8. Potential Energy and Conservation of Energy

Potential Energy:

When an object has potential to have work done on it, it is said to have potential energy, e.g. a ball in your hand has more potential energy than a ball on the ground. If you release the ball, gravity will perform work on the ball and its kinetic energy will increase. If a spring with a block attached is compressed, the block has potential energy because if the block is released, the spring will perform work on the block and give it kinetic energy.

- Potential energy is energy associated with the configuration of two or more objects, e.g. the ball and the earth or the block and the spring.
- A force is acting between objects. When the configuration of the system changes and work is performed on one of the objects, the potential energy of the system is transferred to the kinetic energy of that object.

- If the system returns to the original configuration, the force between the objects removes kinetic energy from the objects and stores it in the potential energy of the system.
- The amount of work done in the original change and the reversal are equal in magnitude but differ by a sign: $W_i = -W_f$
- The force in the system is known as a conservative force.

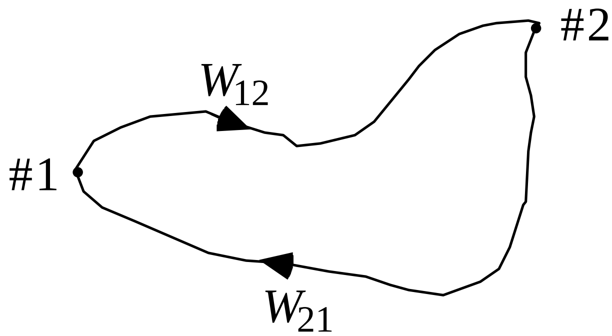
Conservative force:

- Can store energy in the system as potential energy.
- Can retrieve that energy and give it to an object in the system as kinetic energy.
- Gravitational and spring forces are conservative forces.
- Friction is a non conservative force. If we let a block scrape along a rough floor, the friction force will take kinetic energy away from the box. However, if we reverse the block and attempt to put it back in its

original position, we do not retrieve the energy out of the system. The friction force has converted the kinetic energy into thermal energy by heating up the block and the floor. The thermal energy can not be turned back into the kinetic energy of the block.

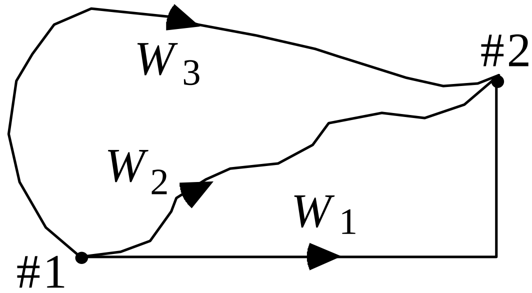
Properties of Conservative Force:

- The net work done by a conservation force as the object moves from point #1 to point #2 and back to point #1 is zero.



$$W_{12} = -W_{21}$$

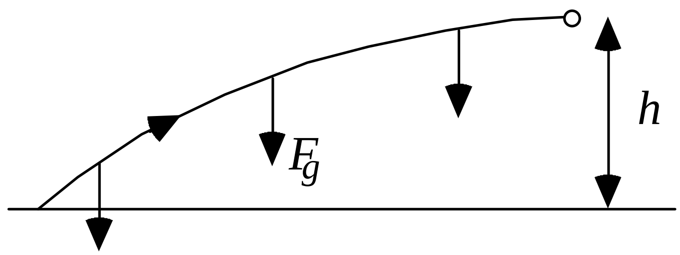
- The work done by a conservative force in moving the object from point #1 to point #2 does not depend on the path between the two points.



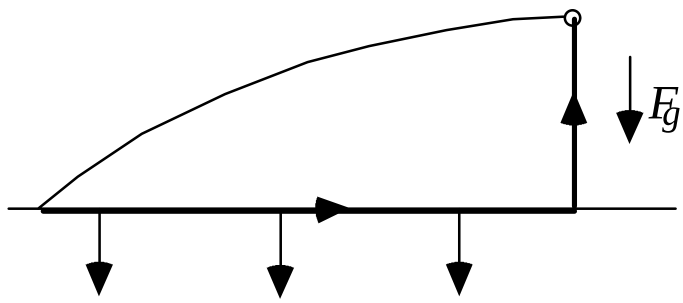
$$W_1 = W_2 = W_3$$

Example:

A 1-kg ball is thrown at an angle with respect to the ground to reach a height of 10 m. What is the work done by gravity between the start of the motion and the peak height?



- In principle, we can calculate the work done along the trajectory. However, this is complicated because the angle between the direction of motion and the gravity is changing along the trajectory. Alternatively, we can use work-energy theorem to calculate the work done if we can calculate the initial and final velocity. However, there is a easier method: use the fact that gravity is a conservative force to calculate the work done along the path shown:



$$
\begin{aligned}
W &= W(\text{horizontal path}) + W(\text{vertical path}) \\
&= 0 - mgh \\
&= -mgh
\end{aligned}
$$

Potential Energy Calculation:

If a positive amount of work is performed on the object (the kinetic energy increases), then the potential energy of the system must decrease:

$$\Delta U = -W$$

Gravitational Potential Energy:

For the ball flying to its maximum height, the change in potential energy:

$$\Delta U = -(-mgh) = mgh$$

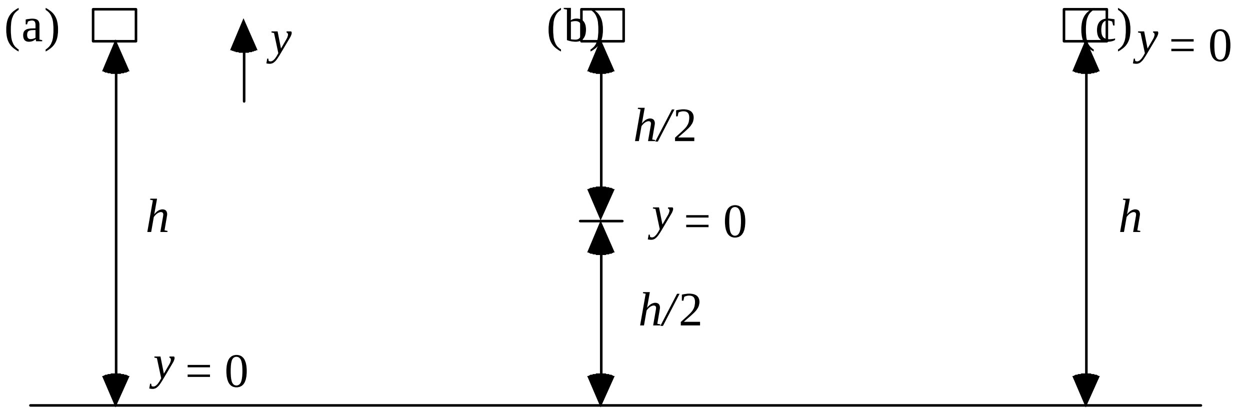
In general, the change in the gravitational potential energy is given by

$$\Delta U_g = mg\left(y_f - y_i\right)$$

This is the potential energy of the ball-earth system. We often say that "the ball has X Joules of potential energy." However, it should be noted:

- We really mean the system has that potential energy.
- Physically, only the change in potential energy between two configurations has any meaning.
- We are free to pick the "zero potential energy" point anywhere we like by arbitrarily assigning certain configuration to have zero potential energy.

Consider a box with weight of 1 N hanging 10 m above the ground. We define zero gravitational potential energy at $y = 0$:



$$U_g = mgh = 10 \text{ J} \quad U_g = mg\frac{h}{2} = 5 \text{ J} \quad U_g = mg(0) = 0 \text{ J}$$

The potential energy depends on the origin of the coordinate system? Only the change in the potential energy has physical meaning! Consider the change in the potential energy if the box fell to the ground:

$$\Delta U_g = U_f - U_i$$

(a) $$\Delta U_g = 0 - mgh = 0 - 10 \text{ J} = -10 \text{ J}$$

(b) $$\Delta U_g = mg\left(-\frac{h}{2}\right) - mg\left(\frac{h}{2}\right) = -5 \text{ J} - 5 \text{ J} = -10 \text{ J}$$

(c) $$\Delta U_g = mg(-h) - 0 = -10 \text{ J}$$

all the changes in the potential energy are identical!

Elastic Potential Energy:

The spring force is a conservative force with its associated elastic potential energy:

$$
\begin{aligned}
U_e &= -W \\
&= -\int_0^x F dx \\
&= +\int_0^x kx dx \\
&= \tfrac{1}{2} kx^2
\end{aligned}
$$

Mechanical Energy:

The mechanical energy of a system is the sum of its potential energy and kinetic energy of the objects:

$$E = K + U$$

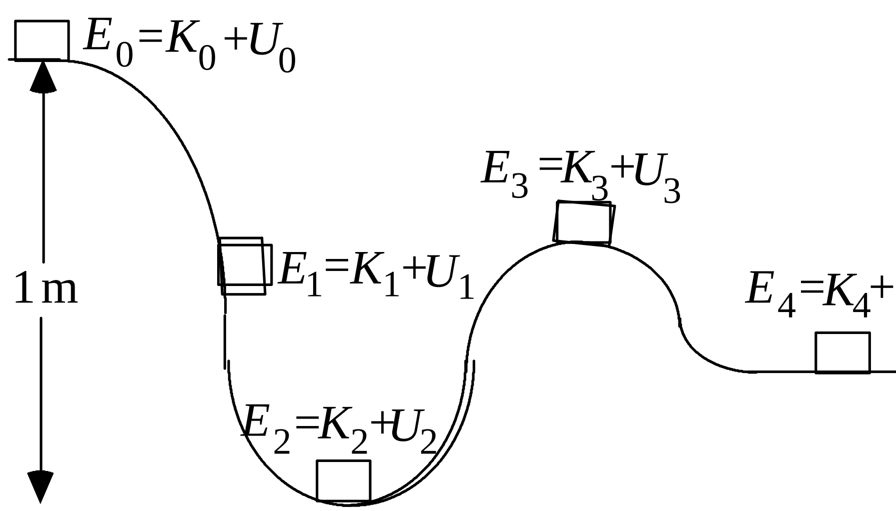
Conservation of Mechanical Energy:

The total mechanical energy remains constant for an isolated system of objects that interact with conservative forces.

★ Not only is the energy at the start of motion equal to the energy at the end of motion, it is equal at **all times** in between.

Example:

A 2 kg block slides down a frictionless surface:



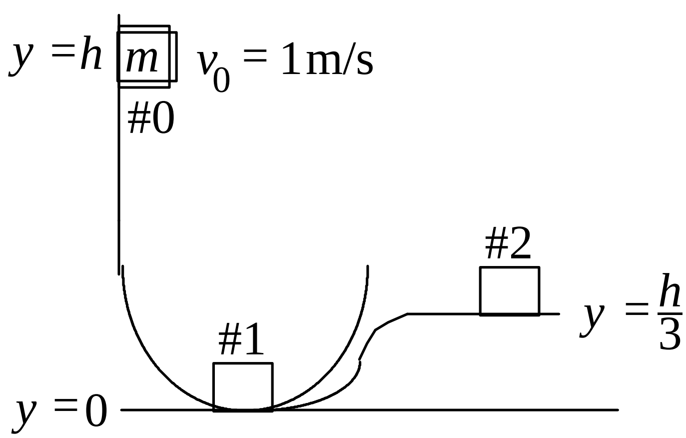
$$E_0 = E_1 = E_2 = E_3 = E_4$$

Position	K.E.	P.E.	M.E.
0	0	20	20
1	12	8	20
2	20	0	20
3	10	10	20
4	15	5	20

- The kinetic energy and potential energy change throughout the motion of the object, i.e. not separately conserved.
- The total mechanical energy remains constant, i.e. conserved.

Example:

A block slides down a frictionless ramp from a height of h = 3 m. The initial velocity is 1 m/s. What is the velocity at the bottom and at the top of the ramp?



- Since gravity is the only force performing work on the object, the total mechanical energy is conserved as gravity is a conservative force.

$$(1) \qquad E_0 = K_0 + U_0 = \tfrac{1}{2}mv_0^2 + mgh$$

$$(2) \qquad = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + mg(0) = \tfrac{1}{2}mv_1^2$$

$$(3) \qquad = K_2 + U_2 = \tfrac{1}{2}mv_2^2 + mg\left(\frac{h}{3}\right)$$

$$(1),(2): \qquad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_0^2 + mgh$$

$$v_1^2 = v_0^2 + 2gh$$

$$v_1 = \sqrt{v_0^2 + 2gh}$$

$$= 7.7 \text{ m / s}$$

- Answer looks like equation of motion, but
  - ★ v_0 is in the vertical direction while v_1 is in the horizontal direction
  - ★ acceleration is not constant
    equation of motion is not valid
- v_1 is independent of mass, similar to the projectile problem.

$$(2),(3): \qquad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_2^2 + mg\left(\frac{h}{3}\right)$$

$$v_2^2 = v_1^2 - \tfrac{2}{3}gh$$

$$v_2 = \sqrt{v_1^2 - \tfrac{2}{3}gh}$$

$$= 6.3 \text{ m / s}$$

- Conservation of energy is a powerful tool: solving impossible problem with ease!

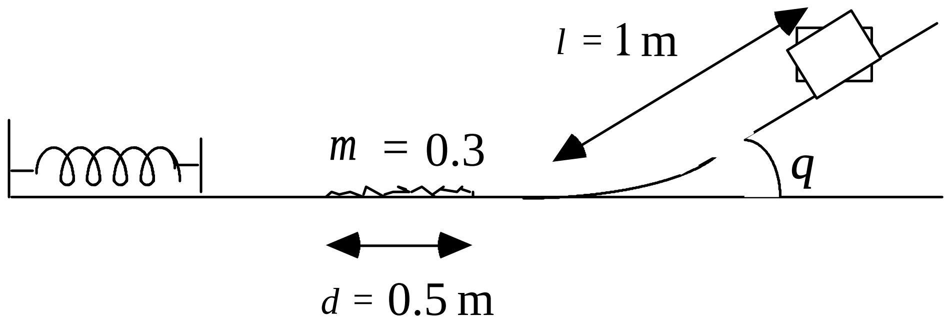
If there is a non conservative force (e.g. friction), then the total mechanical energy is not conserved. The work done by the non conservative force changes the mechanical energy:

$$\Delta E = W_{non\,conservative}$$

So if we know the work done by a non conservative force, we can determine how much the mechanical energy changes.

Example:

A 1-kg block starts from rest 1 m up a frictionless 30° ramp. On the horizontal surface is a 0.5 m rough surface with a coefficient of kinetic friction of 0.3. After the rough stretch, the horizontal surface is again frictionless and there is a spring with a spring constant 500 N/m. How far is the spring compressed?



- To calculate the spring compression:

  need to know the kinetic energy of the block before impact

  need to know the total mechanical energy and the energy lost due to friction

$$E_i = mgl\sin q$$

$$W_{friction} = -f_k d$$

$$= -mNd$$

$$= -mmgd$$

$$= \Delta E$$

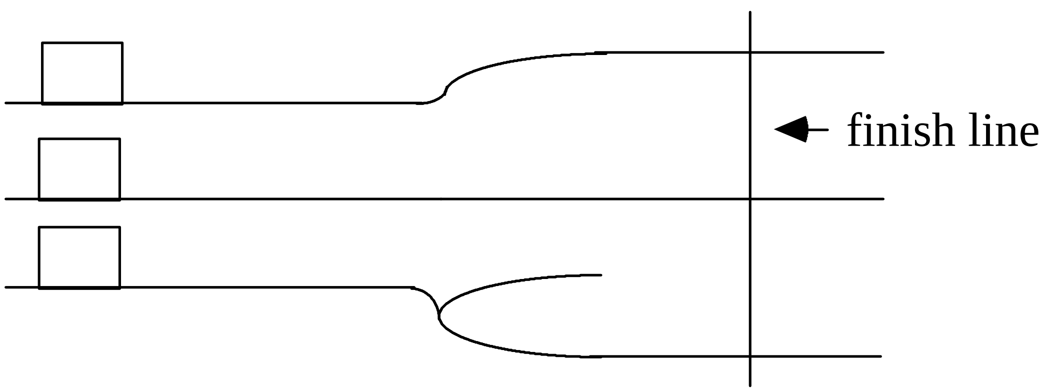
$$= E_f - E_i$$

$$
\begin{aligned}
E_f &= E_i - mmgd \\
&= mgl\sin q - mmgd \\
&= mg(l\sin q - md) \\
&= E_s \\
&= \tfrac{1}{2}kx^2
\end{aligned}
$$

$$
\begin{aligned}
x &= \sqrt{\frac{2mg(l\sin q - md)}{k}} \\
&= 0.12\text{ m}
\end{aligned}
$$

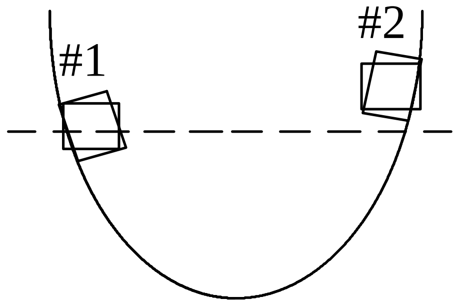
Conceptual Problem:

(1) Three blocks are launched with the same initial speed at the same instant along the three tracks of different elevations. Which will reach the finish line first?



(2) I tell you that I have run an experiment in my lab. I started a box from rest at point # 1. I tell

you that it slides down the ramp and up the other side to point #2. Am I telling the truth?



Gravitational Potential Energy:

We define the gravitational potential energy:

$$U_g = mgh$$

This is valid only near the surface of the planet (e.g. earth, moon) based on the assumption that the gravitational force is constant. The more general form for the force is:

$$F_g = \frac{GM_p m}{r^2}$$

where M_p is the mass of the planet and m is the mass of the object, and r is the distance between the two objects. The work done in moving the object from infinity to r yields a change in the potential energy:

$$U_g = -\int_\infty^r \vec{F}_g \cdot d\vec{r}$$

$$= \int_\infty^r F_g dr$$

$$= \int_\infty^r \frac{GM_p m}{r^2} dr$$

$$= -\left[\frac{GM_p m}{r}\right]_\infty^r$$

$$U_g = -\frac{GM_p m}{r}$$

- Zero potential energy corresponds to when the distance between the objects is infinite.

Escape Velocity:

The speed that an object must have to escape the gravitational attraction of a planet. The speed can be calculated using conservation of energy:

At the surface of the planet:

$$E_i = \tfrac{1}{2}mv^2 + U_g$$

$$= \tfrac{1}{2}mv^2 - \frac{GM_p m}{r}$$

As the object moves away from the planet, the potential energy increase and the speed decrease: at infinity, both the potential energy and speed are zero:

$$E_f = 0$$

From conservation of energy:

$$E_i = E_f = \tfrac{1}{2} m v^2 - \frac{G M_p m}{r} = 0$$

$$\tfrac{1}{2} v^2 = \frac{G M_p}{r}$$

$$v_{escape} = \sqrt{\frac{2 G M_p}{r}}$$

- 11.2 km / s for the planet earth.