Isospin

- Isospin is a continuous symmetry invented by Heisenberg:
  - Explain the observation that the strong interaction does not distinguish between neutron and proton.
    - Example: the mass difference between the two is very small:

      $(m_n - m_p)/m_n \approx 10^{-3}$
  - Heisenberg's thought was that if you could turn off electromagnetism then $m_n = m_p$.
  - We now believe that that isospin symmetry is due the near equality of the up and down quarks:

    $m_u \approx m_d$
- We postulate that isospin is conserved in the strong interaction, but not in the electromagnetic or weak.
  - Strong interaction does not feel (or "couple") to electric charge.
    - ☞ Expect the strong interaction of the proton and neutron to be the same.
    - ☞ Isospin operator (I) commutes with the strong Hamiltonian, but not electromagnetic Hamilatonian.
      - $[H_s,I] = 0$
      - $[H_{EM},I] \neq 0$
- When constructing the wavefunction of a system under the strong interaction
  - we must take isospin into consideration to make sure we have correct (boson or fermion) symmetry.
  - ☞ This generalizes the Pauli Principle.
- When constructing baryons (3 quark states) and mesons (quark anti-quark states)
  - we take into account the isospin of the quarks:
    - u: $I = 1/2$, $I_3 = +1/2$
    - d: $I = 1/2$, $I_3 = -1/2$
    - s, c, t, b: $I = 0$

Isospin Combination

- Mathematically, isospin is identical to spin.
  - ☞ We combine isospin the same way we combine angular momentum in quantum mechanics.
  - Like angular momentum, isospin can be integral or half integral:

Particles	Isospin (I)
Λ^0 or Ω^-	0
(p,n) or (K^0, K^+)	1/2
π^+, π^0, π^-	1
Δ^{++}, Δ^+, Δ^0, Δ^-	3/2

Always $2I + 1$ states

  - Like the proton and neutron:
    - the three pion states (π^+, π^0, π^-) are really one particle under the strong interaction.
      - Split by the electromagnetic interaction.
- Isospin states are labeled by the total isospin (I) and the third component of isospin (I_3).
  - Just like ordinary angular momentum states.

Particles	Isospin state $\|I,I_3>$
Λ^0 (uds) or Ω^- (sss)	$\|0,0>$
p (uud) or $K^+(u\bar{s})$	$\|1/2,1/2>$
n (udd) or $K^0(d\bar{s})$	$\|1/2,-1/2>$
π^+ ($u\bar{d}$)	$\|1,1>$
π^0 $(u\bar{u} - d\bar{d})/\sqrt{2}$	$\|1,0>$
π^- ($d\bar{u}$)	$\|1,-1>$

Clebsch-Gordan Coefficients

- Isospin is useful for understanding low energy (≈ 1 GeV) strong interaction scattering cross sections.
  - Consider the two reactions (d = deuterium):
    $pp \to d\pi^+$
    $pn \to d\pi^0$
    - Deuterium is an "iso-singlet", $|0,0>$.
    - In terms of isospin states we have:
      $pp = |1/2,1/2>|1/2,1/2>$ $\qquad d\pi^+ = |0,0>|1,1>$
      $pn = |1/2,1/2>|1/2,-1/2>$ $\qquad d\pi^0 = |0,0>|1,0>$
    - We can use the same techniques as is used to combine angular momentum in QM.
      - For pp, $d\pi^+$, and $d\pi^0$ there is only one way to combine the spin states:
        $pp = |1/2,1/2>|1/2,1/2> = |1,1>$
        $d\pi^+ = |0,0>|1,1> = |1,1>$
        $d\pi^0 = |0,0>|1,0> = |1,0>$
      - The pn state is tricky since it is a combination of $|0,0>$ and $|1,0>$.
        - ☞ The amount of each state is given by the Clebsch-Gordan coefficients ($1/\sqrt{2}$ in this cases).

$$|j_1,m_1>|j_2,m_2> = \sum_{j=|j_1-j_2|}^{j_1+j_2} C^{J,J_1,J_2}_{m,m_1,m_2} |j,m> \quad with \quad m = m_1 + m_2$$

$$|1/2,+1/2>|1/2,-1/2> = \frac{|0,0>}{\sqrt{2}} + \frac{|1,0>}{\sqrt{2}}$$

Clebsch-Gordan Coefficients

32. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2			
m_1	m_2	Coefficients		
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$1/2 \times 1/2$

m_1	m_2		
		J: 1	
		M: +1	
+1/2	+1/2	1	
		J: 1	0
		M: 0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2
		J: 1	
		M: −1	
−1/2	−1/2	1	

$1 \times 1/2$

m_1	m_2		
		J: 3/2	
		M: +3/2	
+1	+1/2	1	
		J: 3/2	1/2
		M: +1/2	+1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3
		J: 3/2	1/2
		M: −1/2	−1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3
		J: 3/2	
		M: −3/2	
−1	−1/2	1	

$2 \times 1/2$

m_1	m_2		
		J: 5/2	
		M: +5/2	
+2	+1/2	1	
		J: 5/2	3/2
		M: +3/2	+3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5
		J: 5/2	3/2
		M: +1/2	+1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5
		J: 5/2	3/2
		M: −1/2	−1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5
		J: 5/2	3/2
		M: −3/2	−3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5
		J: 5/2	
		M: −5/2	
−2	−1/2	1	

$3/2 \times 1/2$

m_1	m_2		
		J: 2	
		M: +2	
+3/2	+1/2	1	
		J: 2	1
		M: +1	+1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4
		J: 2	1
		M: 0	0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2
		J: 2	1
		M: −1	−1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4
		J: 2	
		M: −2	
−3/2	−1/2	1	

2×1

m_1	m_2			
		J: 3		
		M: +3		
+2	+1	1		
		J: 3	2	
		M: +2	+2	
+2	0	1/3	2/3	
+1	+1	2/3	−1/3	
		J: 3	2	1
		M: +1	+1	+1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	6/15	−1/2	1/10
		J: 3	2	1
		M: 0	0	0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10
		J: 3	2	1
		M: −1	−1	−1
0	−1	6/15	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5
		J: 3	2	
		M: −2	−2	
−1	−1	2/3	1/3	
−2	0	1/3	−2/3	
		J: 3		
		M: −3		
−2	−1	1		

1×1

m_1	m_2			
		J: 2		
		M: +2		
+1	+1	1		
		J: 2	1	
		M: +1	+1	
+1	0	1/2	1/2	
0	+1	1/2	−1/2	
		J: 2	1	0
		M: 0	0	0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3
		J: 2	1	
		M: −1	−1	
0	−1	1/2	1/2	
−1	0	1/2	−1/2	
		J: 2		
		M: −2		
−1	−1	1		

$3/2 \times 1$

m_1	m_2			
		J: 5/2		
		M: +5/2		
+3/2	+1	1		
		J: 5/2	3/2	
		M: +3/2	+3/2	
+3/2	0	2/5	3/5	
+1/2	+1	3/5	−2/5	
		J: 5/2	3/2	1/2
		M: +1/2	+1/2	+1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6
		J: 5/2	3/2	1/2
		M: −1/2	−1/2	−1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2
		J: 5/2	3/2	
		M: −3/2	−3/2	
−1/2	−1	3/5	2/5	
−3/2	0	2/5	−3/5	
		J: 5/2		
		M: −5/2		
−3/2	−1	1		

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$2 \times 3/2$

m_1	m_2				
		J: 7/2			
		M: +7/2			
+2	+3/2	1			
		J: 7/2	5/2		
		M: +5/2	+5/2		
+2	+1/2	3/7	4/7		
+1	+3/2	4/7	−3/7		
		J: 7/2	5/2	3/2	
		M: +3/2	+3/2	+3/2	
+2	−1/2	1/7	16/35	2/5	
+1	+1/2	4/7	1/35	−2/5	
0	+3/2	2/7	−18/35	1/5	
		J: 7/2	5/2	3/2	1/2
		M: +1/2	+1/2	+1/2	+1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10
		J: 7/2	5/2	3/2	1/2
		M: −1/2	−1/2	−1/2	−1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5
		J: 7/2	5/2	3/2	
		M: −3/2	−3/2	−3/2	
0	−3/2	2/7	18/35	1/5	
−1	−1/2	4/7	−1/35	−2/5	
−2	+1/2	1/7	−16/35	2/5	
		J: 7/2	5/2		
		M: −5/2	−5/2		
−1	−3/2	4/7	3/7		
−2	−1/2	3/7	−4/7		
		J: 7/2			
		M: −7/2			
−2	−3/2	1			

$3/2 \times 3/2$

m_1	m_2				
		J: 3			
		M: +3			
+3/2	+3/2	1			
		J: 3	2		
		M: +2	+2		
+3/2	+1/2	1/2	1/2		
+1/2	+3/2	1/2	−1/2		
		J: 3	2	1	
		M: +1	+1	+1	
+3/2	−1/2	1/5	1/2	3/10	
+1/2	+1/2	3/5	0	−2/5	
−1/2	+3/2	1/5	−1/2	3/10	
		J: 3	2	1	0
		M: 0	0	0	0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4
		J: 3	2	1	
		M: −1	−1	−1	
+1/2	−3/2	1/5	1/2	3/10	
−1/2	−1/2	3/5	0	−2/5	
−3/2	+1/2	1/5	−1/2	3/10	
		J: 3	2		
		M: −2	−2		
−1/2	−3/2	1/2	1/2		
−3/2	−1/2	1/2	−1/2		
		J: 3			
		M: −3			
−3/2	−3/2	1			

2×2

m_1	m_2					
		J: 4				
		M: +4				
+2	+2	1				
		J: 4	3			
		M: +3	+3			
+2	+1	1/2	1/2			
+1	+2	1/2	−1/2			
		J: 4	3	2		
		M: +2	+2	+2		
+2	0	3/14	1/2	2/7		
+1	+1	4/7	0	−3/7		
0	+2	3/14	−1/2	2/7		
		J: 4	3	2	1	
		M: +1	+1	+1	+1	
+2	−1	1/14	3/10	3/7	1/5	
+1	0	3/7	1/5	−1/14	−3/10	
0	+1	3/7	−1/5	−1/14	3/10	
−1	+2	1/14	−3/10	3/7	−1/5	
		J: 4	3	2	1	0
		M: 0	0	0	0	0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	1	8/35	−2/5	1/14	1/10	−1/5
−2	2	1/70	−1/10	2/7	−2/5	1/5
		J: 4	3	2	1	
		M: −1	−1	−1	−1	
+1	−2	1/14	3/10	3/7	1/5	
0	−1	3/7	1/5	−1/14	−3/10	
−1	0	3/7	−1/5	−1/14	3/10	
−2	1	1/14	−3/10	3/7	−1/5	
		J: 4	3	2		
		M: −2	−2	−2		
0	−2	3/14	1/2	2/7		
−1	−1	4/7	0	−3/7		
−2	0	3/14	−1/2	2/7		
		J: 4	3			
		M: −3	−3			
−1	−2	1/2	1/2			
−2	−1	1/2	−1/2			
		J: 4				
		M: −4				
−2	−2	1				

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 32.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.

Cross Section Calculation using Isospin

- We now want to calculate the ratio of scattering cross sections for these two reactions.
- Fermi's Golden Rules tells us that a cross section is proportional to the square of a matrix element:

  $\sigma \propto |\langle f|H|I\rangle|^2$
  - I = initial state
  - f = final state
  - H = Hamiltonian
  - If H conserves isospin (strong interaction)
    - The initial and final states must have the same I and I_3.
    - Assuming isospin conservation we have:

      $|\langle d\pi^+|H|pp\rangle|^2 = |\langle 1,1||1,1\rangle|^2 = 1$

      $|\langle d\pi^0|H|pn\rangle|^2 = |\langle 1,0|(1/\sqrt{2})(|0,0\rangle + |1,0\rangle)|^2 = 1/2$
    - The ratio of cross section is expected to be:

$$\frac{\sigma_{pp\to d\pi^+}}{\sigma_{pn\to d\pi^0}} = \frac{\left|\left\langle d\pi^+\middle|H\middle|pp\right\rangle\right|^2}{\left|\left\langle d\pi^0\middle|H\middle|pn\right\rangle\right|^2} = \frac{2}{1}$$

      - ★ This ratio is consistent with experimental measurement!

Another Cross Section Calculation Example

- Another example of isospin invariance can be found in pion nucleon scattering.
  - Consider the following two-body reactions:

State	Isospin decomposition
$\pi^+ p$	$\lvert 1,1\rangle \lvert \tfrac{1}{2},\tfrac{1}{2}\rangle = \lvert \tfrac{3}{2},\tfrac{3}{2}\rangle$
$\pi^- p$	$\lvert 1,-1\rangle \lvert \tfrac{1}{2},\tfrac{1}{2}\rangle = \sqrt{\tfrac{1}{3}}\lvert \tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\lvert \tfrac{1}{2},-\tfrac{1}{2}\rangle$
$\pi^0 n$	$\lvert 1,0\rangle \lvert \tfrac{1}{2},-\tfrac{1}{2}\rangle = \sqrt{\tfrac{2}{3}}\lvert \tfrac{3}{2},-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\lvert \tfrac{1}{2},-\tfrac{1}{2}\rangle$

  - If at a certain energy the scattering particles form a bound state with $I = 3/2$
    - ☞ only the $I = 3/2$ components will contribute to the cross section:

$$\langle \tfrac{1}{2}, I_3 \lvert H \rvert \tfrac{1}{2}, I_3\rangle = 0 \text{ or very small}$$

☞

$$\pi^+ p \to \pi^+ p \qquad = \langle \tfrac{3}{2},\tfrac{3}{2} \lvert H \rvert \tfrac{3}{2},\tfrac{3}{2}\rangle$$

$$\pi^- p \to \pi^- p \qquad = \tfrac{1}{3}\langle \tfrac{3}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{3}{2},-\tfrac{1}{2}\rangle + \tfrac{2}{3}\langle \tfrac{1}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{1}{2},-\tfrac{1}{2}\rangle = \tfrac{1}{3}\langle \tfrac{3}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{3}{2},-\tfrac{1}{2}\rangle$$

$$\pi^- p \to \pi^0 n \qquad = \sqrt{\tfrac{1}{3}}\sqrt{\tfrac{2}{3}}\langle \tfrac{3}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\sqrt{\tfrac{1}{3}}\langle \tfrac{1}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{1}{2},-\tfrac{1}{2}\rangle = \tfrac{\sqrt{2}}{3}\langle \tfrac{3}{2},-\tfrac{1}{2} \lvert H \rvert \tfrac{3}{2},-\tfrac{1}{2}\rangle$$

  - The cross sections depend on the square of the matrix element.
  - If we assume that the strong interaction is independent of I_3 then we get the following relationships:

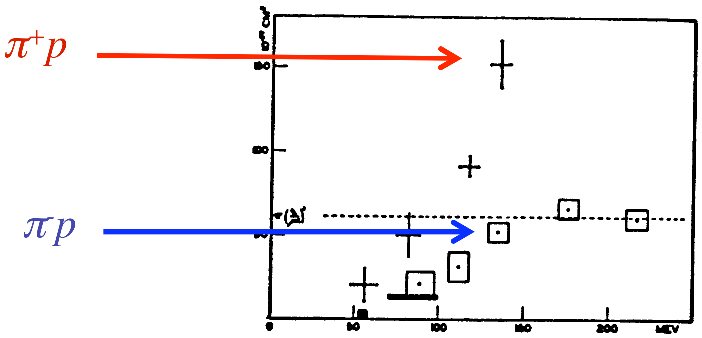
$$\sigma_{\pi^+ p \to \pi^+ p} : \sigma_{\pi^- p \to \pi^o n} : \sigma_{\pi^- p \to \pi^- p} = 9:2:1$$

$$\frac{\sigma_{\pi^+ p}}{\sigma_{\pi^- p}} = \frac{\sigma_{\pi^+ p \to \pi^+ p}}{\sigma_{\pi^- p \to \pi^o n} + \sigma_{\pi^- p \to \pi^- p}} = 3$$

Isospin Prediction vs. Data

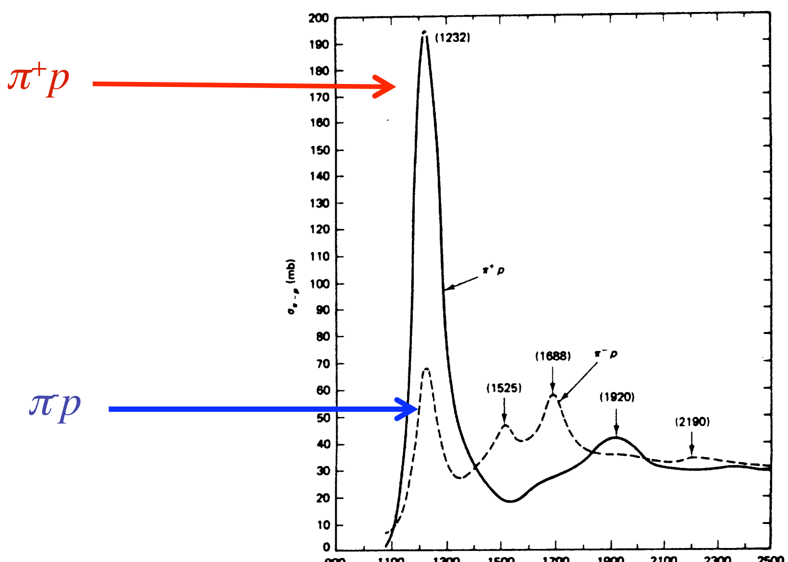
- The three predictions are in good agreement with the data!

$$\frac{\sigma_{\pi^+ p}}{\sigma_{\pi^- p}} = 3$$



Beam Energy

Data from 1952 paper by Fermi's group. They measured the cross section for $\pi^- p$ and $\pi^+ p$ as a function of beam energy.



mass of πp system

Modern compilation of data from many experiments giving the cross section for $\pi^- p$ and $\pi^+ p$ as a function of the πp invariant mass.

Discrete Symmetries

- An example of a discrete transformation is the operation of inverting all angles:

  $\theta \rightarrow -\theta$

  - A rotation by an amount θ is a continuous transformation.
  - Discrete symmetries give multiplicative quantum numbers (e.g. parity, charge conjugation).
  - Continuous symmetries give additive quantum numbers (e.g. charge, spin).
- Three most important discrete symmetries:

Parity (P)	$(x,y,z) \rightarrow (-x,-y,-z)$
Charge Conjugation (C)	particles $\rightarrow$ anti-particles
Time Reversal (T)	time $\rightarrow$ -time

- Other not so common discrete symmetries include G parity:
  - G parity is important for pions under the strong interaction.
- Discrete transformations do not have to be unitary transformations !
  - P and C are unitary transformations
  - T is not a unitary transformation, T is an anti-unitary operator!
    - Operator T is not Hermitian.

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Parity

- Parity and nature:
  - The strong and electromagnetic interactions conserve parity.
  - The weak interaction does not.
    - ☞ If we consider a Hamiltonian to be made up of several pieces:

      $H = H_s + H_{EM} + H_W$

      - ☞ The parity operator (P) commutes with H_s and H_{EM} but not with H_W.
      - ☞ The fact that $[P, H_W] \neq 0$ constrains the functional form of the Hamiltonian.
- What does parity do to some common operations?

vector or polar vector	$\boldsymbol{x} \rightarrow -\boldsymbol{x}$ or $\boldsymbol{p} \rightarrow -\boldsymbol{p}$
axial or pseudo vectors	$\boldsymbol{J} = \boldsymbol{x}\times\boldsymbol{p} \rightarrow$ J
time	$t \rightarrow t$

- What is the parity of scalar, vector etc.?

Name	**Form**	**Parity**
scalar	$\boldsymbol{r}\bullet\boldsymbol{r}$	+
pseudoscalar	$\boldsymbol{x}\bullet(\boldsymbol{y}\times\boldsymbol{z})$	-
vector	$\boldsymbol{r}$	-
axial vector	$\boldsymbol{r}\times\boldsymbol{p}$	+
Tensor	F_{uv}	indefinite

- Special relativity: the Hamiltonian or Lagrangian of any interaction must transform like a Lorentz scalar.
  - ★ If H conserves parity then it should transform as like a scalar.
  - ★ If H does not conserve parity then it must contain some pseudoscalar terms.

Parity

- Fermi's original theory of weak interactions (β-decay):
  - Hamiltonian was made up of bilinear combination of vector operators (V,V).
  - The observation of parity violation showed that this was wrong !
- A more general form of a weak Hamiltonian that does not conserve parity:
  $H_W = (S,S) + (S,PS) + (V,V) + (V,AV)\ldots$
  - Experimental fact: weak interactions where a charged lepton turns into a neutrino ("charged current") can be described by a Hamiltonian of the form:
    $H_W = (V,V) + (V,AV)$

    "V-A" interaction
    - This is parity violating since (V,V) has + parity but (V,AV) has - parity.
- In QED the current is of the form:
  $J^\mu = \bar{u}\gamma^\mu u$
  - transforms like a vector.
- In weak interactions the charged current (involves a W boson) is of the form:
  $J^\mu = \bar{u}\gamma^\mu(1-\gamma^5)v = \bar{u}\gamma^\mu v - \bar{u}\gamma^\mu\gamma^5 v \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
  - contains both vector and axial vector terms
    - ☞ does not conserve parity!