Transistors and Amplifiers

Hybrid Transistor Model for Small AC Signals

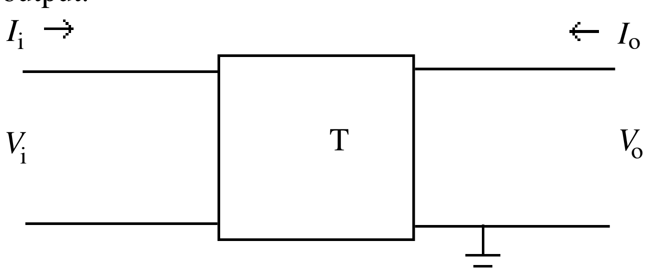
The previous model for a transistor used one parameter (β, the current gain) to describe the transistor. This model is naive and doesn't explain many of the features of the three common forms of transistor amplifiers (common emitter etc.) For example, we could not calculate the output impedance of the common emitter amp with the one parameter model.

Very often in electronics we describe complex circuits in terms of an equivalent circuit or model. For the transistor we wish to have a model that relates the input currents and voltages to the output currents and voltages. We also wish this model to be linear in the currents and voltages. For a transistor this condition of linearity is true for *small* signals.

The most general linear model of the transistor is a 4-terminal "black box".

In this model we assume that the transistor is biased on properly and often we do not even show the biasing circuit.

For the transistor where there are only 3 legs, one of the terminals is common between the input and output.



There are 4 variables in the problem, I_i, V_i, I_o, and V_o. The subscript i refer to the input side while the subscript o refers to the output side. We now *assume* that we know I_i and V_o. Kirchhoff's laws relate all the currents and voltages:

$$V_i = V_i(I_i, V_o)$$
$$I_o = I_o(I_i, V_o)$$

Since we have a linear model of the transistor we can write the following for small changes in I_i and V_o:

$$dV_i = \left(\frac{\partial V_i}{\partial I_i}\right)_{V_o} dI_i + \left(\frac{\partial V_i}{\partial V_o}\right)_{I_i} dV_o$$

$$dI_o = \left(\frac{\partial I_o}{\partial I_i}\right)_{V_o} dI_i + \left(\frac{\partial I_o}{\partial V_o}\right)_{I_i} dV_o$$

The partial derivatives are called the hybrid (or h) parameters. We rewrite the above equations as:

$$dV_{\mathrm{i}} = h_{\mathrm{ii}} dI_{\mathrm{i}} + h_{\mathrm{io}} dV_{\mathrm{o}}$$

$$dI_{\mathrm{o}} = h_{\mathrm{oi}} dI_{\mathrm{i}} + h_{\mathrm{oo}} dV_{\mathrm{o}}$$

The parameters h_{oi} and h_{io} are unitless while h_{oo} has units $1/\Omega$ (mhos) and h_{ii} has units Ω.

The four h parameters are easily measured. For example to measure h_{ii} hold V_{o} (the output voltage) constant and measure $V_{\mathrm{in}}/I_{\mathrm{in}}$. Unfortunately the h parameters are not constant. For example Figs. 11-14 of the 2N3904 spec sheet show the variation of the four parameters with collector current (I_C).

There are 3 sets of the 4 hybrid parameters, one for each type of amp (common emitter, common base, common collector). In order to differentiate one set of parameters from another the following notation is used:

First subscript
- i = input impedance
- o = output admittance
- r = reverse voltage ratio
- f = forward current ratio

Second subscript
- e = common emitter
- b = common base
- c = common collector

Thus for a common emitter amplifier we would write:

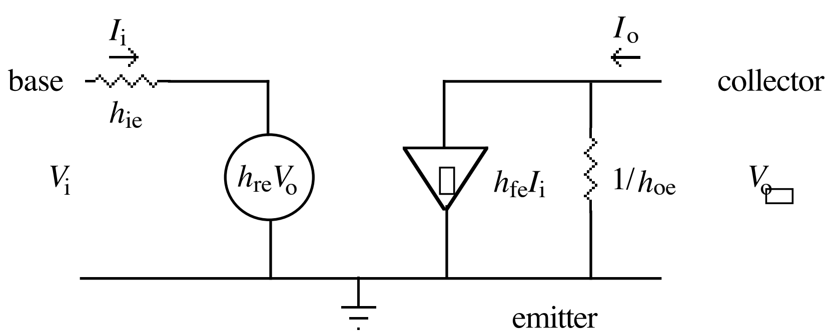
$$dV_{\mathrm{i}} = h_{\mathrm{ie}} dI_{\mathrm{i}} + h_{\mathrm{re}} dV_{\mathrm{o}}$$

$$dI_{\mathrm{o}} = h_{\mathrm{fe}} dI_{\mathrm{i}} + h_{\mathrm{oe}} dV_{\mathrm{o}}$$

Typical values for the h parameters for a 2N3904 transistor in the common emitter configuration with I_C = 1 mA are as follows:

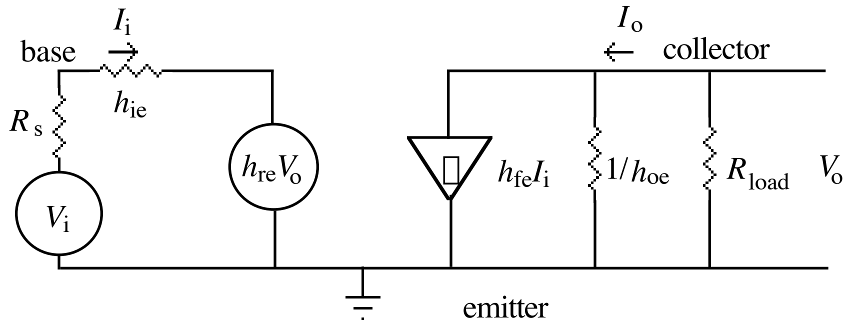
$$h_{\mathrm{fe}} = 120,\ h_{\mathrm{oe}} = 8.7\times10^{-6}\ \Omega^{-1},\ h_{\mathrm{ie}} = 3700\ \Omega,\ \ h_{\mathrm{re}} = 1.3\times10^{-4}$$

The equivalent circuit for a transistor in the common emitter configuration looks like:



The circle is a voltage source, i.e. the voltage across this element is always equal to $h_{\mathrm{re}}V_{\mathrm{o}}$ independent of the current through it. The triangle is a current source, the current through this element is always $h_{\mathrm{fe}}I_{\mathrm{in}}$ independent of the voltage across the device.

We can calculate the voltage and current gain, and the input and output impedance of a common emitter amp using this model. Below is the equivalent circuit (without biasing network) for a CE amp attached to a voltage source (with resistance R_s) and load resistor (R_{load}).



Current gain: The current gain (G_I) is defined as: $G_I = I_o/I_{in}$.

Using Kirchhoff's current law at the output side we have:

$$h_{fe}I_{in} + V_o h_{oe} = I_o$$

Using Kirchhoff's voltage rule at the output we have:

$$V_o = -I_o R_{load}$$

Putting the two equations together we get:

$$h_{fe}I_{in} = h_{oe}I_o R_{load} + I_o$$

$$G_I = I_o / I_{in} = h_{fe} / (1 + h_{oe}R_{load})$$

For typical CE amps, $h_{oe}R_{load} << 1$ and the gain reduces to familiar form:

$$G_I \approx h_{fe} = \beta$$

Voltage gain: The voltage gain (G_V) is defined as: $G_V = V_o/V_{in}$.

This gain can be derived in a similar fashion as the current gain. The result is:

$$G_V = V_o / V_{in} = -h_{fe}R_{load} / (\Delta R_{load} + h_{ie})$$

with $\Delta = h_{ie}h_{oe} - h_{fe}h_{re} \approx 10^{-2}$

This reduces to a familiar form for most cases where $\Delta R_{load} << h_{ie}$

$$G_V = -h_{fe} R_{load} / h_{ie} = -R_{load} / r_{BE}$$

Input Impedance: The input impedance (Z_i) is defined as: $Z_i = V_{in}/I_{in}$.

$$Z_i = (\Delta R_{load} + h_{ie}) / (1 + h_{oe}R_{load})$$

This reduces to a familiar form for most cases where $\Delta R_{load} << h_{ie}$ and $h_{oe} R_{load} << 1$

$$Z_i = h_{ie} = h_{fe}r_{BE}$$

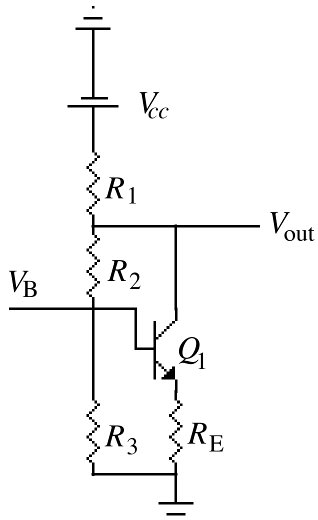
Output Impedance: The output impedance (Z_o) is defined as: $Z_o = V_o/I_o$.

$$Z_o = (R_s + h_{ie}) / (\Delta + h_{oe}R_s)$$

Z_o does not reduce to a simple expression. As the denominator is small, Z_o is as advertised large.

Feedback and Amplifiers

• Consider the following common emitter amplifier:



This amp differs slightly from the CE amp we saw before in that the bias resistor R_2 is connected to the collector resistor R_1 instead of directly to V_{cc}.

•How does this effect V_{out}?

a) If V_{out} *decreases* (moves away from V_{cc}) then I_2 increases which means that V_B decreases (gets closer to ground). However, if V_B decreases, then V_{out} will increase since $\Delta V_{out} = -\Delta V_B R_1/R_E$.

b) If V_{out} *increases* (moves towards V_{cc}) then I_2 decreases which means that V_B increases (moves away from ground). However, if V_B increases, then V_{out} will decrease since $\Delta V_{out} = -\Delta V_B R_1/R_E$.

This is an example of NEGATIVE FEEDBACK

Negative Feedback is *good*:

Stabilizes amplifier against oscillation

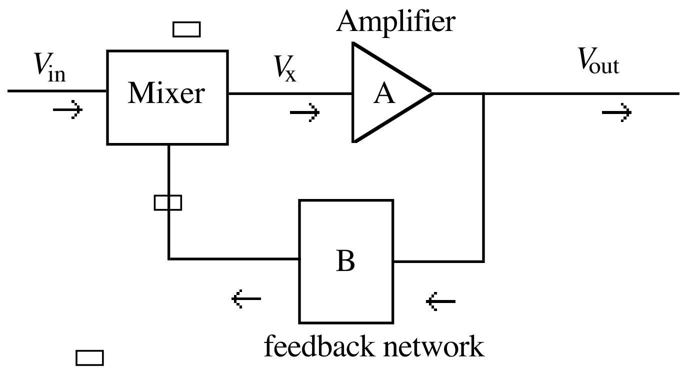
Increases the input impedance of the amplifier

Decreases the output impedance of the amplifier

Positive Feedback is *bad*:

Causes amplifiers to oscillate.

Feedback Fundamentals:



The above is a general feedback circuit. Without feedback the output and input are related by:

$$V_{out} = AV_{in}$$

The feedback (the box with the B) returns a portion of the output voltage to the amplifier through the "mixer".

Note: The feedback network on the AM radio is the collector to base resistors (R_3, R_5)

The input to the amplifier is:

$$V_x = V_{in} + BV_{out}$$

The gain with feedback is:

$$V_{out} = AV_x = A(V_{in} + BV_{out})$$

$$G = V_{out} / V_{in} = A / (1 - AB)$$

Some definitions:

A is the open loop gain, AB is the loop gain, G is the closed loop gain.

Lets define $A > 0$ (positive). Then there are two cases to consider depending on whether or not AB is positive or negative.

1) $AB > 0$. This is positive feedback. As $AB \to 1$, $G \to \infty$. The circuit is unstable and oscillates if $AB = 1$.

2) $AB < 0$. This is negative feedback. As $A \to \infty$, an amazing thing happens: $|AB| \to \infty$, $|G| \to |1/B|$.

For large amplifier gain (A) the circuit properties are determined by the feedback loop.

For example: $A = 10^5$ and $B = -0.01$ then $G = 100$.

The stability of the gain is also determined by the feedback loop (B) and not the amplifier (A). For example if B is held fixed at 0.01 and A varies:

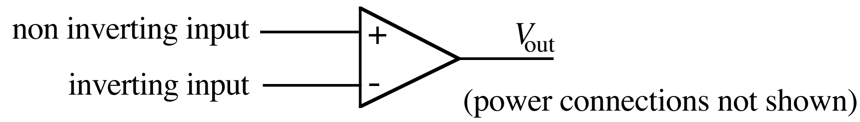
A	Gain
5×10^3	98.3
1×10^4	99.0
2×10^4	99.6

Thus circuits can be made stable with respect to variations in the transistor characteristics as long as B is stable. B can be made from precision components such as resistors.

Operational Amplifiers (Op Amps)

• Op amps are very high gain ($A = 10^5$) differential amplifiers.

A differential amp has two inputs (V_1, V_2) and output $V_{out} = A(V_1 - V_2)$ where A is the amplifier gain.



If an op amp is used without feedback and $V_1 \neq V_2$, then V_{out} saturates at the power supply voltage (either positive or negative supply).

• Op amps are almost always used with negative feedback. The output is connected to the (inverting) input.

• Op amps come in "chip" form. They are made up of complex circuits with 20-100 transistors.

	Ideal Op Amp	Real Op Amp μA741
Voltage gain (open loop)	∞	10^5
Input impedance	∞	2 MΩ
Output impedance	0	75 Ω
Slew rate*	∞	0.5 V/μsec
Power consumption	0	50 mW
V_{out} with $V_{in} = 0$	0	2 mV (unity gain)
Price	0$	$0.25

* slew rate is how fast the output can change

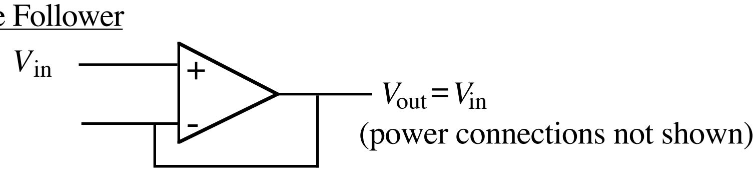
• When working with op amps using negative feedback the following two simple rules (almost) always apply:

1) **<u>No current goes into the op amp.</u>**
2) **<u>Both input terminals of the op amp have the same voltage.</u>**

The first rule reflects the high input impedance of the op amp. The second rule has to do with the actual circuitry making up the op amp.

• Some examples of op amp circuits with negative feedback:

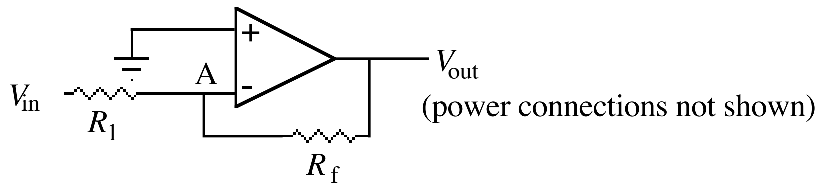
<u>Voltage Follower</u>



The feedback network is just a wire connecting the output to the input. By rule #2, the inverting (-) input is also at V_{in}. Thus $V_{out} = V_{in}$.

What good is this circuit? It is mainly used as a buffer as it has high input impedance (MΩ) and low output impedance (100 Ω).

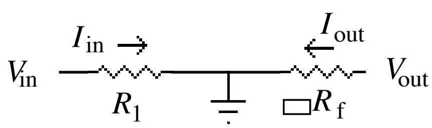
Inverting Amplifier:



By rule #2, point A is at ground.

By Rule #1, no current is going into the op amp.

We can redraw the circuit as:

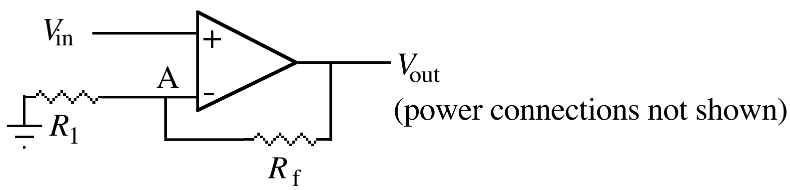


$$V_{in} / R_1 + V_{out} / R_f = 0$$

$$V_{out} / V_{in} = -R_f / R_1$$

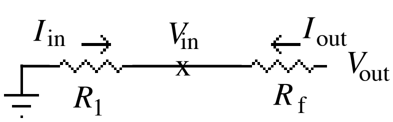
Thus the closed loop gain is R_f/R_1. The minus sign in the gain means that the output has the opposite polarity as the input. See page 8 for more details.

Non-Inverting Amplifier:



By rule #2, point A is V_{in}.

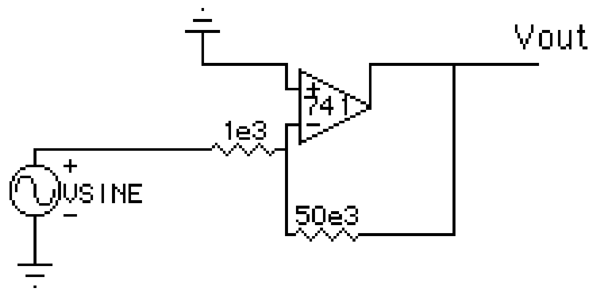
By Rule #1, no current is going into the op amp. We can redraw the circuit as:



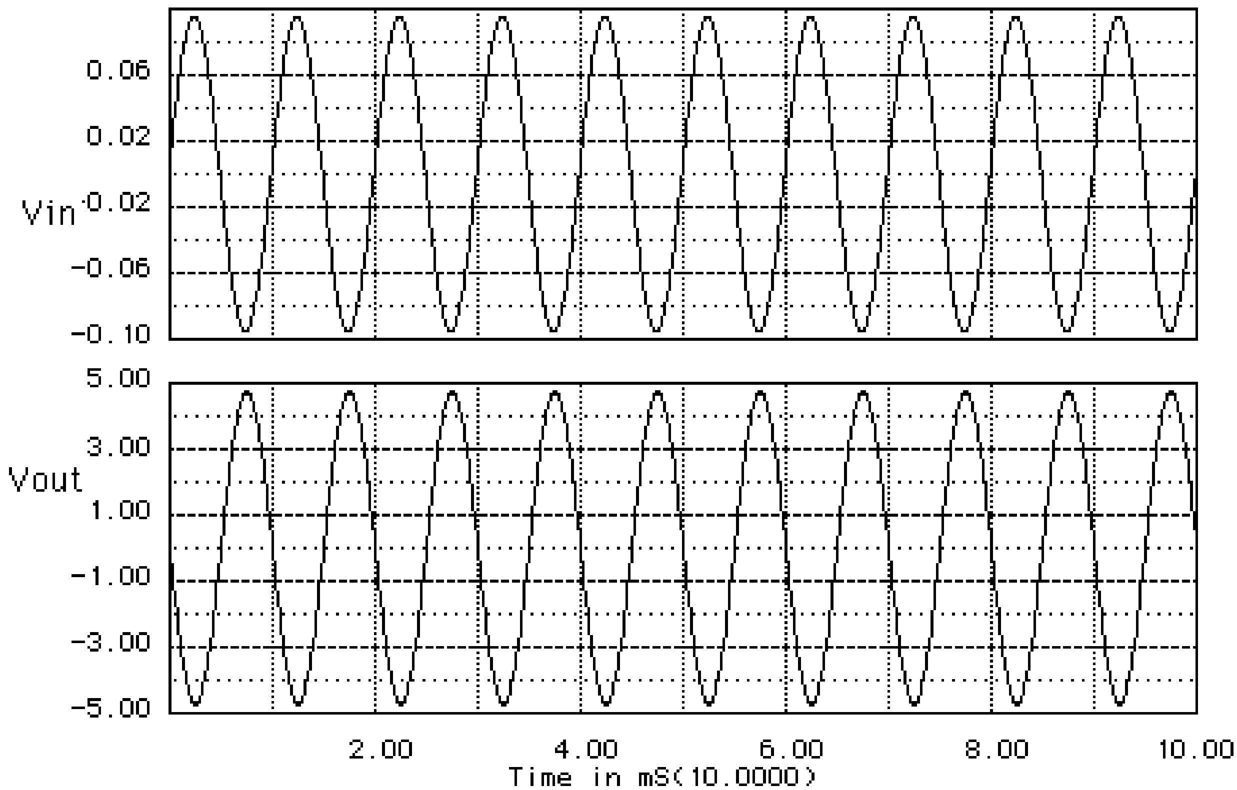
$$V_{in} / R_1 + (V_{in} - V_{out}) / R_f = 0$$

$$V_{out} / V_{in} = (R_1 + R_f) / R_1$$

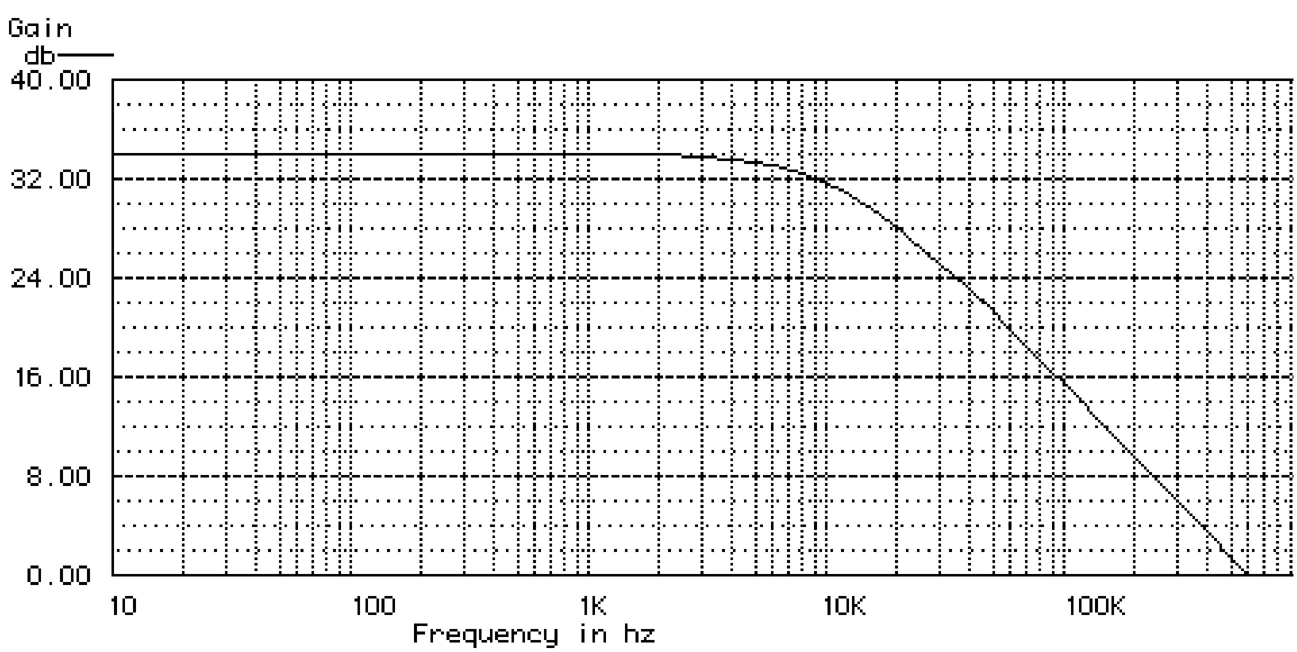
Thus the closed loop gain is $(R_1 + R_f) / R_1$. For this circuit the output has the same polarity as the input.



Vout
741
1e3
VSINE
50e3

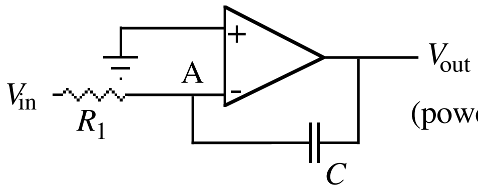


0.06
0.02
Vin -0.02
-0.06
-0.10
5.00
3.00
Vout
1.00
-1.00
-3.00
-5.00
2.00
4.00
6.00
8.00
10.00
Time in mS(10.0000)



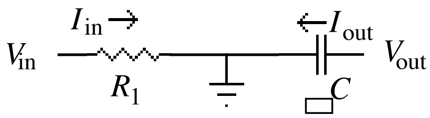
Gain
db
40.00
32.00
24.00
16.00
8.00
0.00
10
100
1K
10K
100K
Frequency in hz

<u>Integrating Amplifier</u>:



(power connections not shown)

Again, using the two rules for op amp circuits we redraw the circuit as:



$$\frac{V_{in}}{R_1} + C\frac{dV_{out}}{dt} = 0$$

Remember $Q = CV$! Solving the above for the voltage gain we obtain:

$$V_{out} = \frac{-1}{CR_1}\int V_{in}dt$$

Thus the output voltage is related to the integral of the input voltage. The negative sign in the gain means that V_{in} and V_{out} have opposite polarity.

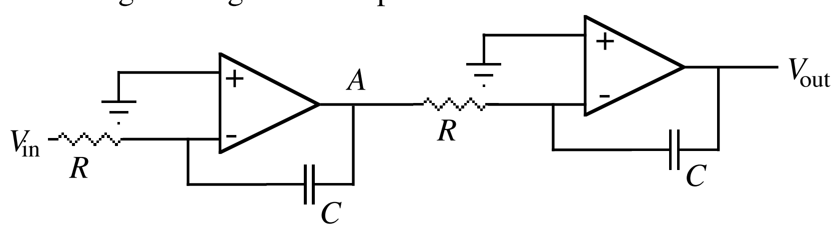
<u>Op Amps and Analog Calculations</u>:

Op amps were invented before transistors to perform analog calculations. Their main function was to solve differential equations in real time. For example suppose we wanted to solve the following:

$$\frac{d^2x}{dt^2} = g$$

This describes a body under constant acceleration (gravity if $g = 9.8$ m/s^2)

The following circuit gives an output which is the solution to the above differential equation:



The input voltage is a constant (= g). For convenience we pick $RC = 1$. At point A we have:

$$V_A = -\int V_{in}dt = -\int \frac{d^2x}{dt^2}dt = -\frac{dx}{dt}$$

The output voltage (V_{out}) is the integral of V_A:

$$V_{out} = -\int V_A dt = \int \frac{dx}{dt}dt = x(t)$$

If we want non-zero boundary conditions (e.g. $V(t = 0) = 1$ m/s) we add a DC voltage at point A.