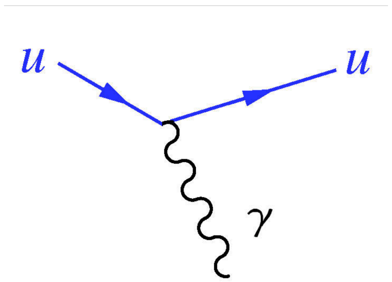
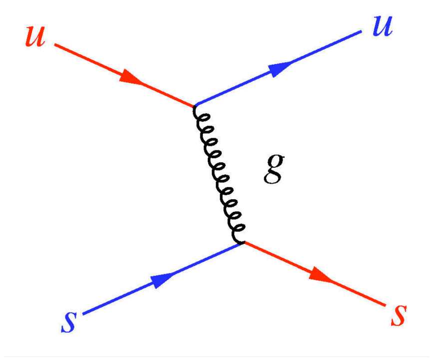
# Quantum Chromodynamics

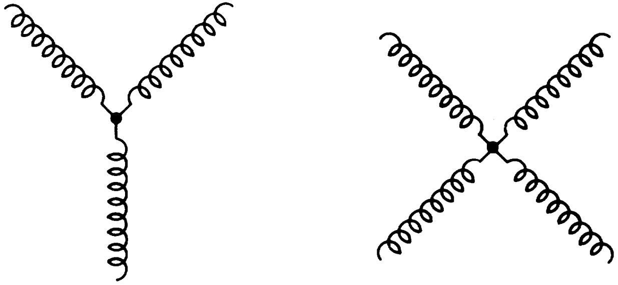
- Quantum Chromodynamics (QCD) is the theory of the strong interaction.
- QCD is a non-abelian gauge theory invariant under SU(3):
  - The interaction is governed by massless spin 1 objects called “gluons”.
  - Gluons couple only to objects that have “color”: quarks and gluons
  - There are three different color charges: red, green, blue.
    - QED: only one electric charge.
  - There are eight different gluons.
    - gluon exchange can change the color of a quark but not its flavor.
    - e.g. a red *u*-quark can become a blue *u*-quark via gluon exchange.

M&S 6.3, 7.1



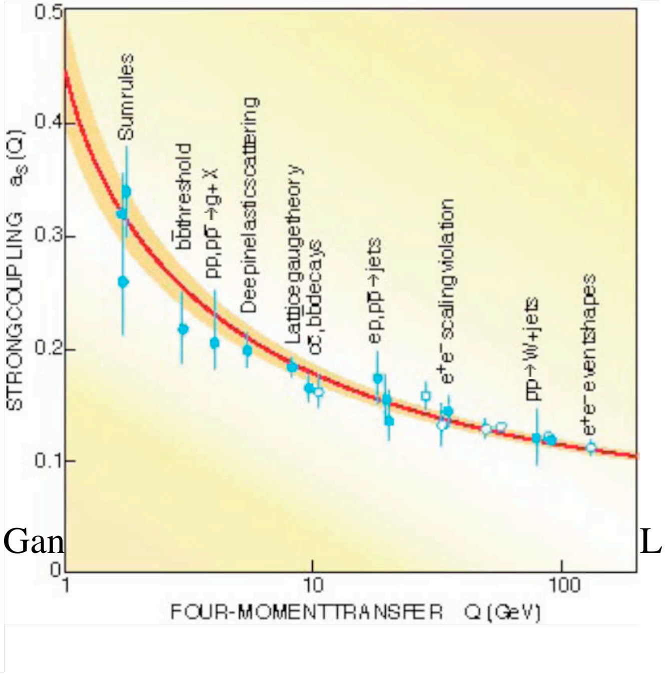


  - Since gluons have color there are couplings involving 3 and 4 gluons.
    - QED: the 3 and 4 photon couplings are absent since the photon does not have an electric charge.



# QCD

- There are several interesting consequences of the SU(3), non-abelian nature of QCD:
  - Quarks are confined in space.
    - We can never "see" a quark the way we can an electron or proton.
    - Explains why there is no experimental evidence for "free" quarks.
  - All particles (mesons and baryons) are color singlets.
    - This "saves" the Pauli Principle.
    - In the quark model the $\Delta^{++}$ consists of 3 up quarks in a totally symmetric state.
      - ☞ Need something else to make the total wavefunction anti-symmetric $\Rightarrow$ color!
  - Asymptotic freedom.
    - The QCD coupling constant changes its value ("runs") dramatically as function of energy.
      - ☞ quarks can appear to be "free" when probed by high energy (virtual) $\gamma$'s and yet be tightly bound into mesons and baryons (low energy).
  - In principle, the masses of mesons and baryons can be calculated using QCD.
    - Reality: very difficult to calculate (almost) anything with QCD.



# QED vs QCD

- QED is an abelian gauge theory with U(1) symmetry: $\psi'(\overline{x},t) = e^{-ief(\overline{x},t)}\psi(\overline{x},t)$
- QCD is a non-abelian gauge theory with SU(3) symmetry: $\psi'(\overline{x},t) = e^{-ig\sum\limits_{i=1}^{8}\frac{\lambda_i\omega_i(\overline{x},t)}{2}}\psi(\overline{x},t)$
- Both are relativistic quantum field theories that can be described by Lagrangians:
  - QED: $L = \overline{\psi}(i\gamma^u\partial_u - m)\psi + \underbrace{e\overline{\psi}\gamma^u A_u\psi}_{\text{electron-}\gamma\text{ interaction}} - \frac{1}{4}F^{uv}F_{uv}$

$m$ = electron mass
$\psi$ = electron spinor

$A_u$ = photon field (1)
$F_{uv} = \partial_u A_v - \partial_v A_u$

  - QCD: $L = \overline{q}_{jk}(i\gamma^u\partial_u - m)q_{jk} + \underbrace{g(\overline{q}_{jk}\gamma^u\lambda_a q_{jk})G_u^a}_{\text{quark-gluon interaction}} - \frac{1}{4}G_{uv}^a G_a^{uv}$

gluon-gluon interaction (3g and 4g)

$m$ = quark mass
$j$ = color (1,2,3)
$k$ = quark type (1-6)
$q$ = quark spinor

$G_u^a$ = gluon field ($a = 1-8$)
$G_{uv}^a = \partial_u G_v^a - \partial_v G_u^a - gf_{abc}G_u^b G_v^c$

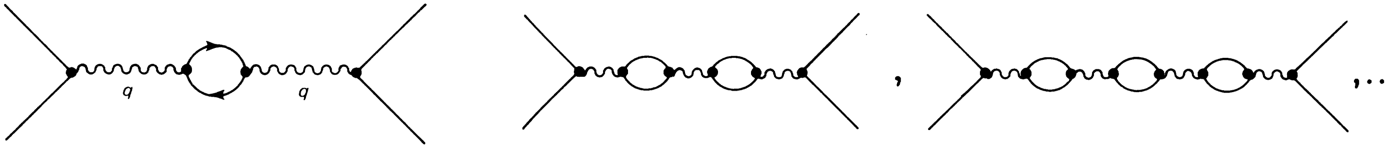
$[\lambda_a, \lambda_b] = if_{abc}\lambda_c$

$\lambda_a$'s ($a$ = 1-8) are the generators of SU(3).
$\lambda_a$'s are 3x3 traceless hermitian matrices.
See M&S Eq. 6.36b for a representation.

$f_{abc}$ are real constants (256)
$f_{abc} \equiv$ structure constants of the group
(M&S Table C.1)

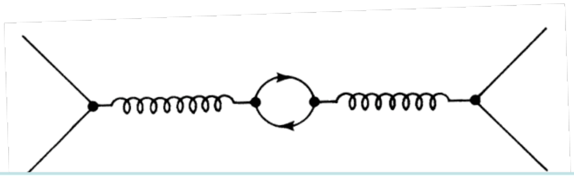
# QED vs QCD

- In QED higher order graphs with virtual fermion loops play an important role in determining the strength of the interaction (coupling constant) as a function of distance scale (or momentum scale).



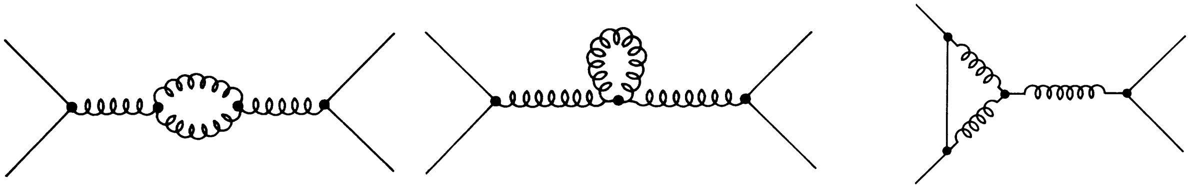
Examples of graphs with virtual fermion loops coupling to virtual photons.

- In QCD similar higher order graphs with virtual loops also play an important role in determining the strength of the interaction as a function of distance scale.



A graph with a virtual fermion loop coupling to virtual gluons.

- In QCD we can also have a class of graphs with gluon loops since the gluons carry color.



- Because of the graphs with gluon loops the QCD coupling constant behaves differently than the QED coupling constant at short distances.

# QED vs QCD

- For both QED and QCD the effective coupling constant $\alpha$ depends on the momentum (or distance) scale that it is evaluated at:

$$\alpha(p^2) = \frac{\alpha(0)}{1 - X(p^2)}$$

$\alpha(0)$ = fine structure constant ≈ 1/137

- For QED it can be shown that:

$$X(p^2) = \left( \sum_{i=1}^{N_f} \left( \frac{q_i}{e} \right)^2 \right) \frac{\alpha(\mu^2)}{3\pi} \ln\left( \frac{p^2}{\mu^2} \right)$$

QED: $X(p^2) > 0$

  - $N_f$: number of fundamental fermions with masses below ½|p|.
  - $m$: mass of the heaviest fermion in the energy region being considered.

- The situation for QCD is very different than QED.
  - Due to the non-abelian nature of QCD:

$$X(p^2) = \frac{\alpha_s(\mu^2)}{12\pi} \ln\left( \frac{p^2}{\mu^2} \right) [2N_f - 11N_c]$$

  - $N_f$: number of quark flavors with masses below ½|p|.
  - $m$: mass of the heaviest quark in the energy region being considered.
  - $N_c$: number of colors (3).
  - For 6 flavors and 3 colors:

$$2N_f - 11N_c < 0$$

☞ $\alpha(p^2)$ decreases with increasing momentum (or shorter distances).

☞ The force between quarks decreases at short distances and increases as the quarks move apart!

asymptotic freedom

# QCD and Confinement

- The fact that gluons carry color and couple to themselves leads to asymptotic freedom:
  - strong force decreases at very small quark-quark separation.
  - strong force increases as quarks are pulled apart.
- Does asymptotic freedom lead to quark confinement?

  Yes!
  - Quark confinement in SU(3) gauge theory can only be proved analytically for 2D (1 space+1 time).
  - Detailed numerical calculations show that even in 4D (3 space+time) quarks configured as mesons and baryons in color singlet states are confined.
- In $SU(3)_{\text{color}}$ there are two conserved quantities (two diagonal generators).

  $Y^c$ = "color hypercharge"

  $I_3^c$ = the 3rd component of "color isospin"
  - The quark color hypercharge and isospin assignments are (M&S p.163)

| | $I_3^c$ | $Y^c$ |
|---|---|---|
| r | 1/2 | 1/3 |
| g | -1/2 | 1/3 |
| b | 0 | -2/3 |

$Y^c$ and $I_3^c$ change sign for anti-quarks

    - By assuming that hadrons are color singlets we are requiring $Y^c = I_3^c = 0$:

$$\text{Mesons}: \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$$

$$\text{Baryons}: \frac{1}{\sqrt{6}}(r_1g_2b_3 - g_1r_2b_3 + b_1r_2g_3 - b_1g_2r_3 + g_1b_2r_3 - r_1b_2g_3)$$

anti-symmetric under color exchange

# QCD and Confinement

- The color part of the quark wavefunction can be represented by:

$$r=\begin{pmatrix}1\\0\\0\end{pmatrix}\quad g=\begin{pmatrix}0\\1\\0\end{pmatrix}\quad b=\begin{pmatrix}0\\0\\1\end{pmatrix}$$

- Using the M&S representation for the SU(3) generators (eq. 6.40b) we have:

$$I_3^c=\frac{1}{2}\lambda_3=\frac{1}{2}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\qquad Y^c=\frac{1}{\sqrt{3}}\lambda_8=\frac{1}{3}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

- As shown in M&S eq. 6.38 only quark and anti-quark combinations with $Y^c = I_3{}^c = 0$ are of the form:

$$(3q)^p(q\bar{q})^n\ \ (p,n\geq 0)$$

- Conventional baryons are the states with $p = 1$, $n = 0$ and mesons are states with $p = 0$, $n = 1$.
  - States such as $qqqq$ and $qq$ are forbidden by the singlet (confinement) requirement.
  - However, the following states are allowed:

$$qqqqqq\quad p=2,n=0$$

$$q\bar{q}q\bar{q}\quad p=0,n=2$$

$$qqqq\bar{q}\quad p=1,n=1$$

What about bound states of gluons -- "glueballs"?

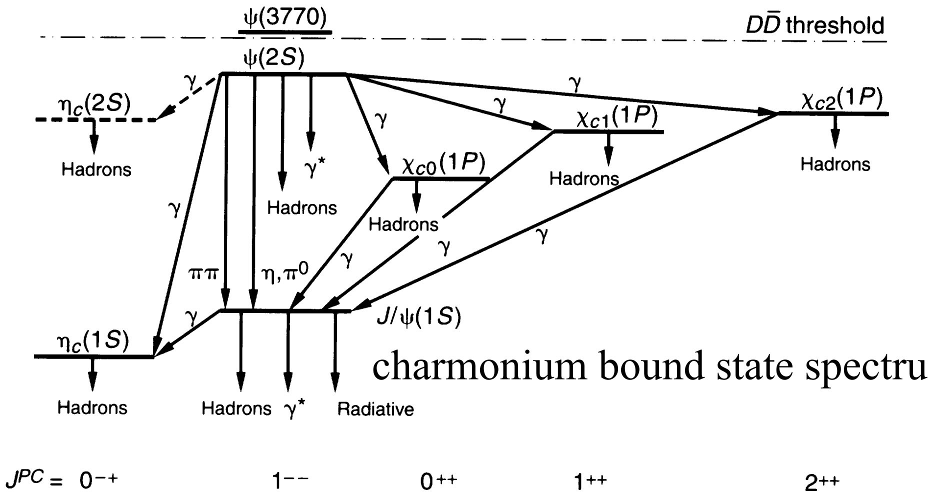
  - There have been many searches for quark bound states other than conventional mesons and baryons.
    - To date there is only evidence for the existence of the conventional mesons and baryons.

# Quark Anti-Quark Bound States and QCD

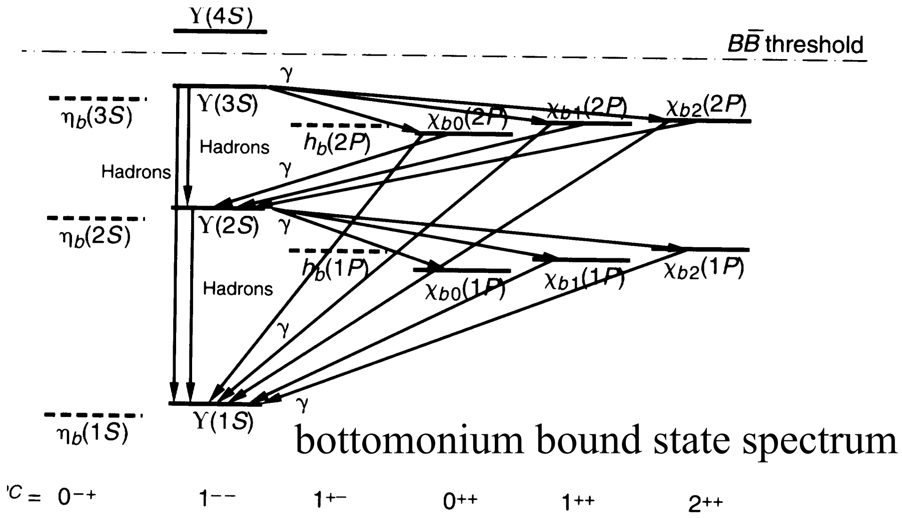
- The spectrum of bounds states of heavy $q\bar{q}$ $(= c\bar{c}$ or $b\bar{b})$ states can be calculated in analogy with positronium ($e^+e^-$).
  - "QCD" calculations work best for non-relativistic (i.e. heavy states) where the potential can be approximated by:

$$V(r) = -\frac{a}{r} + br$$

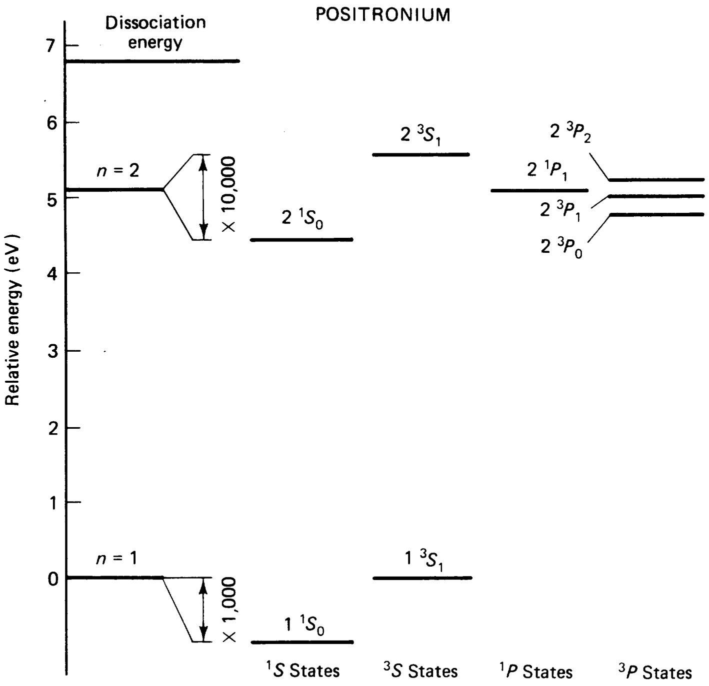
  - Both $a$ and $b$ are constants calculated from fitting data or a model.



charmonium bound state spectrum



bottomonium bound state spectrum



positronium bound state spectrum

# What is the Evidence for Color?

- One of the most convincing arguments for color comes from a comparison of the cross sections for the two processes:

  $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to q\bar{q}$

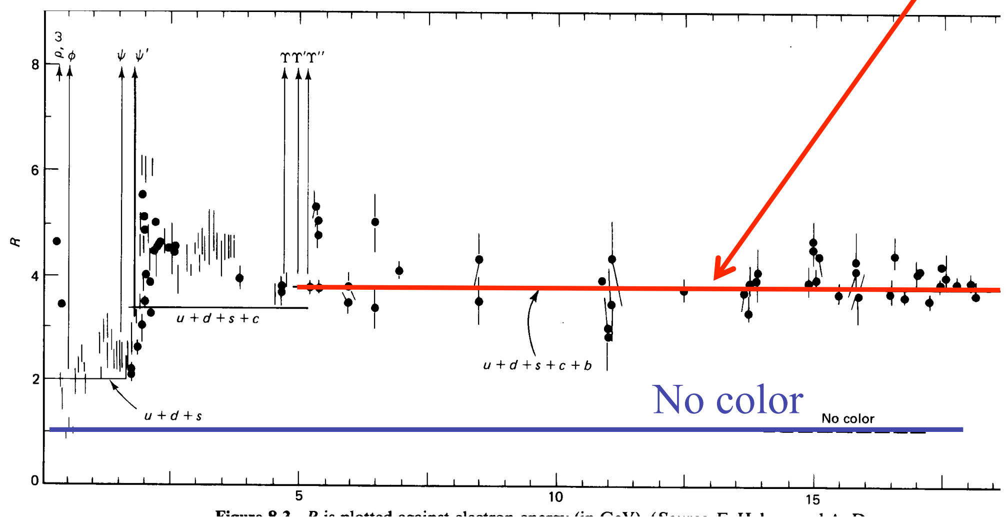
- If we ignore how the quarks turn into hadrons
  - ☞ amplitudes for the two reactions only differ by the charge of final state fermions (muons or quarks):

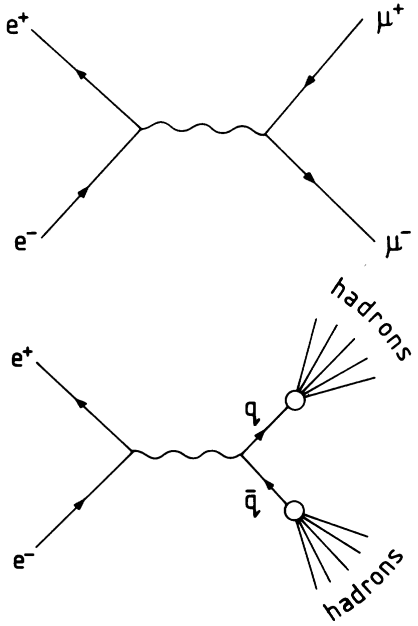
$$R \equiv \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)} = n_c \sum_{i=1}^{n} Q_i^2$$

Assume the CM energy of the reaction is large compared to the fermion masses.

  - ■ $n_c = 1$ if the quarks have no color
  - ■ $n_c = 3$ if each quark has three colors
  - ■ For example, above *b*-quark threshold but below top-quark threshold we would expect:

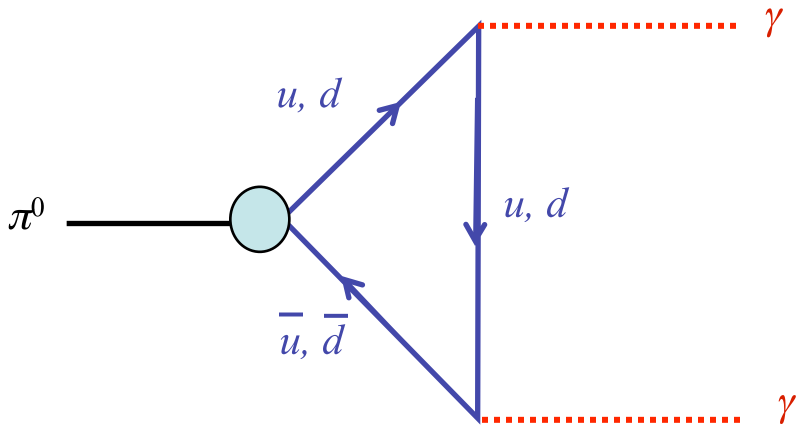
$$R = 3\left[\left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = \frac{33}{9} = 3.67$$





# QCD, Color, and the Decay of the $\pi^0$

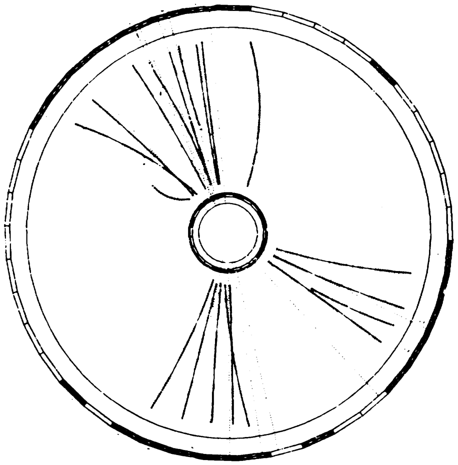
- 1949-50 The decay $\pi^0 \to \gamma\gamma$ calculated and measured by Steinberger.
- 1967 Veltman calculates the $\pi^0$ decay rate using modern field theory and finds that $\pi^0$ does no decay!
- 1968-70 Adler, Bell, and Jackiw "fix" field theory and now $\pi^0$ decays but decay rate is off by factor of 9.
- 1973-4 Gell-Mann and Fritzsch (+others) use QCD with 3 colors and calculate the correct $\pi^0$ decay rate.



Triangle Diagram:
Each color contributes one amplitude. Three colors changes the decay rate by 9.

# QCD and Jets

- QCD predicts that we will not see isolated quarks.
  - hadrons produced in a high energy collision should have some "memory" of its parent quark (or gluon).
  - About 30 years ago someone used the term "jet" to describe the collimation of a group of hadrons as they are Lorentz boosted along the direction of the parent quark.

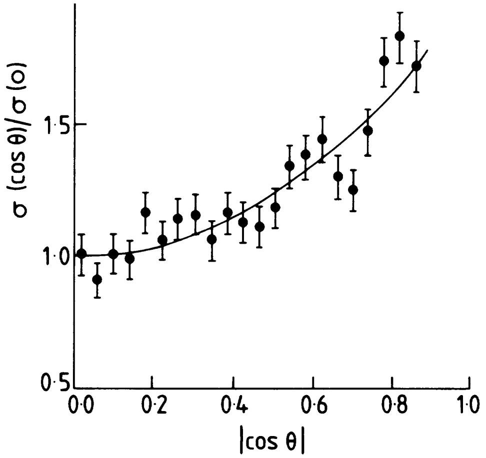


An example of $e^+e^- \rightarrow$ hadrons with three jets. The lines represent the trajectories of charged tracks in the magnetic field of the central detector of the JADE experiment. The beams are ⊥ to the page.

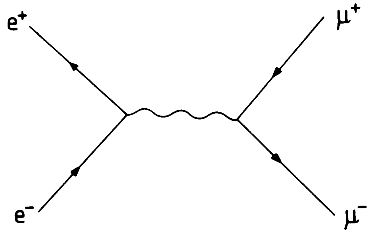
- Again we can compare muon pair production to quark production.
  - The angular distribution ($\cos\theta$) with respect to the beam line ("z-axis") for the $\mu^-$ is:

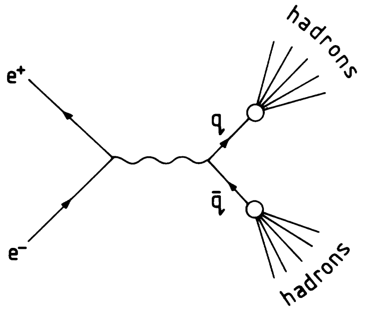
$$\frac{d\sigma}{d\cos\theta} = A(1+\cos^2\theta) \text{ for } e^+e^- \rightarrow \mu^+\mu^-$$

  - Since the parent quarks are also spin 1/2 fermions
    - ☞ expect the quarks to have the same angular distribution as muons.
  - We would also expect the momentum vector of the quark jets to have this same angular distribution.



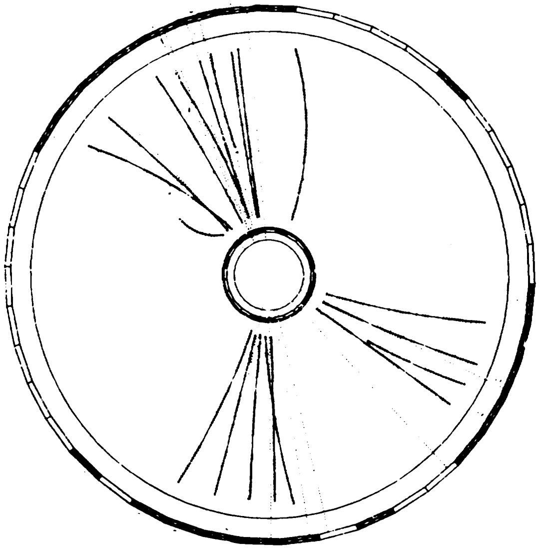
Data from "2 jet" events.
Solid line is $1+\cos^2\theta$.
Good agreement!!

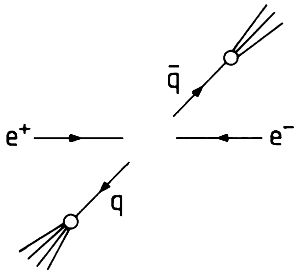


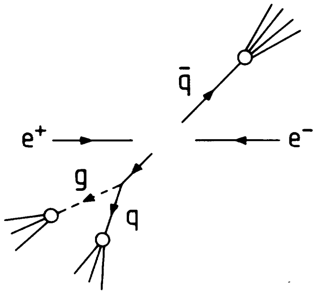


# QCD and Jets

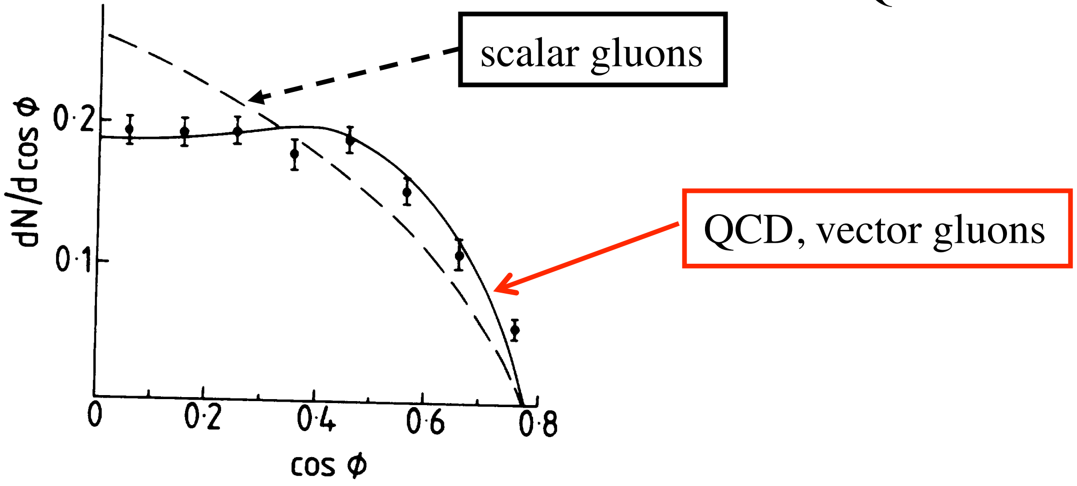
- Perhaps even more interesting than the “two jet” events are “three jet” events.
    - One of the jets is due to a gluon forming hadrons, the other two jets are from the parent quarks.







- Using QCD it is possible to calculate the angular distribution (cos$\theta$) of three jets with respect to “z-axis”.
    - Data (from the TASSO experiment) is in very good agreement with the QCD.



scalar gluons

QCD, vector gluons