Weak Interactions

Chapter 8 M&S

- Some weak interaction basics:
  - Weak force is responsible for β decay e.g. $n \rightarrow pev$ (1930's).
  - Interaction involves both quarks and leptons.
  - Not all quantum numbers are conserved in weak interaction:
    - parity, charge conjugation, CP
    - isospin
    - flavor (strange, charm, bottom, top)
  - Weak (+EM) are "completely" described by the Standard Model.
- Weak interactions has a very rich history:
  - 1930's: Fermi's theory described β decay.
  - 1950's: V-A (vector-axial vector) theory:
    - Yang & Lee describe parity violation.
    - Feynman and Gell-Mann describe muon decay and decay of strange mesons
  - 1963: N. Cabibbo proposes "quark mixing":
    - "explains" why rates for decays with $\Delta S = 0 > \Delta S = 1$:

$$BR(K^- \rightarrow \mu^- \bar{\nu}_\mu) = 63.5\%$$

$$BR(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) = 99.9\%$$

    - Quarks in strong interaction are not the same as the ones in the weak interaction:
      - weak interaction basis different than strong interaction basis: $(K^0, \bar{K}^0) \Leftrightarrow (K_S, K_L)$

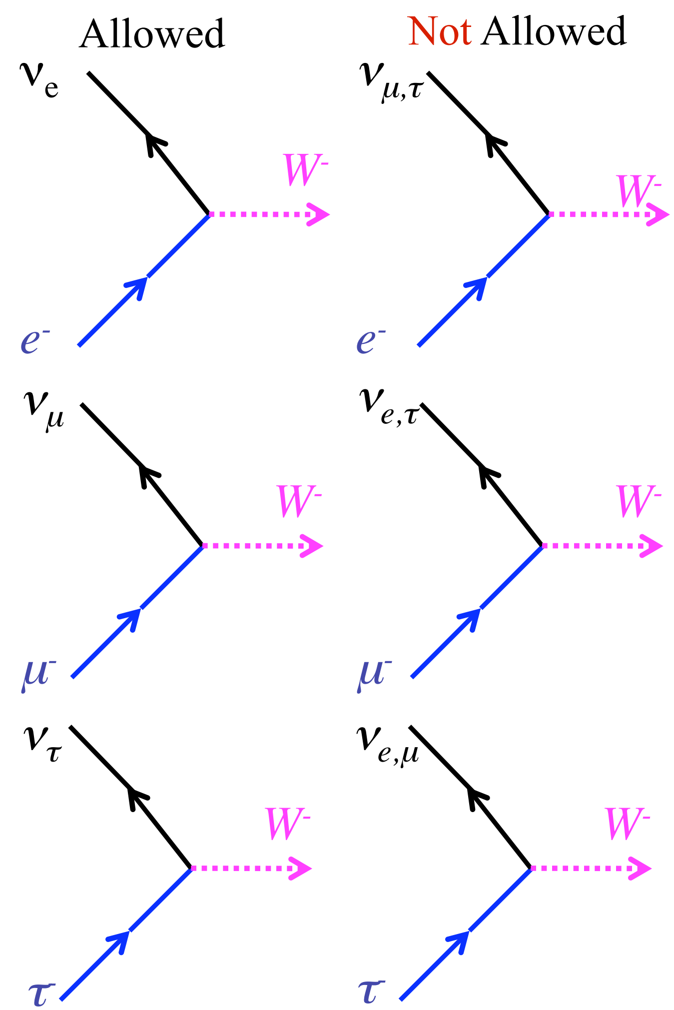
Weak Interactions

- Weinberg-Salam-Glashow (Standard Model 1970's-today):
  - unify weak and EM forces
  - predict neutral current (Z) reactions
  - gives relationship between mass of W and Z
  - predict/explain lots of other stuff, e.g. no flavor changing neutral currents
  - predict existence of Higgs ("generates" mass in Standard Model)
  - construct renormalizable gauge theory
- The picture is still incomplete:
  - must input lots of parameters into the Standard Model (e.g. masses)
  - where's the Higgs and how many are there?
  - how many generations of quarks and leptons are there?
  - how to explain the mass pattern of quarks and leptons?
    - neutrinos have mass!
  - *CP* violation observed with quarks!
    - is there *CP* violation with leptons?

Weak Interactions

- Classification of weak interactions

Type	Comment	Examples
Leptonic	leptons only	$\mu^- \to e^-\bar{\nu}_e\nu_\mu$
		$\nu_e e^- \to \nu_e e^-$
Semileptonic	leptons and quarks	$n \to pe^-\bar{\nu}_e\ (\Delta s = 0)$
		$K^+ \to \mu^+\nu_\mu\ (\Delta s = 1)$
		$B^- \to D^0\mu^-\bar{\nu}_\mu\ (\Delta b = 1)$
Non-leptonic	quarks only	$\Lambda \to \pi^- p\ \&\ K^+ \to \pi^+\pi^0$



- Quarks and leptons are grouped into doublets (SU(2)).
  - Sometimes called families or generations.
  - For every quark doublet there is a lepton doublet:

$$\begin{array}{cccc} & & & Q(|e|) \\ \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} & \begin{matrix} \frac{2}{3} \\ -\frac{1}{3} \end{matrix} \\ \begin{pmatrix} e \\ \nu_e \end{pmatrix} & \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} & \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} & \begin{matrix} -1 \\ 0 \end{matrix} \end{array}$$

- Charged current interaction involves the exchange of a *W* boson.
  - *W*'s couple to leptons in the same doublet.
  - *W* coupling to leptons/quarks is a combination of vector and axial vector (V-A) terms:

$$J^\mu = u\gamma^\mu(1- \gamma^5)u$$

    - parity violating charged current

Cabibbo Model

- Cabibbo's conjecture (1963):
  - quarks participating in weak interaction are a mixture of quarks that participate in the strong interaction
  - explain certain decay patterns in the weak interactions
  - originally had only to do with the *d* and *s* quarks:

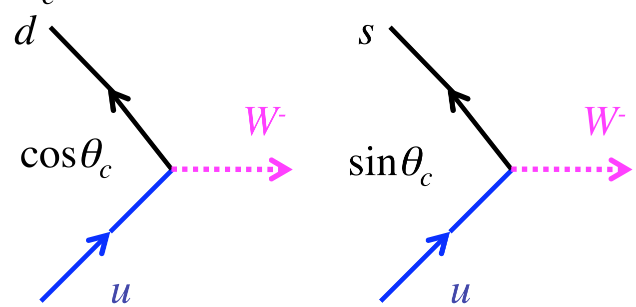
    $$d' = d\cos\theta + s\sin\theta$$

    - the form of the interaction (charged current) has an extra factor for *d* and *s* quarks

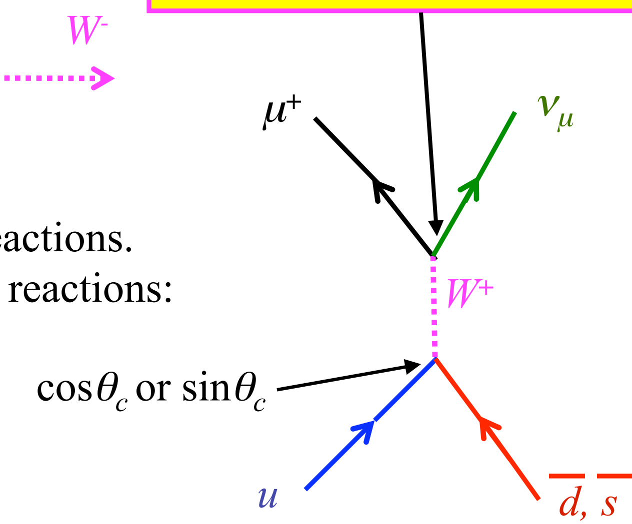
      d quark : $J^u \propto \gamma^u(1-\gamma^5)\cos\theta_c$

      s quark : $J^u \propto \gamma^u(1-\gamma^5)\sin\theta_c$

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$



Purely leptonic decays (e.g. muon decay) do not contain the Cabibbo factor



- The Cabibbo angle is important for determining the rate of many reactions.
  - The Cabibbo angle can measured using data from the following reactions:

    $$\frac{BR(K^+ \to \mu^+ \nu_\mu)}{BR(\pi^+ \to \mu^+ \nu_\mu)} = \frac{\sin^2\theta_c}{\cos^2\theta_c}\frac{m_K}{m_\pi}\left[\frac{1-(m_\mu/m_K)^2}{1-(m_\mu/m_\pi)^2}\right]^2$$

    - From the above branching ratio's we find:

      $\theta_c = 0.27$ radians
  - We can check the above by measuring the rates for $K^- \to \pi^o e^- \overline{\nu}_e$ and $\pi^- \to \pi^o e^- \overline{\nu}_e$:

    $\theta_c = 0.25$ radians

Extension to Cabibbo Model

- Cabibbo's model could easily be extended to 4 quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}$$

$$\begin{pmatrix} c \\ s \end{pmatrix} \to \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- Adding a fourth quark actually solved a long standing puzzle in weak interactions:
  - "absence" (i.e. very small BR) of decays with a "flavor" (e.g. strangeness) changing neutral current:

$$\frac{BR(K^0 \to \mu^+\mu^-)}{BR(K^+ \to \mu^+\nu_\mu)} = \frac{7\times 10^{-9}}{0.64} \approx 10^{-8}$$

M&S section 8.2.1

- Cabibbo's model could not incorporate CP violation
  - 1977: there was evidence for 5 quarks!

Cabibbo's name was added to make "CKM"

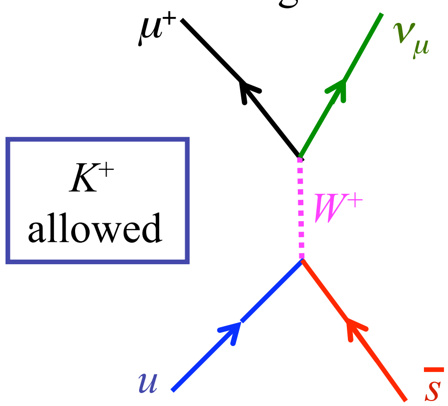
- CKM model:
  - Kobayashi and Maskawa extended Cabibbo's idea to six quarks in 1972.
    - This is two years before discovery of charm!
    - 2x2 → 3x3 matrix that mixes the weak quarks and the strong quarks
    - The matrix is unitary.
      - 1 parameter (Cabibbo angle) → 3 angles (generalized Cabibbo angles) + 1 phase
      - The phase allows for CP violation.
      - Just like θ_c, the matrix elements of the CKM matrix must also be obtained from experiment.

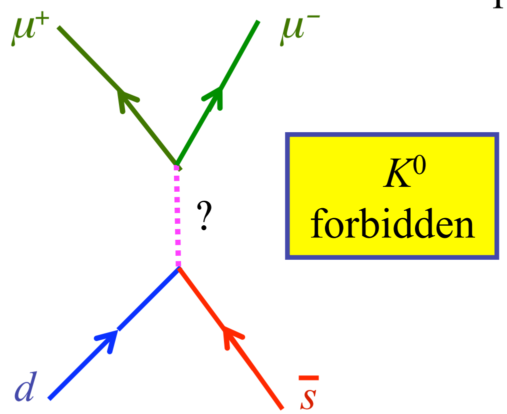
GIM Mechanism

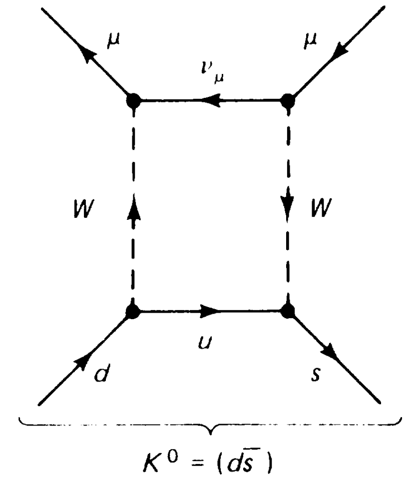
- In 1969-70, Glashow, Iliopoulos, and Maiani (GIM) proposed a solution to the $K^0 \to \mu^+\mu^-$ rate puzzle:

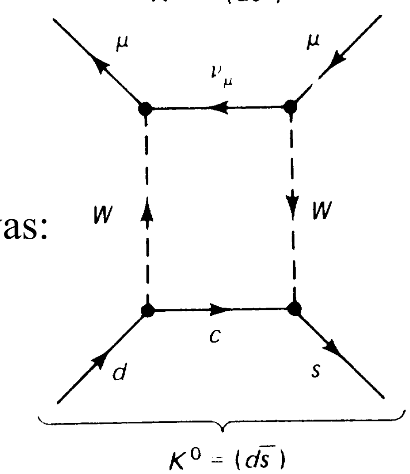
$$\frac{BR(K^0 \to \mu^+\mu^-)}{BR(K^+ \to \mu^+\nu_\mu)} = \frac{7\times10^{-9}}{0.64} \approx 10^{-8}$$

  - The branching fraction for $K^0 \to \mu^+\mu^-$ was expected to be small.
  - The first order diagram is forbidden as this is not a allowed W coupling.









  - The 2nd order diagram ("box") was calculated:

    amplitude $\propto \sin\theta_c\cos\theta_c$

    - Found to give a rate higher than the experimental measurement!
  - GIM proposed that a 4th quark existed and its coupling to the s and d quark was:

    $s' = s\cos\theta - d\sin\theta$

    - The new quark would produce a second "box" diagram with

      amplitude $\propto -\sin\theta_c\cos\theta_c$

    - These two diagrams almost cancel each other out.
      - The amount of cancellation depends on the mass of the new quark.
      - A quark mass of $\approx$ 1.5 GeV is necessary to get good agreement with the experimental data.
      - First "evidence" for Charm quark!

CKM Matrix

- The CKM matrix can be written in many forms:
  - In terms of three angles and phase:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This matrix is not unique, many other 3x3 forms in the literature. This one is from PDG2000.

    - The four real parameters are δ, θ_{12}, θ_{23}, and θ_{13}.
    - s = sin, c = cos, and the numbers refer to the quark generations, e.g. $s_{12} = \sin\theta_{12}$.
  - In terms of coupling to charge 2/3 quarks:

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{sb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

    - Best for illustrating physics!
  - In terms of the sine of the Cabibbo angle (θ_{12}).

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

"Wolfenstein" representaton

    - This representation uses the fact that $s_{12} >> s_{23} >> s_{13}$.
    - $\lambda = \sin\theta_{12}$, and A, ρ, η are all real and approximately one.
    - This representation is very good for relating *CP* violation to specific decay rates.

CKM Matrix

- The magnitudes of the measured CKM elements (PDG2000):

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & (2-5)\times 10^{-3} \\ 0.219-0.225 & 0.9734-0.9749 & (3.7-4.3)\times 10^{-2} \\ (0.4-1.4)\times 10^{-2} & (3.5-4.3)\times 10^{-2} & 0.9990-0.9993 \end{pmatrix}$$

  - There are several interesting patterns:
    1) The CKM matrix is almost diagonal (off diagonal elements are small).
    2) The further away from a family, the smaller the matrix element (e.g. $V_{ub} << V_{ud}$).
    3) Using 1) and 2), we see that certain decay chains are preferred:
       - $c \to s$ over $c \to d$:
         BR($D^0 \to K^-\pi^+$) ~ 3.8%
         BR($D^0 \to \pi^-\pi^+$) ~ 0.15%
       - $b \to c$ over $b \to u$:
         BR($B^0 \to D^-\pi^+$) ~ $3\text{x}10^{-3}$
         BR($B^0 \to \pi^-\pi^+$) ~ $1\text{x}10^{-5}$
    4) Since the matrix is supposed to be unitary there are lots of constraints among the matrix elements:

$$V_{ud}^* V_{ud} + V_{cd}^* V_{cd} + V_{td}^* V_{td} = 1$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

       - So far experimental results are consistent with expectations from a unitary matrix.
       - As precision of experiments increases, we might see deviations from unitarity.

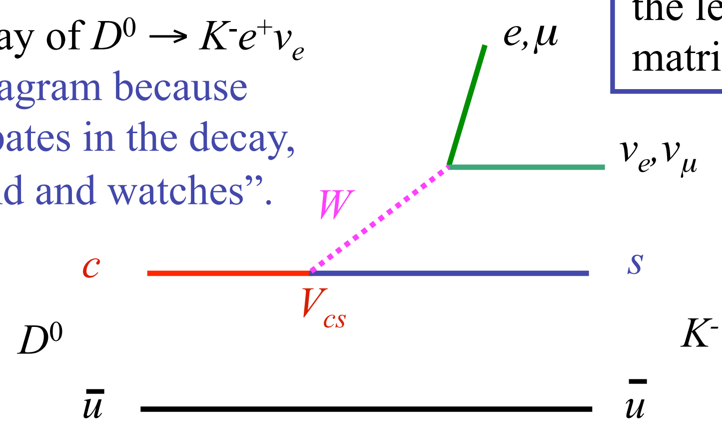
Measuring the CKM Matrix

- No one knows how to calculate the values of the CKM matrix.
  - Cleanest way to measure the CKM elements is to use interactions or decays involving leptons.
    - CKM factors are only present at one vertex in decays with leptons.

V_{ud}	neutron decay	$n \to pe^-\bar{\nu}_e$	$d \to ue\nu$
V_{us}	kaon decay	$K^0 \to \pi^+ e^- \bar{\nu}_e$	$s \to ue\nu$
V_{bu}	B - meson decay	$B^- \to (\rho^+ \text{ or } \pi^+) e^- \bar{\nu}_e$	$b \to ue\nu$
V_{bc}	B - meson decay	$B^- \to D^0 e^- \bar{\nu}_e$	$b \to ce\nu$
V_{cs}	charm decay	$D^0 \to K^- e^+ \nu_e$	$c \to se\nu$
V_{cd}	neutrino interaction	$\nu_\mu d \to \mu^- c$	$d \to c$

For massless neutrinos the lepton "CKM" matrix is diagonal

"Spectator" model decay of $D^0 \to K^- e^+ \nu_e$
Called a "spectator" diagram because only one quark participates in the decay, the other "stands around and watches".



Decay rate $\propto |V_{cs}|^2$