Parity Operator and Eigenvalue

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- The parity operator acting on a wavefunction:
  $P\Psi(x, y, z) = \Psi(-x, -y, -z)$
  $P^2\Psi(x, y, z) = P\Psi(-x, -y, -z) = \Psi(x, y, z)$
  - ☞ $P^2 = I$
  - ☞ Parity operator is unitary.
  - ■ If the interaction Hamiltonian (H) conserves parity
    - ☞ $[H,P] = 0$
    - ☞ $P_i = P_f$
  - ■ What is the eigenvalue (P_a) of the parity operator?
    $P\Psi(x, y, z) = \Psi(-x, -y, -z) = P_a\Psi(x, y, z)$
    $P^2\Psi(x, y, z) = P_aP\Psi(x, y, z) = (P_a)^2\Psi(x, y, z) = \Psi(x, y, z)$
    $P_a = 1$ or -1
    - ☞ The quantum number P_a is called the intrinsic parity of a particle.
      - ◆ If $P_a = 1$ the particle has even parity.
      - ◆ If $P_a = -1$ the particle has odd parity.
  - ■ If the overall wavefunction of a particle (or system of particles) contains spherical harmonics
    - ☞ we must take this into account to get the total parity of the particle (or system of particles).
  - ■ For a wavefunction containing spherical harmonics:
    $$P\Psi(r,\theta,\phi) = PR(r)Y_m^l(\theta,\phi) = (-1)^l R(r)Y_m^l(\theta,\phi)$$
    - ☞ The parity of the particle: $P_a(-1)^l$
    - ★ Parity is a multiplicative quantum number.

Parity of Particles

- The parity of a state consisting of particles *a* and *b*:
  $(-1)^L P_a P_b$
  - L is their relative orbital momentum.
  - P_a and P_b are the intrinsic parity of the two particles.
- Strictly speaking parity is only defined in the system where the total momentum $\boldsymbol{p} = 0$ since the parity operator (P) and momentum operator anticommute, $P\boldsymbol{p} = -\boldsymbol{p}$.
- How do we know the parity of a particle?
  - By convention we assign positive intrinsic parity (+) to spin 1/2 fermions:
    +parity: proton, neutron, electron, muon (μ^-)
    - ☞ Anti-fermions have opposite intrinsic parity.
  - Bosons and their anti-particles have the same intrinsic parity.
  - What about the photon?
    - Strictly speaking, we can not assign a parity to the photon since it is never at rest.
    - By convention the parity of the photon is given by the radiation field involved:
      - electric dipole transitions have + parity.
      - magnetic dipole transitions have – parity.
- We determine the parity of other particles (π, K...) using the above conventions and assuming parity is conserved in the strong and electromagnetic interaction.
    - Usually we need to resort to experiment to determine the parity of a particle.

Parity of Pions

- Example: determination of the parity of the π using $\pi d \rightarrow nn$.
  - For this reaction we know many things:
    - $s_\pi = 0$, $s_n = 1/2$, $s_d = 1$, orbital angular momentum $L_d = 0$, $J_d = 1$
    - We know (from experiment) that the π is captured by the d in an s-wave state.
      - ☞ The total angular momentum of the initial state is just that of the d ($J = 1$).
    - The isospin of the nn system is 1 since d is an isosinglet and the π^- has $I = |1,-1>$
      - ○ $|1,-1>$ is symmetric under the interchange of particles. (see below)
    - The final state contains two identical fermions
      - ☞ Pauli Principle: wavefunction must be anti-symmetric under the exchange of the two neutrons.
  - Let's use these facts to pin down the intrinsic parity of the π .
    - Assume the total spin of the nn system = 0.
      - ☞ The spin part of the wavefunction is anti-symmetric:
        $|0,0> = (2)^{-1/2}[|1/2,1/2>|1/2-1/2> - |1/2,-1/2>|1/2,1/2>]$
      - ☞ To get a totally anti-symmetric wavefunction L must be even (0, 2, 4…)
      - ☞ Cannot conserve momentum ($J = 1$) with these conditions!
    - Assume the total spin of the nn system = 1.
      - ☞ The spin part of the wavefunction is symmetric:
        $|1,1> = |1/2,1/2>|1/2,1/2>$
        $|1,0> = (2)^{-1/2}[|1/2,1/2>|1/2-1/2> + |1/2,-1/2>|1/2,1/2>]$
        $|1,-1> = |1/2,-1/2>|1/2,-1/2>$
      - ☞ To get a totally anti-symmetric wavefunction L must be odd (1, 3, 5…)
      - ★ $L = 1$ consistent with angular momentum conservation: nn has $s = 1$, $L = 1$, $J = 1 \rightarrow {}^3P_1$
      - ☞ Parity of the final state: $P_n P_n (-1)^L = (+)(+)(-1)^1 = -$
      - ☞ Parity of the initial state: $P_\pi P_d (-1)^L = P_\pi(+)(-1)^0 = P_\pi$
      - ☞ Parity conservation: $P_\pi = -$

Spin Parity of Particles

- There is other experimental evidence that the parity of the π is -:
  - The reaction $\pi d \to nn\pi^0$ is not observed.
  - The polarization of γ's from $\pi^0 \to \gamma\gamma$.
- Spin-parity of some commonly known particles:

State	Spin	Parity	Particle
pseudoscalar (0^-)	0	-	π, K
scalar (0^+)	0	+	a_0, Higgs (not observed yet)
vector (1^-)	1	-	$\gamma, \rho, \omega, \phi, \psi, Y$
pseudovector (axial vector) (1^+)	1	+	a_1

$\theta-\tau$ Puzzle

- How well is parity conserved?
  - Very well in strong and electromagnetic interactions (10^{-13})
  - Not at all in the weak interaction!
- In the mid-1950's it was noticed that there were 2 charged particles that had (experimentally) consistent masses, lifetimes, and spin = 0, but very different weak decay modes:

  $\theta^+ \to \pi^+\pi^0$

  $\tau^+ \to \pi^+\pi^-\pi^+$

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  - The parity of θ^+ = + while the parity of τ^+ = -.
  - Some physicists said the θ^+ and τ^+ were different particles, and parity was conserved.
  - Lee and Yang said they were the same particle but parity was not conserved in weak interaction!
    - Awarded Nobel Prize when parity violation was discovered.

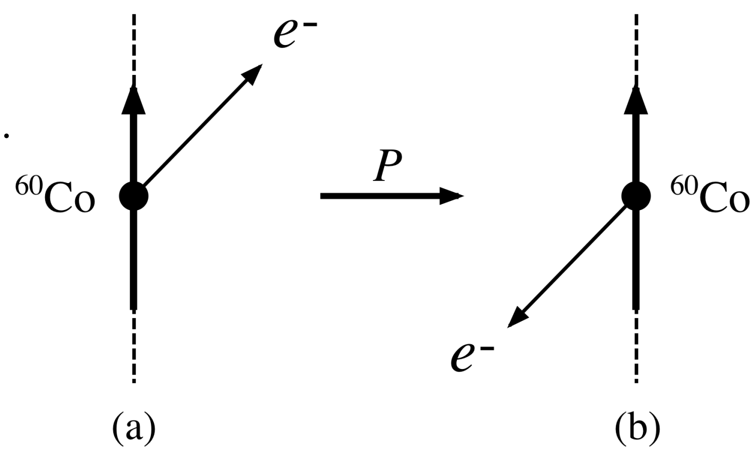
Parity Violation in β-decay

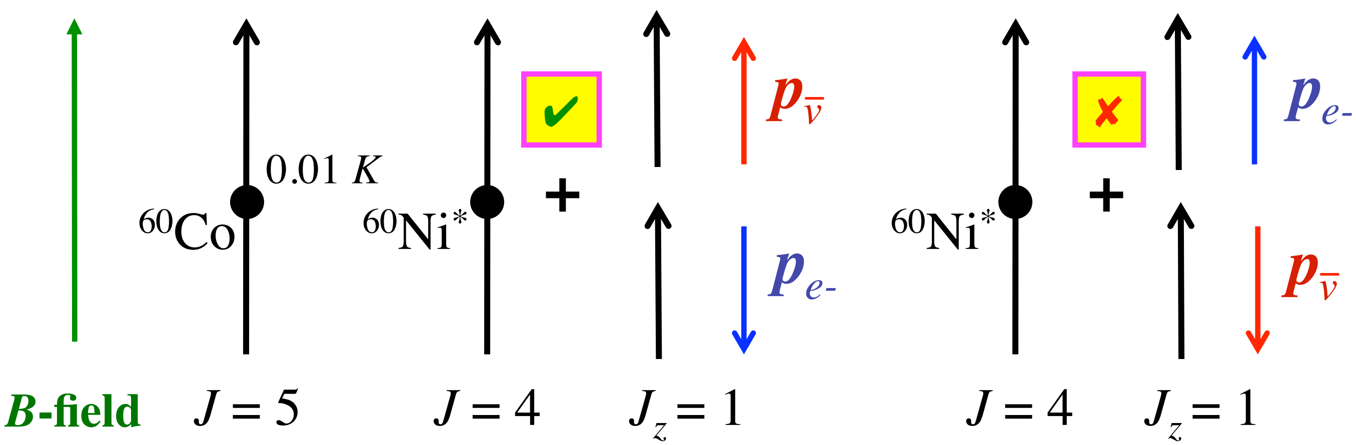
- Classic experiment of Wu et. al. (Phys. Rev. V105, Jan. 15, 1957) looked at β spectrum:

$$^{60}_{27}Co \rightarrow ^{60}_{28}Ni^* + e^- + \bar{\nu}_e$$

$$^{60}_{28}Ni^* \rightarrow ^{60}_{28}Ni^* + \gamma(1.17) + \gamma(1.33)$$

  - Parity transformation reverses all particle momenta while leaving spin angular momentum (***r***x***p***) unchanged.
    - Parity invariance requires equal rate for (a) and (b).
    - Fewer electrons are emitted in the forward hemisphere.
      - An forward-backward asymmetry in the decay.
    - 3 other papers reporting parity violation published within a month of Wu et. al.!





β counting rate depends on <***J***>·$\boldsymbol{p}_e$ which is – under a parity transformation

Charge Conjugation

- Charge Conjugation (C) turns particles into anti-particles and visa versa.
  - C(proton) → anti-proton C(anti-proton) → proton
  - C(electron) → positron C(positron) → electron

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- The operation of Charge Conjugation changes the sign of all intrinsic additive quantum numbers: electric charge, baryon #, lepton #, strangeness, etc.
  - Variables such as momentum and spin do not change sign under C.
- The eigenvalues of C are ± 1:

$$C|\psi\rangle = |\overline{\psi}\rangle = C_\psi|\psi\rangle$$

$$C^2|\psi\rangle = C_\psi^2|\psi\rangle = |\psi\rangle$$

$$\Rightarrow C_\psi^2 = 1$$

  - C_ψ is sometimes called the "charge parity" of the particle.
  - Like parity, C_ψ is a multiplicative quantum number.
  - If an interaction conserves C
    - ☞ C commutes with the Hamiltonian:

$$[H,C]|\psi\rangle = 0$$

    - Strong and electromagnetic interactions conserve C.
    - Weak interaction violates C conservation.

Charge Conjugation of Particles

- Most particles are not eigenstates of *C*.
  - Consider a proton with electric charge *q*.
    - Let *Q* be the charge operator:

      $Q|q> = q|q>$

      $CQ|q> = qC|q> = q|-q>$

      $QC|q> = Q|-q> = -q|-q>$

      $[C,Q]|q> = [CQ - QC]|q> = 2q|-q>$

      ☞ *C* and *Q* do not commute unless $q = 0$.
    - We get the same result for all additive quantum numbers!

      ☞ Only particles that have all additive quantum numbers = 0 are eigenstates of *C*.
      - e.g. γ, ρ, ω, ϕ, ψ, Υ
        - These particle are said to be "self conjugate".

Charge Conjugation of Photon and π^o

- How do we assign C_ψ to particles that are eigenstates of C?
  - Photon: Consider the interaction of the photon with the electric field.
    - As we previously saw the interaction Lagrangian of a photon is:

      $$L_{EM} = J_u A_u$$

      - J_u is the electromagnetic current density and A_u the vector potential.
    - By definition, C changes the sign of the EM field.
      - In QM, an operator transforms as:

        $$CJC^{-1} = -J$$
    - Since C is conserved by the EM interaction:

      $$CL_{EM}C^{-1} = L_{EM}$$
      $$CJ_uA_uC^{-1} = J_uA_u$$
      $$CJ_uC^{-1}CA_uC^{-1} = -J_uCA_uC^{-1} = J_uA_u$$
      $$CA_uC^{-1} = -A_u$$

      - The photon (as described by A) has $C = -1$.
        - A state that is a collection of n photons has $C = (-1)^n$.
  - π^o: Experimentally we find that the π^o decays to 2 γs and not 3 γs.

    $$\frac{BR(\pi^0 \to \gamma\gamma\gamma)}{BR(\pi^0 \to \gamma\gamma)} < 4\times10^{-7}$$

    - This is an electromagnetic decay so C is conserved:

      $$C_\pi = (-1)^2 = +1$$
    - Particles with the same quantum numbers as the photon ($\gamma, \rho, \omega, \phi, \psi, Y$) have $C = -1$.
    - Particles with the same quantum numbers as the π^o (η, η') have $C = +1$.

Handedness of Neutrinos

- Assuming massless neutrinos, we find experimentally:
  - All neutrinos are left handed.
  - All anti-neutrinos are right handed.
  - Left handed: spin and z component of momentum are anti-parallel.
  - Right handed: spin and z component of momentum are parallel.
  - This left/right handedness is illustrated in $\pi^+ \to l^+\nu_l$ decay:

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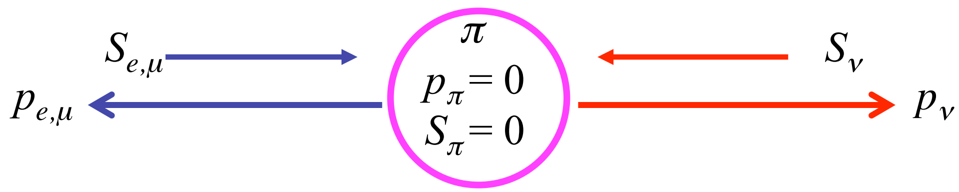
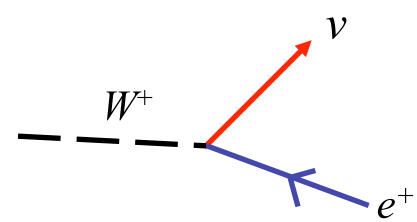
$$\frac{BR(\pi^+ \to e^+\nu_e)}{BR(\pi^+ \to \mu^+\nu_\mu)} = 1.23\times 10^{-4}$$

- ★ If neutrinos were not left handed, the ratio would be > 1!





- ◆ Angular momentum conservation forces the charged lepton (e, μ) to be in "wrong" handed state:
  - ☞ a left handed positron (e^+).
    - ❍ The probability to be in the wrong handed state ~ m_l^2

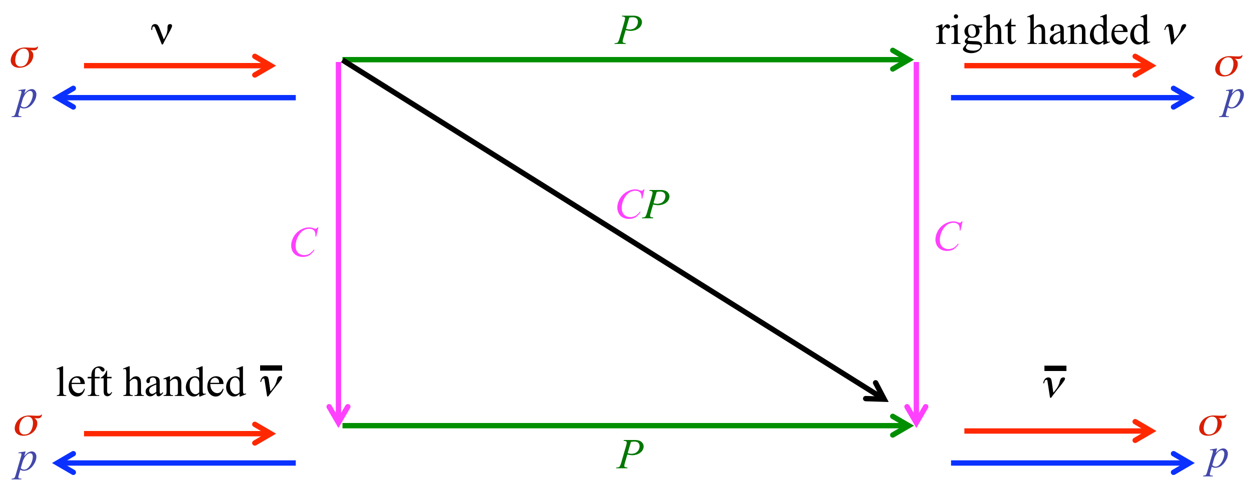
$$\frac{\mathrm{BR}(\pi^+ \to \mathrm{e}^+\nu_e)}{\mathrm{BR}(\pi^+ \to \mu^+\nu_\mu)} = \frac{m_e^2}{m_\mu^2}\left[\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right]^2 = (1.230 \pm 0.004)\times 10^{-4}$$

Handedness ~ 2x10^{-5}

Phase space ~ 5

Charge Conjugation and Parity

- In the strong and EM interaction *C* and *P* are conserved separately.
  - In the weak interaction we know that *C* and *P* are not conserved separately.
    - The combination of *CP* should be conserved!
- Consider how a neutrino (and anti-neutrino) transforms under *C, P*, and *CP*.
  - Experimentally we find that all neutrinos are left handed and anti-neutrinos are right handed.



☞ *CP* should be a good symmetry.

Neutral Kaons and CP violation

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- In 1964 it was discovered that the decay of neutral kaons sometimes (10^{-3}) violated CP!
  - ☞ The weak interaction does not always conserve CP!
  - In 2001 CP violation was observed in the decay of B-mesons.
- CP violation is one of the most interesting topics in physics:
  - The laws of physics are different for particles and anti-particles!
  - What causes CP violation?
    - It is included into the Standard Model by Kobayashi and Maskawa (Nobel Prize 2008).
    - Is the CP violation observed with B's and K's the same as the cosmological CP violation?
- To understand how CP violation is observed with K's and B's need to discuss mixing.
  - Mixing is a QM process where a particle can turn into its anti-particle!
  - As an example, lets examine neutral kaon mixing first (B-meson mixing later):

    $K^0 = \bar{s}d \quad \bar{K}^0 = s\bar{d}$
    - In terms of quark content these are particle and anti-particle.
    - The K^0 has the following additive quantum numbers:

      strangeness = +1

      charge = baryon # = lepton # = charm = bottom = top = 0

      $I_3 = -1/2$
      - The K^0's isospin partner is the K^+ (I_3 changes sign for anti-particles.)
    - The K^0 and $\bar{K}^0$ are produced by the strong interaction and have definite strangeness.
      - ☞ They cannot decay via the strong or electromagnetic interaction.

Neutral Kaons, Mixing, and CP violation

- The neutral kaon decays via the weak interaction, which does not conserve strangeness.
  - Let's assume that the weak interaction conserves CP.
    - ☞ The K^0 and $\bar{K}^0$ are not the particles that decay weakly since they are not CP eigenstates:

$$P|K^0\rangle = -|K^0\rangle \quad \text{and} \quad P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$C|K^0\rangle = -|\bar{K}^0\rangle \quad \text{and} \quad C|\bar{K}^0\rangle = -|K^0\rangle$$

$$CP|K^0\rangle = |\bar{K}^0\rangle \quad \text{and} \quad CP|\bar{K}^0\rangle = |K^0\rangle$$

    - ◆ We can make CP eigenstates out of a linear combination of K^0 and $\bar{K}^0$:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

This is just a QM system in two different basis.

$$CP|K_1\rangle = |K_1\rangle \quad \text{and} \quad CP|K_2\rangle = -|K_2\rangle$$

    - ◆ If CP is conserved in the decay of K_1 and K_2 then we expect the following decay modes:
      $K_1 \to$ two pions ($\pi^+\pi^-$ or $\pi^0\pi^0$) ($CP = +1$ states)
      $K_2 \to$ three pions ($\pi^+\pi^-\pi^0$ or $\pi^0\pi^0\pi^0$) ($CP = -1$ states)
      - ★ In 1964 it was found that every once in a while ($\approx$ 1/500) $K_2 \to$ two pions!

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Neutral Kaons

- The K^0 and $\overline{K}^0$ are eigenstates of the strong interaction.
  - These states have definite strangeness, are not CP eigenstates
  - They are particle/anti-particle.
  - They are produced in strong interactions (collisions) e.g.
    $\pi^- p \to K^0 \Lambda$ or $K^0 \Sigma^0$
- The K_1 and K_2 are eigenstates of the weak interaction, assuming CP is conserved.
  - These states have definite CP but are not strangeness eigenstates.
  - Each is its own anti-particle.
  - These states decay via the weak interaction and have different masses and lifetimes.
    $K_1 \to \pi^0 \pi^0$ $\quad$ $K_2 \to \pi^0 \pi^0 \pi^0$
- 1955: Gell-Mann/Pais pointed out there was more decay energy (phase space) available for K_1 than K_2.
  - ☞ K_1 and K_2 lifetimes should be very different.
    $m_K - 2m_\pi \approx 219$ MeV/c^2 $\quad$ $m_K - 3m_\pi \approx 80$ MeV/c^2

    $m_2 - m_1 = 3.5\text{x}10^{-6}$ eV
  - ☞ Expect K_1 to have the shorter lifetime.
  - The lifetimes were measured to be:
    $\tau_1 \approx 9\text{x}10^{-11}$ sec (1947-53)
    $\tau_2 \approx 5\text{x}10^{-8}$ sec (Lande et. al., Phys. Rev. V103, 1901 (1956))
- We can use the lifetime difference to produce a beam of K_2's from a beam of K^0's.
  - Produce K^0's using $\pi p \to K^0 \Lambda$.
  - Let the beam of K^0's propagate in vacuum until the K_1 component dies out.
    1 GeV/c K_1 travels on average $\approx$ 5.4 cm
    1 GeV/c K_2 travels on average $\approx$ 3100 cm
    - Need to put detector 100-200 m away from target.

Mixing and *CP* violation

- How do we look for *CP* violation with a K_2 beam?
  - Look for decays that have $CP = +1$:
    $K_2 \to \pi^+\pi^-$ or $\pi^0\pi^0$
  - Experimentally we find:
    - $K_2 \to \pi^+\pi^- \sim 0.2\%$ of the time
    - $K_2 \to \pi^0\pi^0 \sim 0.1\%$ of the time

Christenson et. al. PRL V13, 138 (1964)

  - Can also look for differences in decays that involve matter and anti-matter:

$$\delta(e) = \frac{BR(K_2 \to \pi^- e^+ \nu_e) - BR(K_2 \to \pi^+ e^- \bar{\nu}_e)}{BR(K_2 \to \pi^- e^+ \nu_e) + BR(K_2 \to \pi^+ e^- \bar{\nu}_e)} = 0.333 \pm 0.014$$

$$\delta(\mu) = \frac{BR(K_2 \to \pi^- \mu^+ \nu_\mu) - BR(K_2 \to \pi^+ \mu^- \bar{\nu}_\mu)}{BR(K_2 \to \pi^- \mu^+ \nu_\mu) + BR(K_2 \to \pi^+ \mu^- \bar{\nu}_\mu)} = 0.303 \pm 0.025$$

    ☞ Nature differentiates between matter and antimatter!

- *CP* violation has recently (2001) been unambiguously measured in the decay of *B*-mesons.
  - This is one of the most interesting areas in HEP (will be discussed later).
  - Observing *CP* violation with *B*-mesons is much more difficult than with kaons!
    - B_S and B_L have essentially the same lifetime
      ☞ No way to get a beam of B_L and look for “forbidden” decay modes.
    - It is much harder to produce large quantities of *B*-mesons than kaons.