Some Definitions:

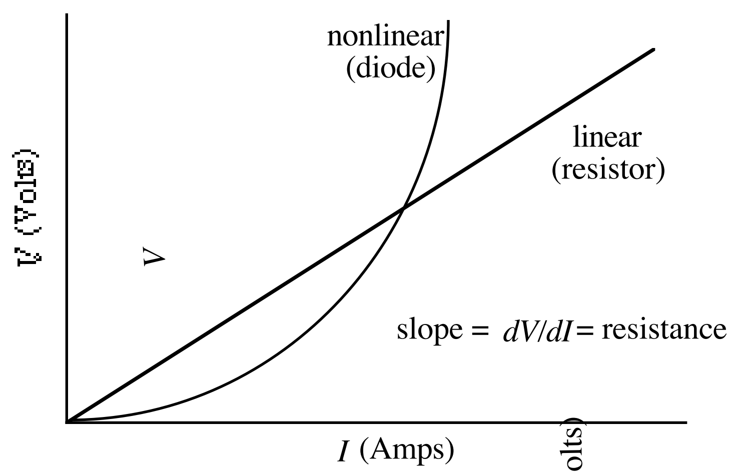
<u>Current (I):</u> Amount of electric charge (Q) moving past a point per unit time.
$I = dQ/dt$ = Coulombs/sec
units = Amps (1 Coul. = $6\text{x}10^{18}$ electrons)

<u>Voltage (V):</u> Work needed to move charge from point a to point b.
Work = $V \cdot Q$
$\Rightarrow$ Volt = Work/Charge = Joules/Coulomb
$\Rightarrow$ Voltage is always measured with respect to something.
$\Rightarrow$ "ground" is defined as zero Volts.

<u>DC:</u> Stands for <u>d</u>irect <u>c</u>urrent. In a DC circuit the current and voltage are constant as a function of time.

<u>Power (P):</u> Rate of doing work.
$P = dW/dt$, units = Watts

<u>Ohms Law:</u> Linear relationship between voltage and current.
$V = I \cdot R$
R = Resistance (Ω), units = Ohms



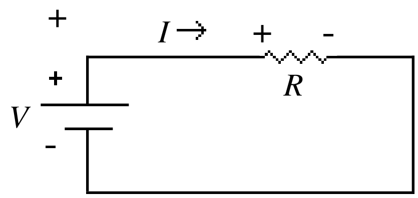
<u>Joules Law:</u> When current flows through a resistor energy is dissipated.
$W = QV$
$P = dW/dt = VdQ/dt + QdV/dt$
but $dV/dt = 0$ for DC circuit and averages to 0 for AC.
$\Rightarrow$ Power = $VdQ/dt = V \cdot I$
Using Ohms law
$\Rightarrow$ $P = I^2R = V^2/R$
100 Watts = 10 V and 10 Amps or 10 V through 1 Ω

Simple Circuits

Symbols:

Battery Resistor Ground

Simple(st) Circuit:

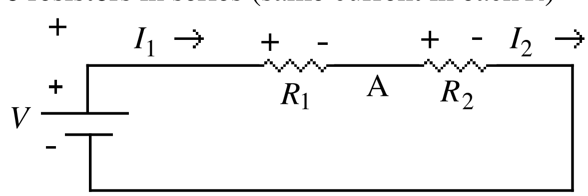


By Convention: Current flow is in the direction of positive charge flow.
When we go across a battery in direction of current (- → +) we get a $+V$.
Voltage drop across a resistor in direction of current (+ → -) gives $-IR$.

Conservation of Energy says that sum of potential drops around the circuit has to add up to zero.

For above circuit: $V - IR = 0$ or $V = IR$!!

Next simple(st) circuit: Two resistors in series (same current in each R)



Conservation of charge says that $I_1 = I_2 = I$ at point A.

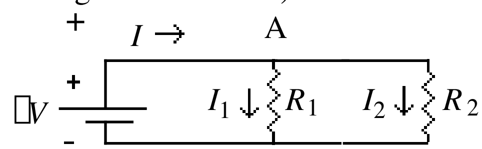
$$V = I(R_1 + R_2) = IR \text{ with } R = R_1 + R_2$$

Resistors in Series Add: $R = R_1 + R_2 + R_3 \ldots$

What's voltage across R_2?

$$V_2 = I_2 R_2 = VR_2/(R_1 + R_2)$$ "**Voltage Divider Equation**"

Two resistors in parallel (same voltage across each R)



At point A: $I = I_1 + I_2$

Also we have: $V = I_1R_1$ and $V = I_2R_2$

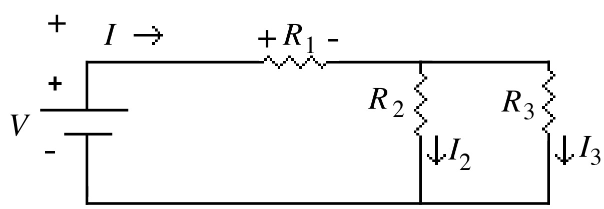
so: $I = V/R_1 + V/R_2 = V/R$

$$1/R = 1/R_1 + 1R_2$$

$$\Rightarrow R = \frac{R_1R_2}{R_1 + R_2}$$

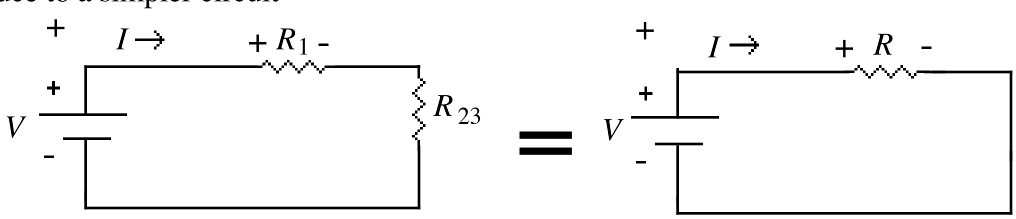
Parallel Resistors add like: $1/R = 1/R_1 + 1/R_2 + 1/R_3 + \ldots$

In a circuit with 3 resistors (series and parallel), what's I_2?



From Ohms law we know $I_2 = V_2/R_2$

Try to reduce to a simpler circuit



Now looks like a series circuit!

$$R_{23} = R_2 \| R_3 = \frac{R_2R_3}{R_2 + R_3}$$

$R = R_1 + R_2 \parallel R_3$ and $I = V/R = V/(R_1 + R_{23})$

Can now find V_2 using $V_2 = IR_{23}$

$$V_2 = \frac{V}{R_1 + \frac{R_2R_3}{R_2 + R_3}} \times \frac{R_2R_3}{R_2 + R_3}$$

$$= \frac{VR_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$I_2 = \frac{V_2}{R_2}$$

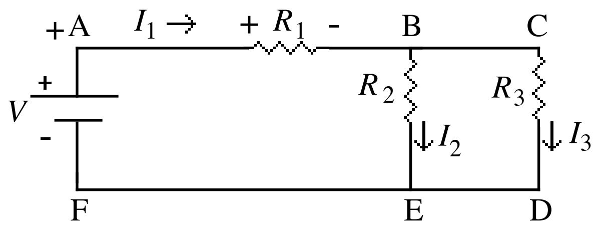
$$= \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

Note: if $R_3 \to \infty$ then $I_2 = I = V/(R_1 + R_2)$ as expected!

Kirchoff's Laws

We can formalize and generalize the previous examples using Kirchoff's Laws:

1) $\Sigma I = 0$ at a node. This is a restatement of conservation of charge.
2) $\Sigma V = 0$ around a closed loop. This is just conservation of energy.



at node B: $I_1 = I_2 + I_3$
at node E: $I_1 = I_2 + I_3$ (Same as above equation, there's no new info here)

Again, as with previous examples solve for I_2

at node B	$I_1 = I_2 + I_3$ or $I_1 - I_2 - I_3 = 0$
from loop ABEF	$V - I_1R_1 - I_2R_2 = 0$
from loop ACDF	$V - I_1R_1 - I_3R_3 = 0$

We have 3 linear equations with 3 unknowns (I_1, I_2, I_3)
We will always wind up with as many linear equations as unknowns!

Can use matrix methods to solve these equations: $\boldsymbol{V} = \boldsymbol{RI}$,
with $\boldsymbol{V}$ and $\boldsymbol{I}$ column vectors and $\boldsymbol{R}$ a matrix (3x3).
If we have n (n = 3 here) linearly independent equations then the determinant of $R \neq 0$. See any decent book on linear algebra or Diefenderfer Appendix A.

$$\begin{pmatrix} V \\ V \\ 0 \end{pmatrix} = \begin{pmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

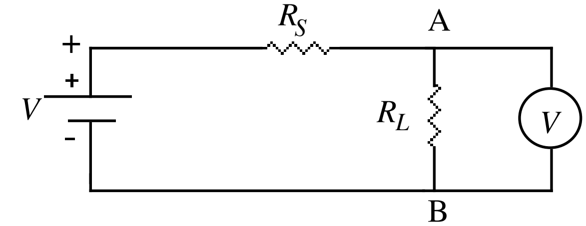
Can solve for I_2 using determinants:

$$I_2 = \frac{\det \begin{pmatrix} R_1 & V & 0 \\ R_1 & V & R_3 \\ 1 & 0 & -1 \end{pmatrix}}{\det \begin{pmatrix} R_1 & R_2 & 0 \\ R_1 & 0 & R_3 \\ 1 & -1 & -1 \end{pmatrix}} = \frac{VR_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

Note: This is the *same* solution as on previous page!

Measuring Things

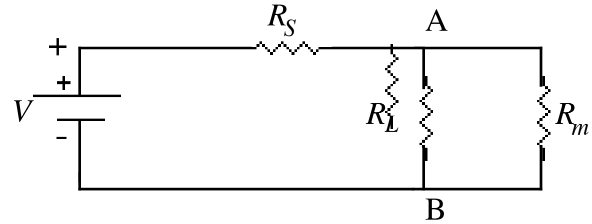
Voltmeter: **Always** put in **parallel** with what you want to measure.



If no voltmeter we would have:

$$V_{AB} = \left(\frac{R_L}{R_S + R_L}\right) V$$

If the voltmeter has a finite resistance R_m then circuit looks like:

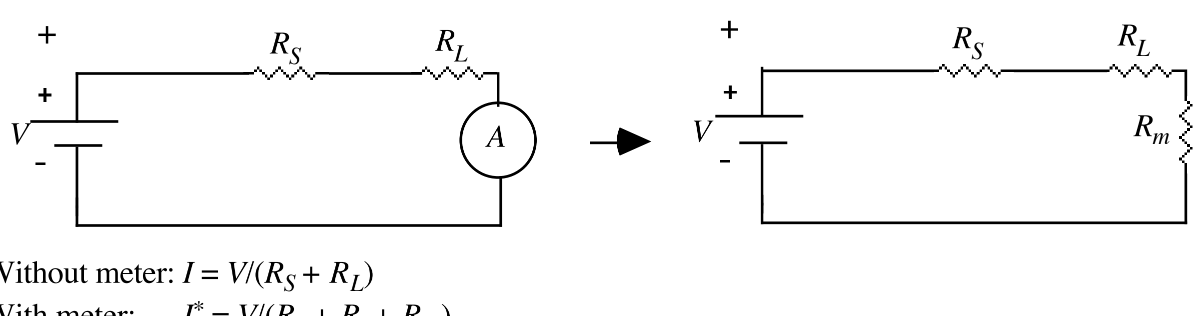


From previous page we have:

$$\begin{aligned} V^*_{AB} &= \left(\frac{R_m \| R_L}{R_S + R_m \| R_L}\right) V \\ &= \frac{V R_m R_L}{R_S R_L + R_m R_L + R_S R_m} \\ &= \frac{V R_L}{R_L + R_S + \frac{R_S R_L}{R_m}} \\ &\approx V_{AB} \quad \text{if } R_L << R_m \end{aligned}$$

A good voltmeter has high resistance ($> 10^6\ \Omega$)

Ammeter: Measures Current. **Always** put in **series** with what you want to measure.



Without meter: $I = V/(R_S + R_L)$

With meter: $I^* = V/(R_S + R_L + R_m)$

So a good ammeter has $R_m << (R_S + R_L)$, i.e. low (0.1-1 Ω) resistance.

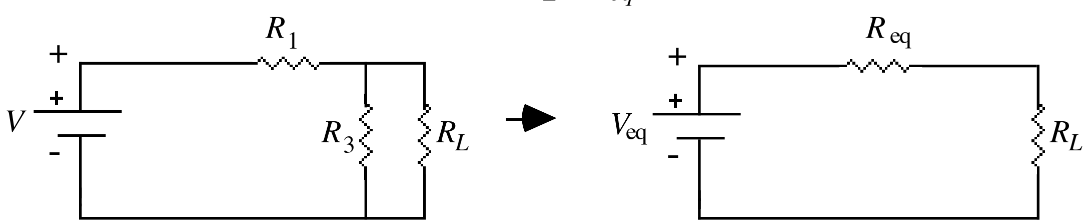
Thevenin's Equivalent circuit Theorem

"Any network of resistors and batteries having 2 output terminals may be replaced by a series combination of resistor and battery."

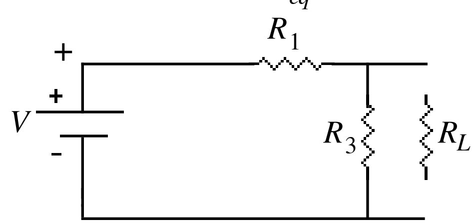
Useful when solving complicated (!?) networks:

Solve problems by finding V_{eq} and R_{eq} for circuit without load, then add load to circuit.
Use basic voltage divider equation:

$$V_L = \frac{V_{eq} R_L}{R_L + R_{eq}}$$

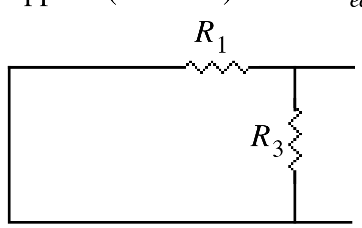


Two rules for using Thevenin's Thereom:
1) Take the load out of the circuit to find V_{eq}:



$$V_{eq} = \frac{VR_3}{R_1 + R_3}$$

2) Short circuit all power supplies (batteries) to find R_{eq}:



$$R_{eq} = \frac{R_1 R_3}{R_1 + R_3}$$

Can now solve for I_L as in previous examples:

$$I_L = \frac{V_{eq}}{R_{eq} + R_L}$$

$$= \left(\frac{VR_3}{R_1 + R_3}\right) \frac{1}{\frac{R_1 R_3}{R_1 + R_3} + R_L}$$

$$= \frac{VR_3}{R_1 R_L + R_1 R_3 + R_L R_3}$$

Same answer as previous examples!