Physics 131 Review

Translational Kinematics:

Position ($\vec{r}$): location relative to an origin

Unit: m

Displacement: change in position

$$\Delta\vec{r} = \vec{r}_1 - \vec{r}_2$$

Unit: m

Velocity: rate of change of the position

$$\vec{v} = \frac{d\vec{r}}{dt}$$

Unit: m / s

Acceleration: rate of change of the velocity

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Unit: m / s^2

Equation of Motion:

When the acceleration vector is constant, the motion of the center of mass of an object is given by:

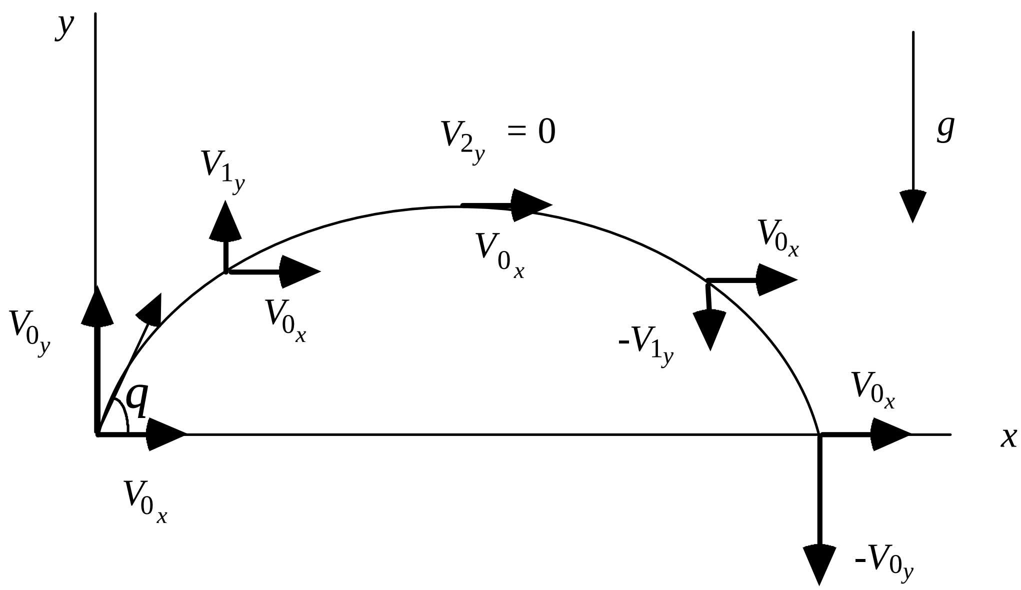
$$x = x_0 + v_{o_x} t + \tfrac{1}{2} a_x t^2$$

$$v_x = v_{o_x} + a_x t$$

$$v_x^2 = v_{0_x}^2 + 2a_x(x - x_0)$$

Similarly for the *y* and *z* directions. The motion in each direction is independent of the motion in other directions.

Projectile Motion:



- Acceleration is in the *y*-direction:

  $$a_y = -g$$

  $$g = 9.8 \text{ m / s}^2$$

- Motion in the *x*-direction is independent of motion in the *y*-direction. Since there is no acceleration in the x-direction, the velocity in that direction is constant.

- At any given height, the speed of the ball is the same.

Forces

Newton's Laws:

1st: An object in motion or an object at rest will remain in motion or at rest if no net force acts on the object.

2nd: Net force is related to the acceleration by:

$$\sum \vec{F} = m\vec{a}$$

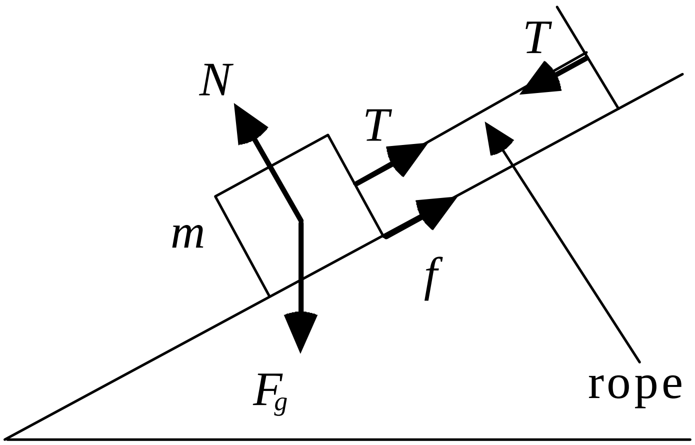
$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

3rd: When one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body.

- The 1st and 2nd laws refer to a single body, while the 3rd refers to a pair of bodies.



Normal Force (N):

The force perpendicular to the two surfaces in contact.

Tension Force (T):

The force from a rope, cable etc. It always acts away from the object to which it is connected.

Gravitational Force:

$$F_g = \frac{GMm}{r^2}$$

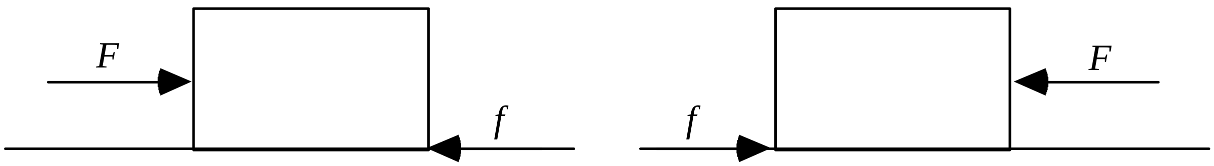
Near the surface of a planet:

$$F_g = mg$$

$$g = \frac{GM}{R^2}$$

Friction:

Frictional force always opposes the direction of motion:



Static Friction:

The friction between two surfaces when there is no relative motion,

$$f_s \le m_s N$$

where m_s is the coefficient of static friction.

Kinetic Friction:

The friction between two surfaces when the objects are moving across each other.

$$f_k = m_k N$$

Spring Force:

The restoring force on an object to an equilibrium position:

$$F = -kx$$

Steps for solving a dynamics (force) problem:

- Draw a free-body diagram.

- Define a coordinate system (It can be different for different objects.)
- Clearly label the forces acting on the body.
- Write down Newton's 2nd Law for each object in terms of the forces acting on the object.
- Determine the relationship between the acceleration of different objects.

Work and Energy

Work:

$$dW = \left(F_{\text{dir. of motion}}\right)dx$$

For a constant force:

$$W = \left(F_{\text{dir. of motion}}\right)\Delta x$$

Kinetic Energy:

$$K = \tfrac{1}{2}mv^2$$

Work-Kinetic Energy Theorem:

The change in the kinetic energy is equal to the work performed on the object:

$$K = \Delta W$$

Power:

Rate at which work is being performed:

$$P = \frac{dW}{dt}$$

For a constant force:

$$P = \left(F_{\text{dir. of motion}}\right)v$$

Potential Energy:

If two objects interact via a conservative force (e.g. gravity, spring force), then the potential energy is related to the work done:

$$U = -\Delta W$$

Gravitational Potential Energy:

$$U_g = -\frac{GMm}{r}$$

For an object near the surface of a planet:

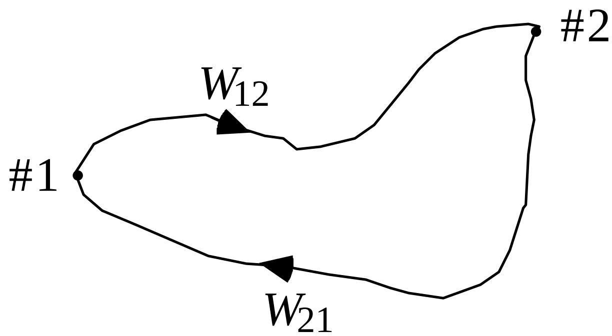
$$U_g = mgh$$

Spring Potential Energy:

$$U_s = \tfrac{1}{2}kx^2$$

where k is the spring constant and x is the distance from the equilibrium position.

The work done by a conservative force is independent of the path taken between two points, and the work done along a closed path is zero.



$$W_{12} = -W_{21}$$

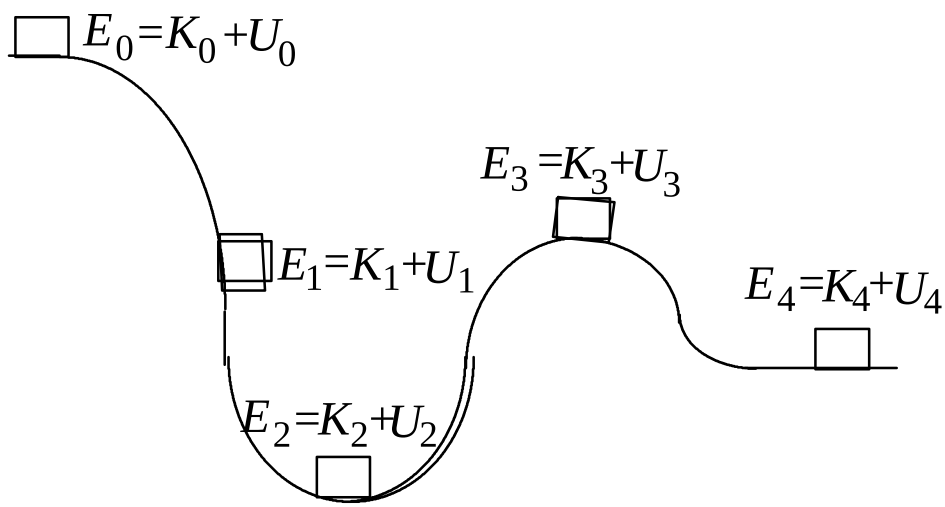
Friction is not a conservative force, and hence there is no potential energy associated with it. The force dissipates energy rather than "storing" it as a spring does.

Conservation of Mechanical Energy:

If only conservative forces act upon an object, the total mechanical energy of the object is conserved:

$$E = K + U_g + U_s$$

The total is the same at all points of the motion. The individual pieces (K, U_g, U_s) are not conserved (constant).



$$E_0 = E_1 = E_2 = E_3 = E_4$$

Systems of Particles and Collisions

Center of Mass:

The center of mass of a body or a system of bodies is the point that moves as though all the mass was concentrated there and all external forces were applied there.

For a system of particles:

$$x_{cm} = \frac{1}{M_{total}} \sum m_i x_i$$

For an extended object:

$$x_{cm} = \frac{1}{M} \int x \, dm$$

The center of mass of a system moves as:

$$\sum F_{x_{ext}} = ma_{x_{cm}}$$

The sum is over the external forces. Internal forces do not effect the motion of the center of mass.

The expressions for the *y* and *z*-coordinates are similar.

Momentum:

$$\vec{p} = m\vec{v}_{cm}$$

For a system of particles:

$$\vec{P}_{sys} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_n$$

Newton's 2nd Law:

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

Impulse:

Impulse from a force that an object feels:

$$\vec{J} = \int \vec{F}(t)\, dt$$

Impulse-Momentum Theorem:

Impulse is related to the change in the momentum of an object:

$$\vec{J} = \Delta\vec{p}$$

Conservation of Momentum:

For a system with no net external force:

$$\frac{d\vec{P}_{sys}}{dt} = 0$$

$$\vec{P}_{sys} = \text{constant}$$

$$\vec{P}_{i_{sys}} = \vec{P}_{f_{sys}}$$

Within a system, if two of more objects collide, the forces that they exert on one another are internal forces. These forces cannot change the momentum of the system. Thus, the momentum is conserved in the collision.

Elastic Collision:

A collision that conserves both momentum and kinetic energy of the system:

$$K_{i_{sys}} = K_{f_{sys}}$$

For two objects colliding elastically:

$$v_{1_f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1_i} + \frac{2m_2}{m_1 + m_2} v_{2_i}$$

$$v_{2_f} = \frac{2m_1}{m_1 + m_2} v_{1_i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2_i}$$

Totally Inelastic Collision:

A collision in which two objects stick together. The maximum amount of kinetic energy is lost into the deformation and heating of the two objects. However, the momentum of the system is still conserved:

$$m_1 v_{1_i} + m_2 v_{2_i} = (m_1 + m_2) v$$

For collisions in two dimensions, momentum is still conserved separately in each dimension.

Rotational Dynamics

Angular Position (q):

The orientation of the object as measured from a reference line fixed to the object to a fixed set of axes.

Angular Velocity:

The rate of change of angular position:

$$w = \frac{dq}{dt}$$

Unit: rad / s

Angular Acceleration:

The rate of change of angular velocity:

$$a = \frac{dw}{dt} = \frac{d^2q}{dt^2}$$

Unit: rad / s^2

Equations of Motion:

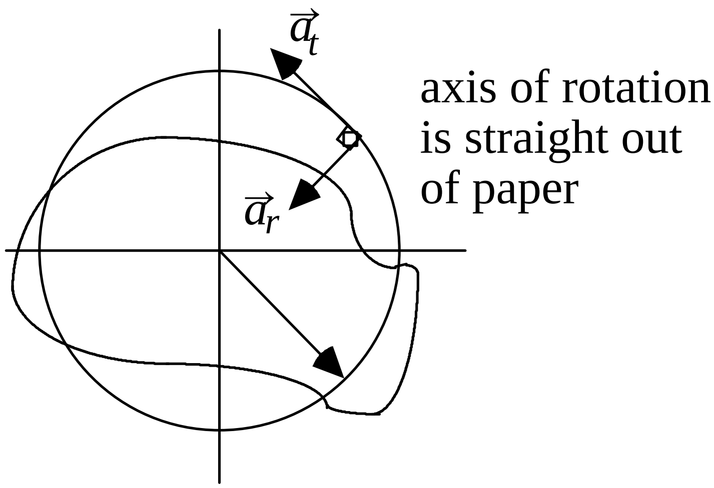
The equations of motion for constant angular acceleration are analogous to those for translational motion:

$$q = q_0 + w_0 t + \tfrac{1}{2} a t^2$$

$$w = w_0 + a t$$

$$w^2 = w_0^2 + 2a(q - q_0)$$

Relationship between Angular and Linear Variables:



Centripetal acceleration: $a_r = \frac{v^2}{r} = w^2 r$

Tangential acceleration: $a_t = r a$

Velocity: $v = r w$

- The angular variables in these equations must be in radians per unit time (or time^2).

Moment of Inertia:

Moment of inertia is similar to mass in translational motion. However, unlike mass, moment of inertia depends on about which axis the object is rotating. For a system of particles:

$$I = \sum m_i r_i^2$$

where is r_i the perpendicular distance between the object and the axis of rotation.

For a continuous object:

$$I = \int r^2 dm$$

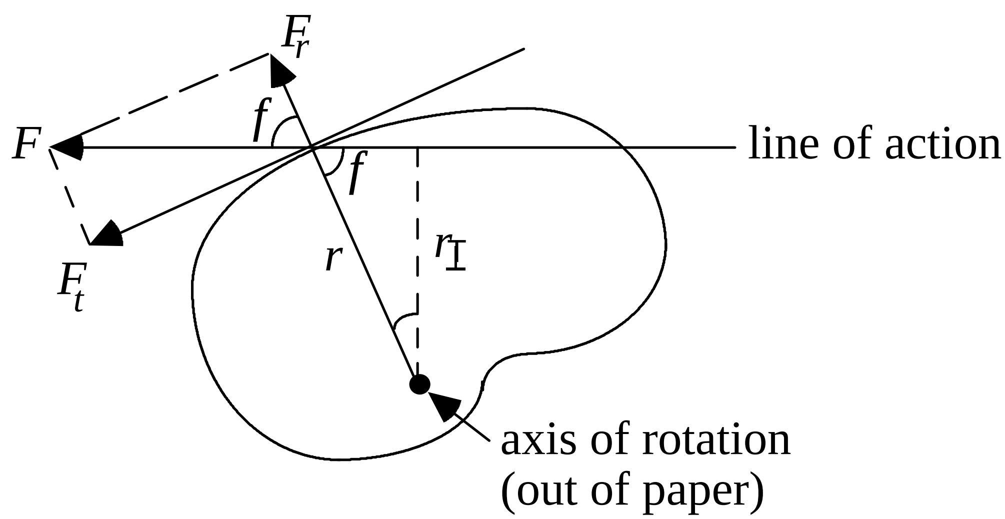
Parallel Axis Theorem:

The moment of inertia about an axis parallel to an axis passing through the center of mass a distance *h* away:

$$I = I_{cm} + mh^2$$

Torque:

A torque causes an object to undergo angular acceleration (*a*):



The torque can be calculated in two methods:

- Use the tangential component of the force:

$$t = F_t r = (F \sin f) r = Fr \sin f$$

★ Only the tangential component of the force contributes to the torque. The radial component cannot cause the object to rotate.

- Use the line of action:

$$t = Fr_{\perp} = Fr\sin f$$

Newton’s 2nd Law for Rotation:

$$\sum t = Ia$$

Work, Energy, and Power:

$$W_{tot} = \Delta K$$

$$dW = t\ dq$$

$$P = \frac{dW}{dt}$$

For a constant torque:

$$W = t\ \Delta q$$

$$P = tw$$

Rolling Motion:

If an object rolls without slipping:

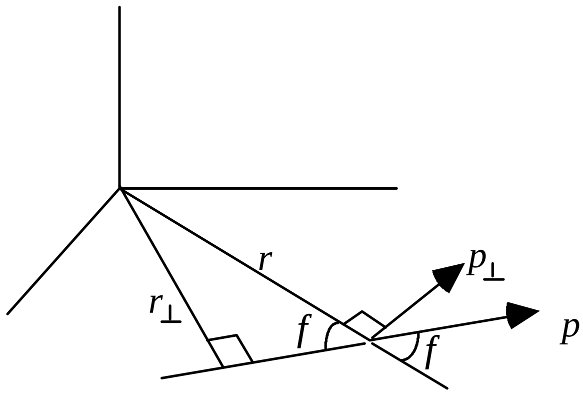
$$v_{cm} = Rw$$

The energy of a rolling object:

$$E = \tfrac{1}{2}mv_{cm}^2 + \tfrac{1}{2}I_{cm}w^2 + U_g + U_s$$

- When no nonconservative force is acting on the object, we can use conservation of energy to solve for the motion of the object.

Angular Momentum:



$$L = rp_\perp = rp\sin f = rmv\sin f$$

$$L = r_\perp p = (r\sin f)p = rmv\sin f$$

- The direction of the angular momentum vector is perpendicular to the plane containing the $\vec{r}$ and $\vec{p}$ vectors, as given by the Right Hand Rule.

For a system of particles:

$$\sum \vec{t}_{ext} = \frac{d\vec{L}}{dt}$$

The component of the angular momentum along a fixed axis of rotation:

$$L_z = Iw$$

Conservation of Angular Momentum:

If a component of the net external torque on a system is zero, then that component of the angular momentum of the system along the axis is conserved,

$$L_i = L_f$$

$$I_i w_i = I_f w_f$$

$$w_f = \frac{I_i}{I_f} w_i$$

- If the moment of inertia becomes larger, the object will spin slower, and vice versa.

Static Equilibrium:

An object is in static equilibrium if its center of mass is stationary. There are no net forces or torques acting upon it:

$$\sum F_x = 0 \qquad \sum t_x = 0$$

$$\sum F_y = 0 \qquad \sum t_y = 0$$

$$\sum F_z = 0 \qquad \sum t_z = 0$$

- Torque can be measured about any point.

Equation Sheet

These equations will be supplied on the test, if they are necessary for the problems.

$$x = x_0 + v_o t + \tfrac{1}{2} a t^2$$

$$v = v_o + at$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$F = -kx$$

$$F_g = \frac{GMm}{r^2}$$

$$U_g = -\frac{GMm}{r}$$

$$U_s = \tfrac{1}{2} k x^2$$

$$P = \left(F_{\text{dir. of motion}}\right) v$$

$$x_{cm} = \frac{1}{M} \int x \, dm$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F}(t)\, dt$$

$$v_{1_f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1_i} + \frac{2m_2}{m_1 + m_2} v_{2_i}$$

$$v_{2_f} = \frac{2m_1}{m_1 + m_2} v_{1_i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2_i}$$

$$q = q_0 + w_0 t + \tfrac{1}{2} a t^2$$

$$w = w_0 + a t$$

$$w^2 = w_0^2 + 2a(q - q_0)$$

$$a_r = \frac{v^2}{r} = w^2 r$$

$$a_t = r a$$

$$v = rw$$

$$I = \int r^2 dm$$

$$I = I_{cm} + mh^2$$

$$t = Fr_{\perp} = F_t r = Fr \sin f$$

$$\sum t = I a$$

$$dW = t \, dq$$

$$E = \tfrac{1}{2} m v_{cm}^2 + \tfrac{1}{2} I_{cm} w^2 + U_g + U_s$$

$$L = rp_{\perp} = r_{\perp} p = rmv \sin f$$

$$L = Iw$$