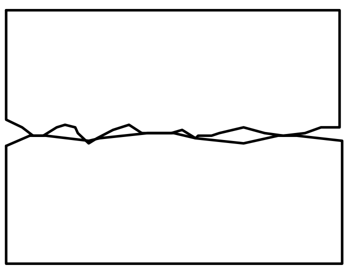
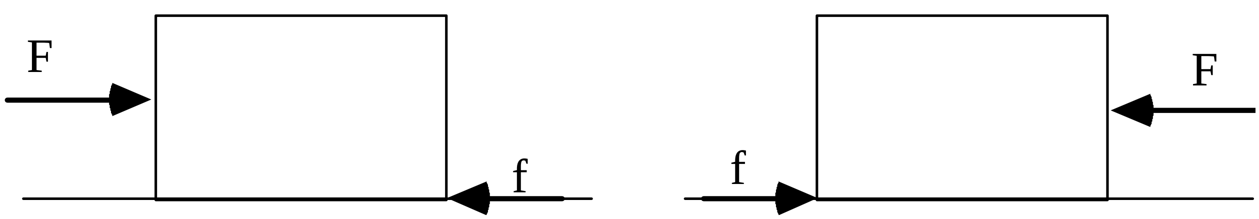
6. Forces and Motion-II

Friction:

- The resistance between two surfaces when attempting to slide one object across the other.
- Friction is due to interactions at molecular level where “rough edges” bond together:

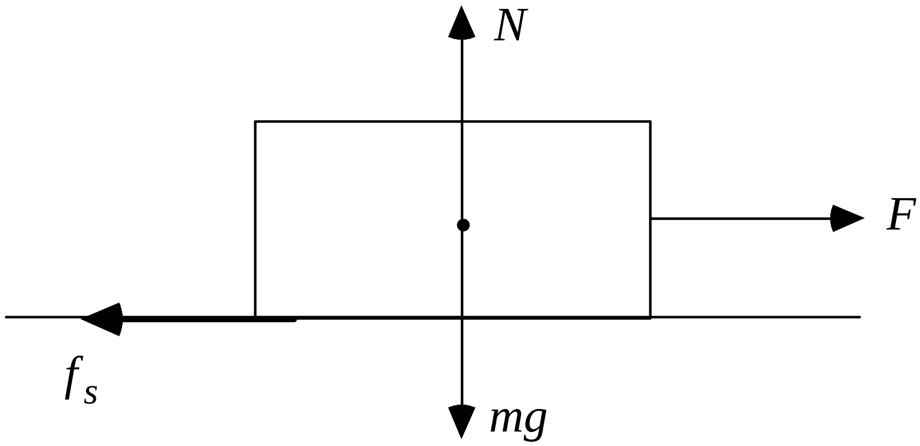


- Friction is always opposite to the direction of motion.



- Friction are both friend and foe of everyday life, for example:
  ★ Allow you to start your car from rest
  ★ Slow the car down
  ★ Allow you to break

Static Friction:



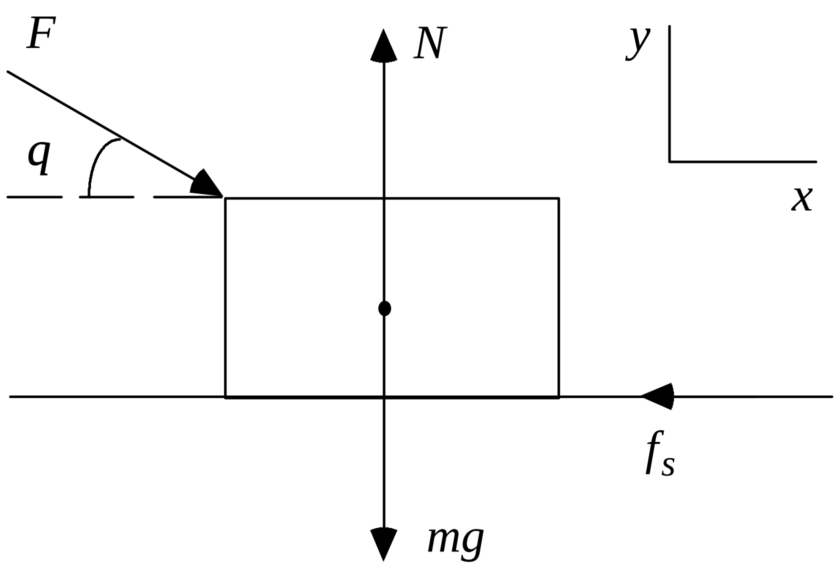
- The frictional force between two surfaces in attempting to slide one object across the other but neither objects are moving with respect to each other.
- The friction is always equal to the net force parallel to the surface.
- If one net force increases or decreases, the friction force will also increase or decrease to compensate.
- Experimentally, the maximum magnitude of static friction is proportional to the magnitude of the normal force:

$$f_s^{\max} = m_s N$$

where m_S is the coefficient of static friction, which is dependent on the two surfaces in contact and must be determined experimentally.

Example:

A Physics 131 student applies a force F at angle q on a box of mass m on a floor. If the coefficient of static friction is m_S, calculate the force required to move the box.



$$\sum F_y = N - mg - F\sin q = 0$$

$$N = mg + F\sin q$$

$$\sum F_x = F\cos q - f_s > 0$$

$$F\cos q - m_s N > 0$$

$$F\cos q - m_s(mg + F\sin q) > 0$$

$$F\cos q - m_s F\sin q > m_s mg$$

$$F > \frac{m_s mg}{\cos q - m_s \sin q}$$

- If we increase the angle to the critical angle, $\cos q = m_s \sin q$, no matter how hard the student pushes, the box will not move, i.e. the vertical component of the force produces a normal force which yields a frictional force that is larger than the horizontal component of the force.

Kinetic Friction:

Once an object is in motion, the friction force changes to kinetic friction force, which is proportional to the magnitude of normal force,

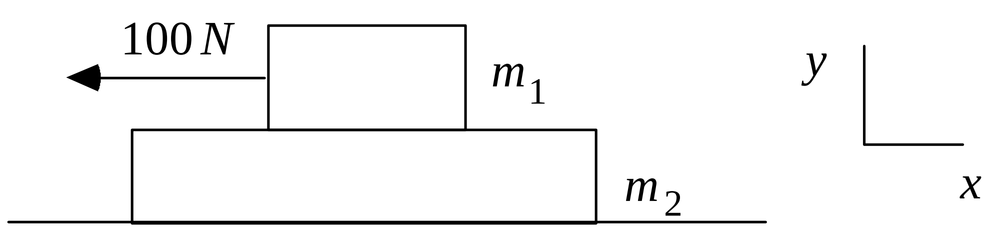
$$f_k = m_k N$$

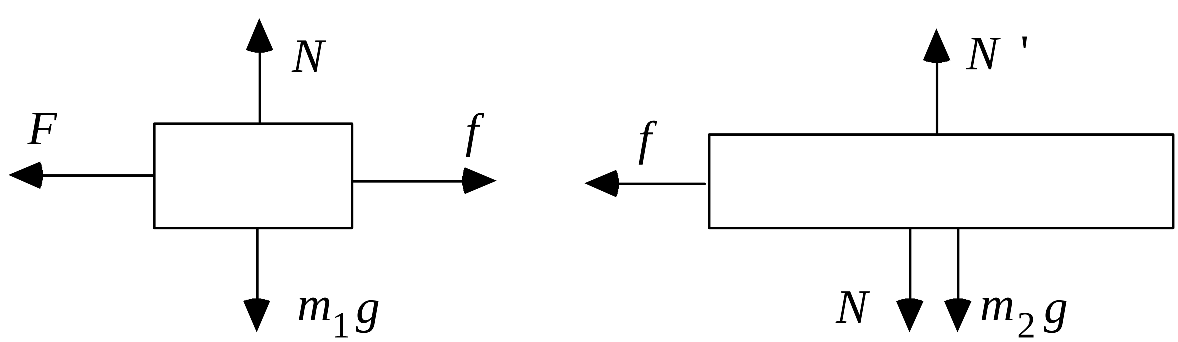
where m_k is the coefficient of kinetic friction.

- Unlike the static friction force, the kinetic friction force does not change in magnitude. It has a single value.
- $m_k < m_s$, i.e. once you start moving an object, it is easier to keep it moving.

Problem 39 (p. 127):

A 10 kg block rests on top of a 40 kg slab which rests on a frictionless floor. The coefficient of static and kinetic friction between the block and the slab are 0.60 and 0.40 respectively. The block is pulled with a force of 100 N. What are the accelerations of (a) the block and (b) the slab?





- Must first decide on whether the block will slide relative to the slab

$$\sum F_y = N - m_1 g = 0$$

$$N = m_1 g$$

$$f_s^{\max} = m_s N$$

$$= m_s m_1 g$$

$$= (0.60)(10 \text{ kg})(9.8 \text{ m / s}^2)$$

$$= 59 \text{ N}$$

- The maximum frictional force is smaller than the applied force.

  The block will move relative to the slab.

Block: $\sum F_x = m_k N - F = m_1 a_1$

$$m_k m_1 g - F = m_1 a_1$$

$$a_1 = m_k g - \frac{F}{m_1}$$

$$= -6.1 \text{ m / s}^2$$

Slab: $$\sum F_x = -m_k N = m_2 a_2$$

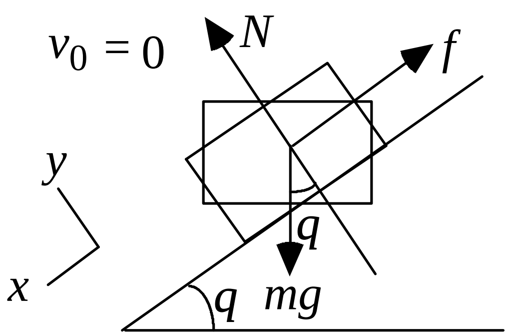
$$-m_k m_1 g = m_2 a_2$$

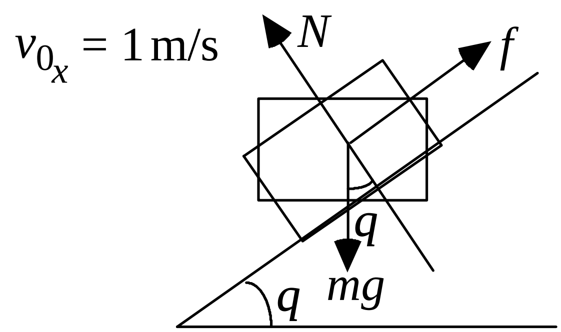
$$a_2 = \frac{-m_k m_1 g}{m_2}$$

$$= -0.98 \text{ m / s}^2$$

Example:

Two identical blocks of 10 kg each are sitting on an incline with an angle q = 30°. One block is released so that its initial speed is zero, while the other block is released with an initial speed of 1 m / s. What are the accelerations of each block just after the release and 10 s later? The coefficients of static and kinetic friction are 0.7 and 0.6.





Block 1: $\sum F_y = N - mg\cos q = 0$

$$N = mg\cos q$$

$$f_s = m_s N$$

$$= m_s mg\cos q$$

$$= 59 \text{ N}$$

$$F_{g_x} = mg\sin q$$

$$= 49 \text{ N}$$

$$< f_s$$

The block will remain stationary.

Block 2: $\sum F_y = N - mg\cos q = 0$

$$N = mg\cos q$$

$$\sum F_x = mg\sin q - f = ma$$

$$mg\sin q - m_k N = ma$$

$$mg\sin q - m_k mg\cos q = ma$$

$$a = g(\sin q - m_k \cos q)$$

$$= -0.19 \text{ m/s}^2$$

- Since the acceleration is in the -*x* direction (up the plane), the block will slow down and come to a stop.

$$v = v_0 + at$$

$$t = \frac{v - v_0}{a}$$

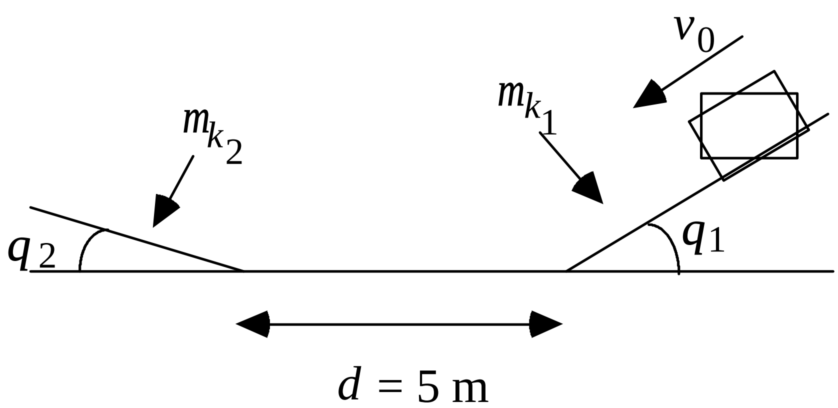
$$= \frac{(0-1)\ \text{m/s}}{-0.19\ \text{m/s}^2}$$

$$= 5.2\ \text{s}$$

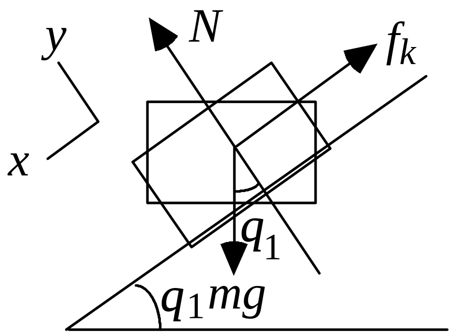
- Since this is before 10 s, the acceleration of both blocks will be zero after 10 s.

Example:

A box at 2 m up a ramp (*q* = 30°) slides down with an initial velocity of 1 m/s. The ramp has a coefficient of kinetic friction of 0.6. The box then slides across a 5 m sheet of ice and up a second ramp (*q* = 5°). The motion from start to where the box comes to rest took 9 s. What is the coefficient of kinetic friction on the second ramp and how far up the second ramp did the box slide?



v_0
μ_{k_1}
μ_{k_2}
θ_2
θ_1
$d = 5$ m



$$\Sigma F_y = N - mg\cos q_1 = 0$$

$$N = mg\cos q_1$$

$$\Sigma F_x = mg\sin q_1 - f_k = ma_{x_1}$$

$$mg\sin q_1 - m_k mg\cos q_1 = ma_{x_1}$$

$$a_{x_1} = g(\sin q_1 - m_k \cos q_1)$$

$$= (9.8\ \mathrm{m/s^2})(\sin 30° - 0.6\cos 30°)$$

$$= -0.19\ \mathrm{m/s^2}$$

$$x = x_0 + v_{0_x} t_1 + \tfrac{1}{2} a_{x_1} t_1^{\,2}$$

$$2 = 0 + 1 \times t_1 - \tfrac{1}{2}(0.19) t_1^{\,2}$$

$$0.092 t_1^{\,2} - t_1 + 2 = 0$$

$$t_1 = 1.6 \text{ or } -12.1\ \mathrm{s}$$

$$v = v_0 + a_{x_1} t_1$$

$$= 1 - 0.19 \times 1.6$$

$$= 0.70\ \mathrm{m/s}$$

$$t_{ice} = \frac{d}{v}$$

$$= \tfrac{5}{0.7}$$

$$= 7.1 \text{ s}$$

$$t_2 = t_{total} - t_1 - t_{ice}$$

$$= 9 - 1.6 - 7.1$$

$$= 0.3 \text{ s}$$

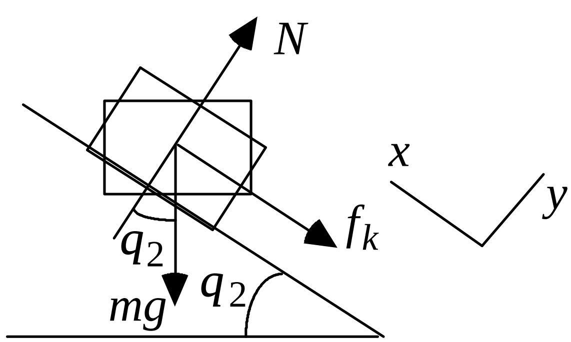
$$v = v_0 + at$$

$$0 = v_0 + a_{x_2} t$$

$$a_{x_2} = -\frac{v_0}{t}$$

$$= -\frac{0.7 \text{ m/s}}{0.3 \text{ s}}$$

$$= -2.33 \text{ m/s}^2$$



$$\sum F_y = N - mg\cos q_2 = 0$$

$$N = mg\cos q_2$$

$$\sum F_x = -f_k - mg\sin q_2 = ma_{x_2}$$

$$-m_{k_2} mg\cos q_2 - mg\sin q_2 = ma_{x_2}$$

$$m_{k_2} g\cos q_2 = -g\sin q_2 - a_{x_2}$$

$$m_{k_2} = \frac{-g\sin q_2 - a_{x_2}}{g\cos q_2}$$

$$= 0.15$$

★ We solve this complicated problem one piece at a time.

$$v^2 = v_0{}^2 + 2a(x - x_0)$$

$$0 = v_0{}^2 + 2a_{x_2}(d - 0)$$

$$d = -\frac{v_0{}^2}{2a_{x_2}}$$

$$= -\frac{0.7^2}{2(-2.33)}$$

$$= 0.11 \text{ m}$$

Centripetal Force:

The force directed toward the center that provides the acceleration for an object in a circular motion:

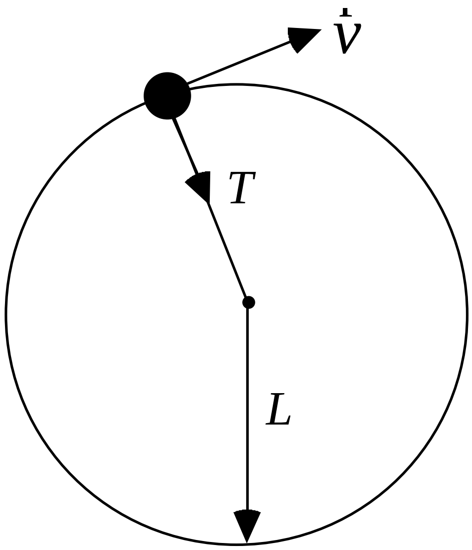
$$F_c = ma = m\frac{v^2}{r}$$

e.g. gravity, tension, normal force.

- Centripetal force provides the centripetal acceleration.
- Centripetal acceleration is **not** a force.

Example:

A 2 kg ball on a string of length $L = 1$ m has a period of 0.5 s. Ignoring the gravitational force, what is the speed, centripetal acceleration, and tension?



$$T = \frac{2pL}{v}$$

$$v = \frac{2pL}{T}$$

$$= 12.5 \text{ m / s}$$

$$a_c = \frac{v^2}{L}$$

$$= 158 \text{ m / s}^2$$

$$T = F_c$$

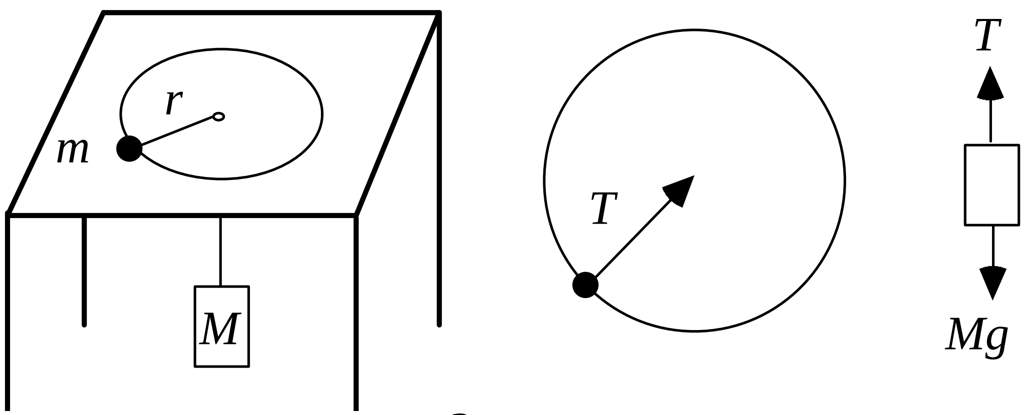
$$= ma_c$$

$$= 316 \text{ N}$$

- Tension is the centripetal force that provides the centripetal acceleration

Problem 57 (p.128)

A mass m on a frictionless table is attached to a hanging mass M by a cord through a hole in the table. Find the speed with which m must move in order for M to stay at rest for a radius r.



$$T = ma_c = m\frac{v^2}{r}$$

$$T - Mg = 0$$

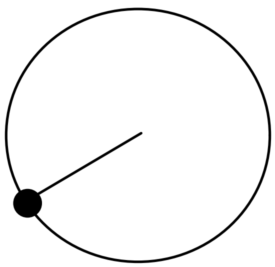
$$Mg = m\frac{v^2}{r}$$

$$v = \sqrt{\frac{Mgr}{m}}$$

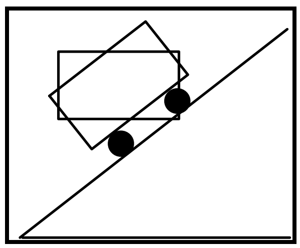
Conceptual Question:

Determine the source of the centripetal force that causes the object to move in a circle:

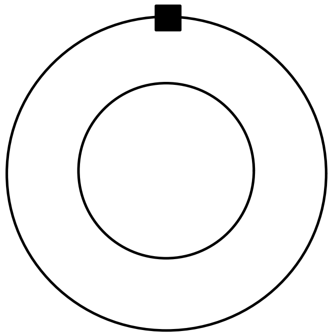
1. A ball on a string:



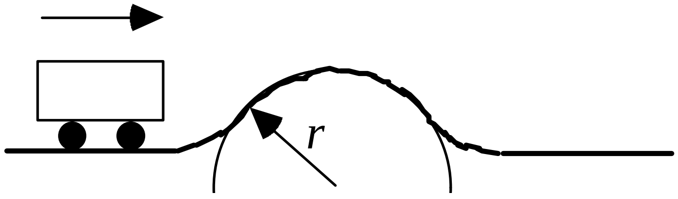
2. A car going around a banked curve:



3. A satellite orbiting the earth:

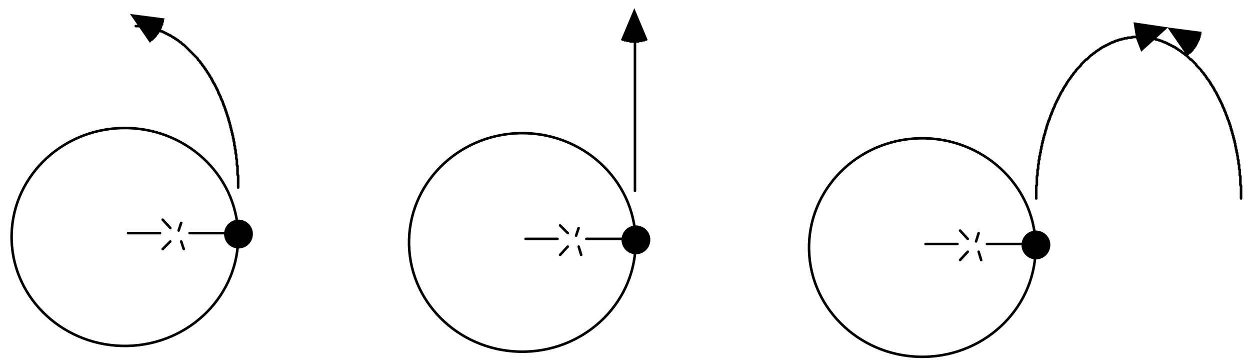


4. A car going over a rounded hill:



Conceptual Question:

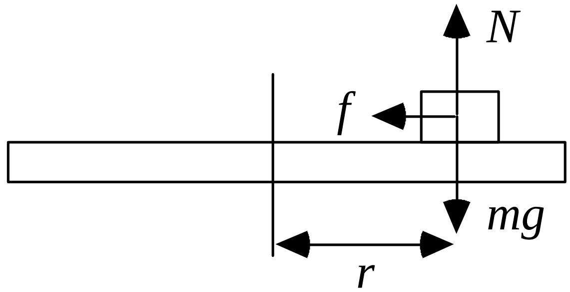
A ball on a string is moving in uniform circular motion. Which diagram represents the path of the ball after the string suddenly breaks?



(a) (b) (c)

Problem 59 (p. 128)

A small coin is placed on a turntable making 3 revolutions in 3.14 s. (a) What are the speed and acceleration of the coin when it rides without slipping at 5.0 cm from the center of the turntable? (b) What is the magnitude of the friction force if the mass of the coin is 2.0 g? (c) What is the coefficient of static friction if the coin is observed to slide off when it is more than 10 cm from the center of the turntable?



(a) $$v = \frac{2pr}{T}$$

$$= \frac{2pr}{\frac{p}{3}}$$

$$= 6r$$

$$= 0.3 \text{ m/s}$$

$$a = \frac{v^2}{r}$$

$$= 1.82 \text{ m/s}^2$$

(b) $$f = ma$$

$$= 3.64 \times 10^{-3} \text{ N}$$

(c) At r = 10 cm, the friction is just enough to provide the centripetal force:

$$mN = \frac{mv^2}{r}$$

$$mmg = \frac{mv^2}{r}$$

$$m = \frac{v^2}{gr}$$

$$= \frac{\left(\frac{2pr}{T}\right)^2}{gr}$$

$$= \frac{4p^2 r}{gT^2}$$

$$= \frac{4p^2 r}{g\left(\frac{p}{3}\right)^2}$$

$$= \frac{36r}{g}$$

$$= 0.37$$