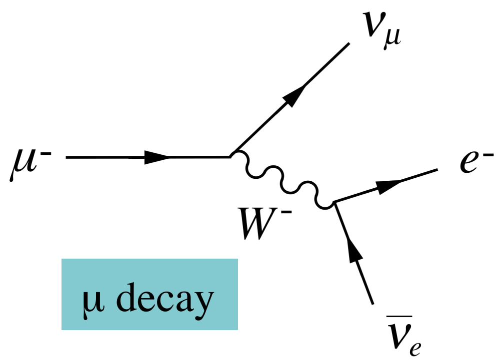
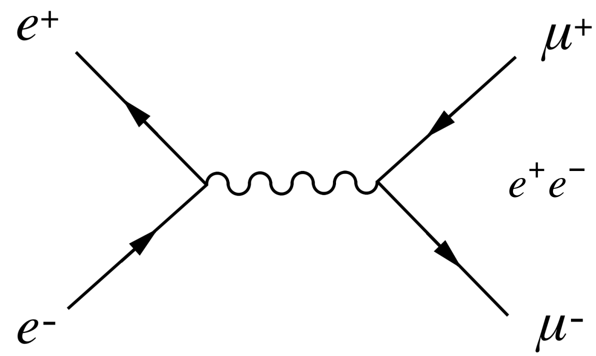
Feynman Diagrams

- Pictorial representations of amplitudes of particle reactions, i.e scatterings or decays.
- Greatly reduce the computation involved in calculating rate or cross section of a physical process, e.g.





$e^+e^- \to \mu^+\mu^-$ scattering

- Like electrical circuit diagrams, every line in the diagram has a strict mathematical interpretation.
- For details of Feynman diagram calculation,
  - take a Advanced Quantum or 880.02 course
  - see Griffiths (e.g. sections 6.3, 6.6, and 7.5)
  - Bjorken & Drell (Relativistic Quantum Mechanics).

Feynman and his diagrams

Scattering Amplitute

- Each Feynman diagram represents an amplitude (M).
  - Quantities such as cross sections and decay rates (lifetimes) are proportional to $|M|^2$.
- The transition rate for a process can be calculated with time dependent perturbation theory using Fermi's Golden Rule:

$$\text{transition rate} = \frac{2\pi}{\hbar}|M|^2 \times [\text{phase space}]$$

In lowest order perturbation theory M is the Fourier transform of the potential, M&S B.20-22, p295, "Born Approximation", M&S 1.32, p20.

- The differential cross section for two body scattering (e.g. $pp \to pp$) in the CM frame is:

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{q_f^2}{v_i v_f}|M|^2$$

M&S B.29, p350

v_i = speed of initial state particle

v_f = speed of final state particle

q_f = final state momentum

- The decay rate (Γ) for a two-body decay (e.g. $K^0 \to \pi^+\pi^-$) in CM is given by:

$$\Gamma = \frac{S|\vec{p}|}{8\pi\hbar m^2 c}|M|^2$$

Griffiths 6.32

m = mass of parent

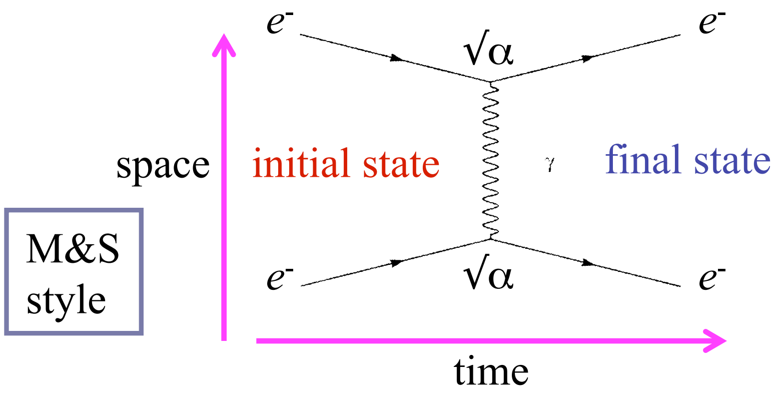
p = momentum of decay particle

S = statistical factor (fermions/bosons)

- $|M|^2$ cannot be calculated exactly in most cases.
- Often M is expanded in a power series.
- Feynman diagrams represent terms in the series expansion of M.

Some Rules of Feynman Diagrams

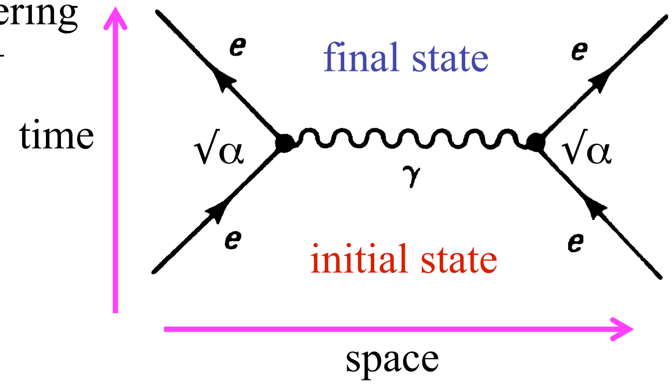
- Feynman diagrams plot time vs space:



Moller Scattering

$e^-e^- \rightarrow e^-e^-$

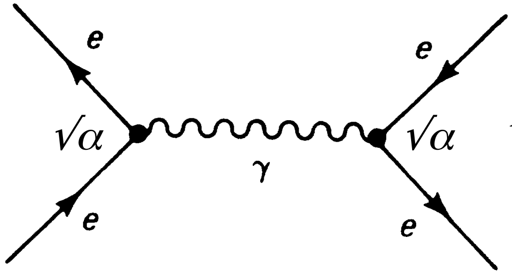
or



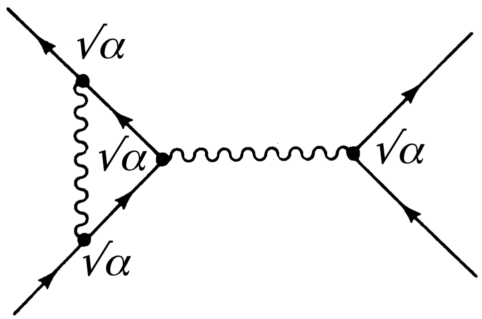
- Solid lines are charged fermions:
  - particle: arrow in same direction as time
  - antiparticle: arrow opposite direction as time
- Wavy (or dashed) lines are photons.
- At each vertex there is a coupling constant.
- Quantum numbers are conserved at a vertex:
  - electric charge, lepton number…
- "Virtual" particles do not conserve E and $\boldsymbol{p}$.
  - Virtual particles are internal to diagram(s).
    - γ: $E^2 - p^2 \neq 0$ (off "mass shell")
  - In all calculations we integrate over the virtual particles 4-momentum (4D integral).
- Photons couple to electric charge.
  - No photons only vertices.

Higher Order Feynman Diagrams

- We classify diagrams by the order of the coupling constant:
  - Bhabha scattering: $e^+e^- \rightarrow e^+e^-$:



Amplitude is of order α.

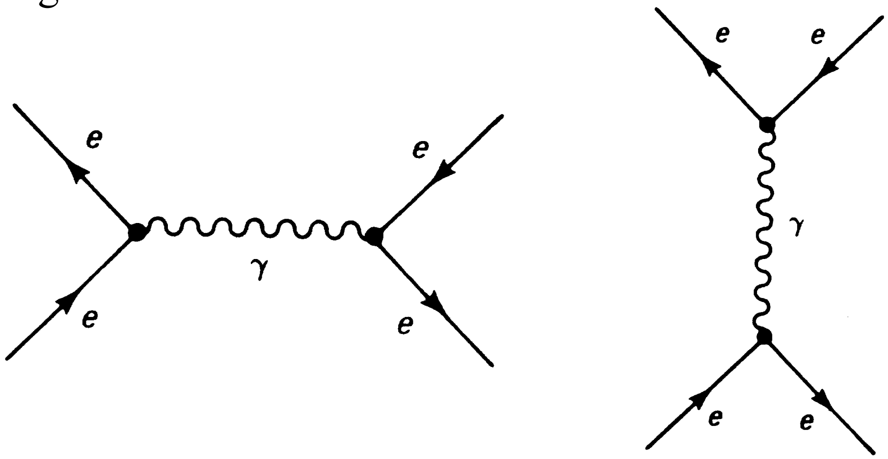


Amplitude is of order α^2.

  - Since $\alpha_{QED} = 1/137$, higher order diagrams should be corrections to lower order diagrams.
    - This is just perturbation Theory!
    - This expansion in the coupling constant works for QED since $\alpha_{QED} = 1/137$.
    - Does not work well for QCD where $\alpha_{QCD} \sim 1$

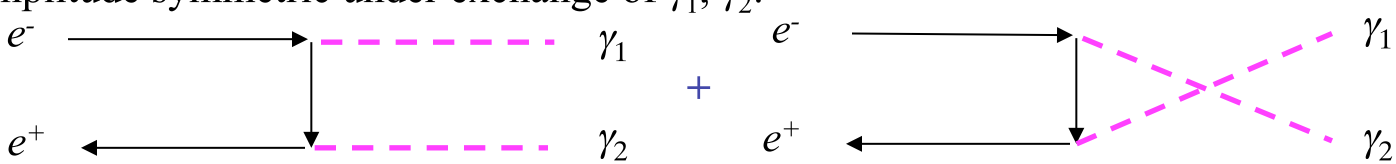
Same Order Feynman Diagrams

- For a given order of the coupling constant there can be many diagrams.
  - Moller scattering $e^-e^- \rightarrow e^-e^-$:

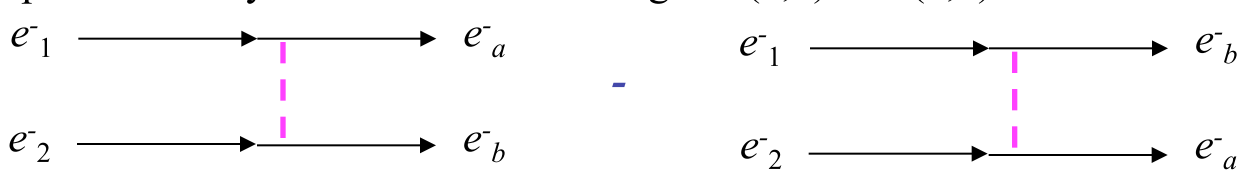


amplitudes can interfere constructively or destructively

  - Must add/subtract diagram together to get the total amplitude.
    - Total amplitude must reflect the symmetry of the process.
    - $e{+}e{-} \rightarrow \gamma\gamma$: identical bosons in final state
      - Amplitude symmetric under exchange of γ_1, γ_2:

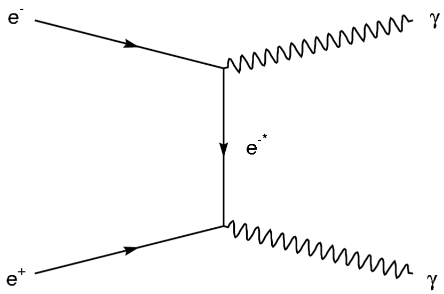


    - Moller scattering $e^-e^- \rightarrow e^-e^-$: identical fermions in initial and final state
      - Amplitude anti-symmetric under exchange of (1,2) and (a,b)

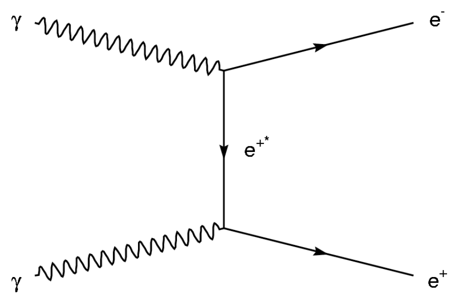


Relationship between Feynman Diagrams

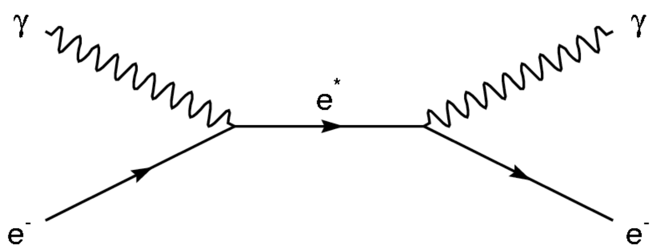
- Feynman diagrams of a given order are related to each other:



$e^+e^- \to \gamma\gamma$ γ's in final state



$\gamma\gamma \to e^+e^-$ γ's in initial state



Compton scattering
$\gamma e^- \to \gamma e^-$

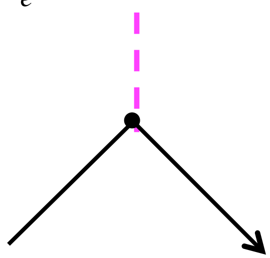
Electron and positron wave functions are related to each other.

Anomalous Magnetic Moment

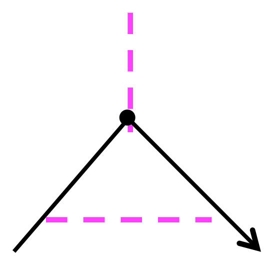
- Calculation of Anomalous Magnetic Moment of electron is one of the great triumphs of QED.
  - In Dirac's theory a point like spin 1/2 object of electric charge q and mass m has a magnetic moment:

    $\boldsymbol{\mu} = q\mathbf{S}/m$.
- In 1948 Schwinger calculated the first-order radiative correction to the naïve Dirac magnetic moment of the electron.
  - Radiation and re-absorption of a single virtual photon contributes an anomalous magnetic moment:

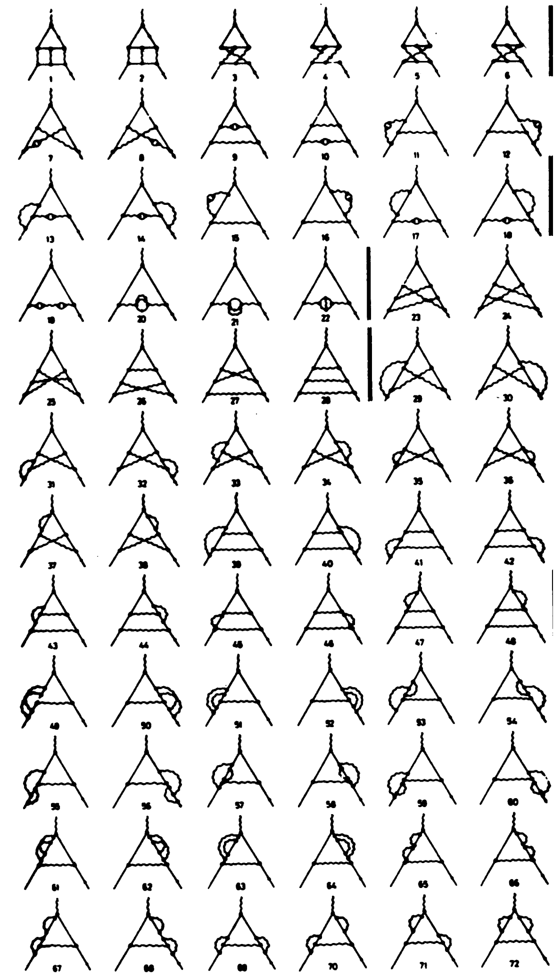
    $a_e = \alpha/2\pi \sim 0.00116$.



Basic interaction with B field photon



First order correction



3rd order corrections

  - Radiation correction modified the Dirac Magnetic Moment to:

    $\boldsymbol{\mu} = g(q\mathbf{S}/m)$
    - The deviation from the Dirac-ness: $a_e = (g_e - 2)/2$
      - The electron's anomalous magnetic moment (a_e) is now known to 4 parts per billion.
      - Current theoretical limit is due to 4th order corrections (> 100 10-dimensional integrals).
    - The muons's anomalous magnetic moment (a_μ) is known to 1.3 parts per million.
      - Recent screw up with sign convention on theoretical a_μ calculations caused a stir!