \frac{L \cos f}{\cos f + \sin f \cot q}$$

$$= 2.93 \text{ m}$$

Example:

One end of a uniform beam that weighs 50.0 lbs and is 3.00 ft long is attached to the wall with a hinge. The other end is supported by a wire. (a) Find the tension in the wire. What are the (b) horizontal and (c) vertical components of the force of the hinge on the beam.



Take the torque about the hinge:

$$\sum t = T(\sin 30^{o})L - W(\cos 30^{o})\frac{L}{2}$$

$$T = \frac{W \cot 30^{o}}{2} = 43.3 \text{ lbs}$$

$$\sum F_x = F_x - T\cos 60^{o} = 0$$

$$F_x = T\cos 60^{o} = 21.7 \text{ lbs}$$

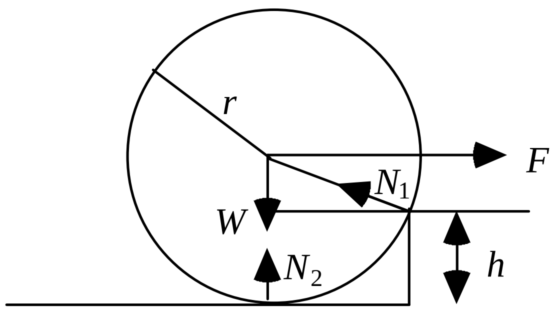
$$\sum F_y = F_y + T\sin 60^{o} - W = 0$$

$$F_y = W - T\sin 60^{o} = 12.5 \text{ lbs}$$

Example:

In the figure below, what magnitude of force, *F*, applied horizontally at the axle of the wheel is necessary to raise the wheel over the obstacle of height *h*? Take *r* as the radius of the wheel and *W* as its weight.

- When the wheel is just starting to rotate, the normal force N_2 will be zero.
- The step exerts a normal force that is directed towards the center of the wheel.



Take the torque about the point of contact with the step:

$$\sum t = WL - F(r-h) = 0$$

$$F = \frac{WL}{r-h}$$

$$= \frac{W\left[r^2 - (r-h)^2\right]^{\frac{1}{2}}}{r-h}$$

$$= \frac{W\left[2rh - h^2\right]^{\frac{1}{2}}}{r-h}$$