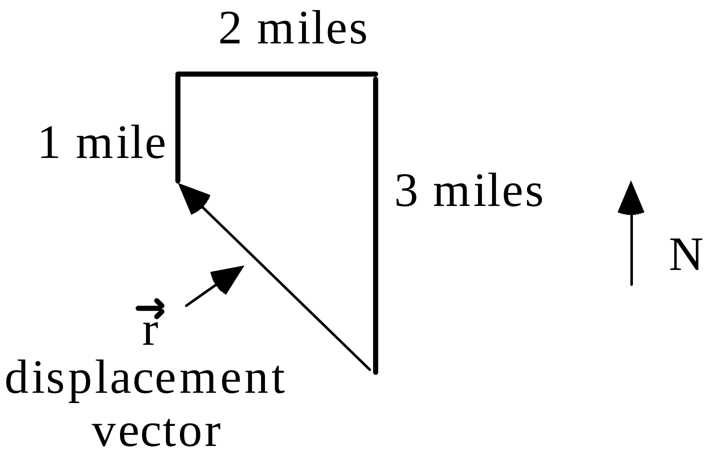
3. Vectors

We can use "+" and "-" to indicate the direction of one-dimensional motion. For motion in 3D, we use the concept of a vector.

A vector has both a magnitude and a direction, represented graphically by an arrow with length proportional to magnitude.
e.g. displacement, velocity, and acceleration.

A scalar has a magnitude but no direction.
e.g. time and speed.

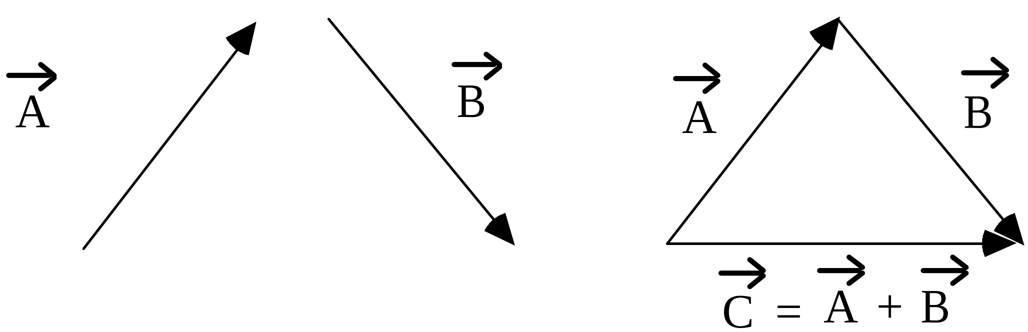
Displacement Vector:
Consider a car travelling north from OSU for 3 miles which then turns west for 2 miles and then south for 1 mile. What is the car's displacement from OSU?



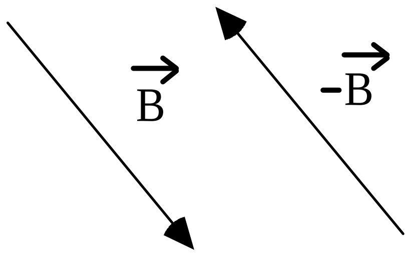
- displacement vectors describe the change in position from start to finish, independent of the path taken.

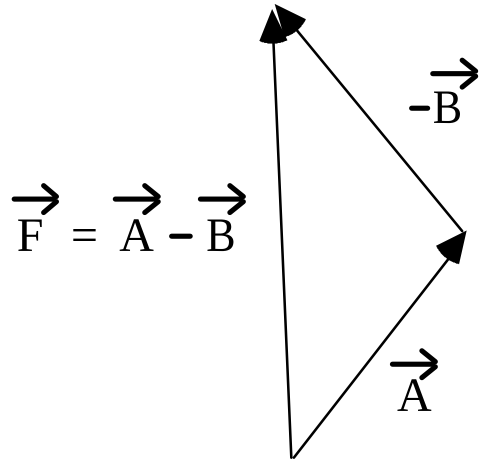
Addition and Subtraction of Vectors:

Adding two vectors graphically by placing the end of one arrow to the tip of the other:

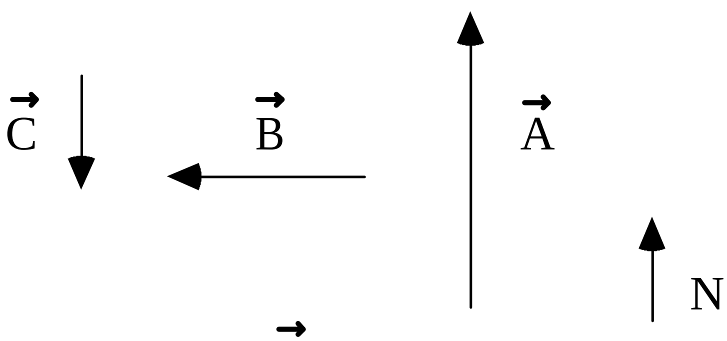


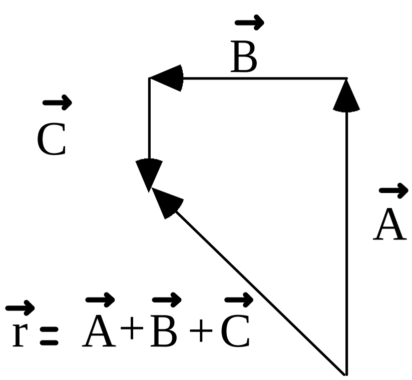
Treat subtraction as addition of a vector with a negative sign:





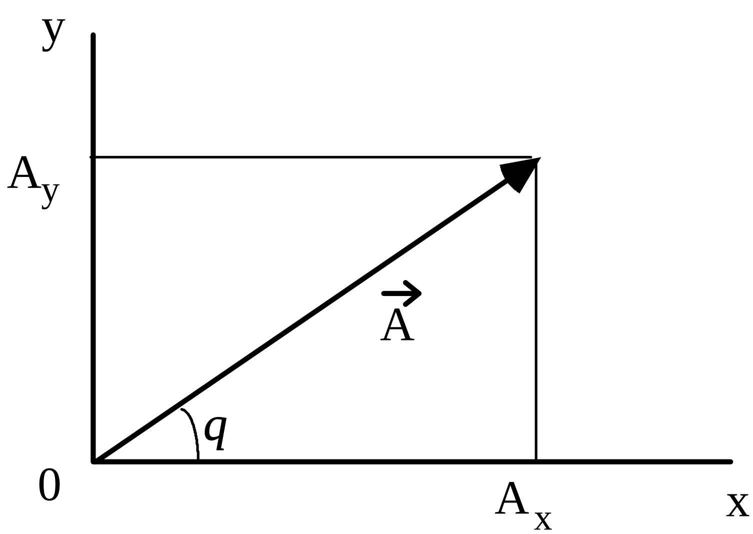
We can also add or subtract a series of vectors. Consider the car trip around OSU:





Vector Components:

We can also add vectors by adding the individual components. To calculate the individual components, choose a Cartesian coordinate system:



$$A_x = A\cos q$$

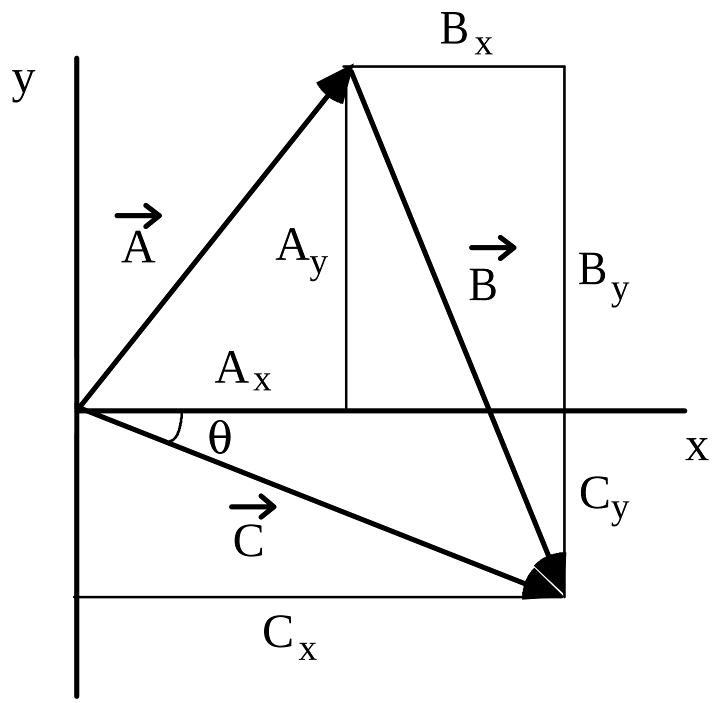
$$A_y = A\sin q$$

Addition of Vectors by Components:

$$\vec{C} = \vec{A} + \vec{B}$$

$$C_x = A_x + B_x$$

$$C_y = A_y + B_y$$



$$|\vec{C}| = \sqrt{C_x^2 + C_y^2}$$

$$q = \tan^{-1}\frac{C_y}{C_x}$$

Subtraction of Vectors by Components:

$$\vec{F} = \vec{A} - \vec{B}$$

$$F_x = A_x - B_x$$

$$F_y = A_y - B_y$$

Similarly, in 3D:

$$\vec{C} = \vec{A} + \vec{B}$$

$$C_x = A_x + B_x$$

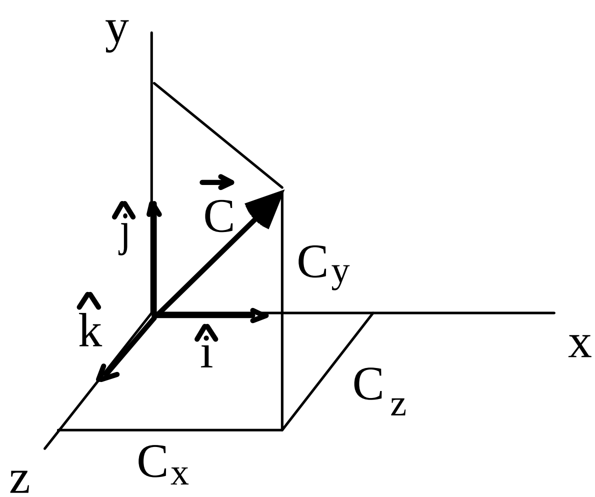
$$C_y = A_y + B_y$$

$$C_z = A_z + B_z$$

$$\left|\vec{C}\right| = C = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

Unit Vectors:

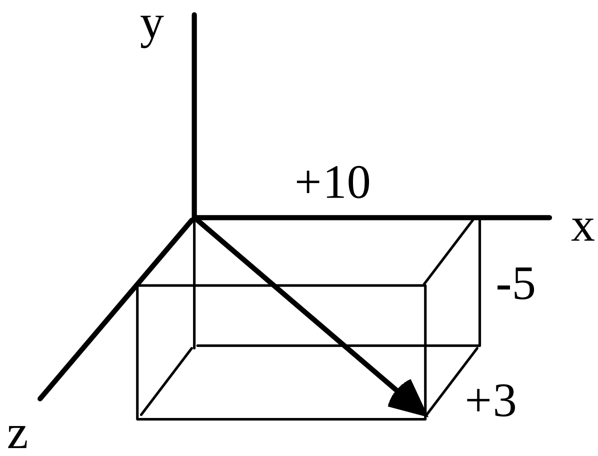
Vector with a length of 1.



Any vector can be written as a sum of unit vectors:

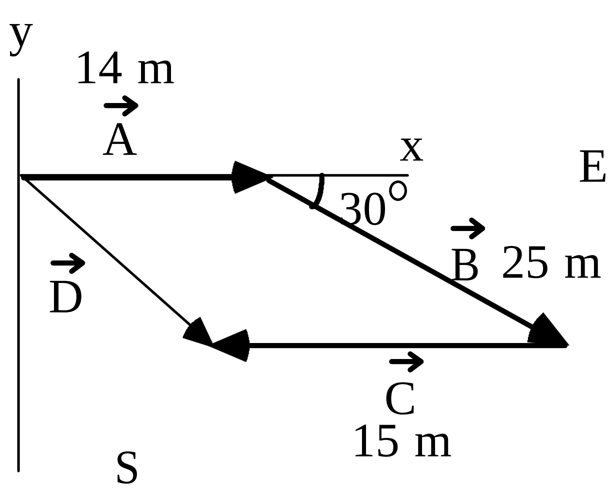
$$\vec{C} = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$

e.g. $\vec{D} = 10\hat{i} - 5\hat{j} + 3\hat{k}$



Example:

A P131 student jogs east for 14 m, then turns south by 30° and runs 25 m, and then turns directly west and runs 15 m. What is the student's displacement?



$$\vec{A} = A_x\hat{i} + A_y\hat{j} = 14 \text{ m } \hat{i}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} = 25\cos 30^{\circ}\text{m } \hat{i} - 25\sin 30^{\circ}\text{m } \hat{j}$$

$$\vec{C} = C_x\hat{i} + C_y\hat{j} = -15 \text{ m } \hat{i}$$

$$D_x = A_x + B_x + C_x = 14 \text{ m} + 22 \text{ m} - 15 \text{ m} = 21 \text{ m}$$

$$D_y = A_y + B_y + C_y = 0 \text{ m} - 12.5 \text{ m} + 0 \text{ m} = -12.5 \text{ m}$$

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(21)^2 + (-12.5)^2} = 24 \text{ m}$$

$$\theta = \tan^{-1}\frac{D_y}{D_x} = \tan^{-1}\left(\frac{-12.5}{21}\right) = 31^{\circ} \text{ (south of east)}$$