Direct and Indirect *CP* violation

- *CP* is not conserved in neutral kaon decay.
  - It makes more sense to use mass (or lifetime) eigenstates rather than $|K_1>$ and $|K_2>$:
    - "*K*-short": short lifetime state with $\tau_S \approx 9\text{x}10^{-11}$ sec.

      $$|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1\rangle + \varepsilon|K_2\rangle\right)$$

    - "*K*-long": long lifetime state with $\tau_L \approx 5\text{x}10^{-8}$ sec.

      $$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

      - ε is a (small) complex number that allows for *CP* violation through mixing.
- There are two types of *CP* violation in K_L decay:
  - Indirect ("mixing"): $K_L \to \pi\pi$ because of its K_1 component
  - Direct: $K_L \to \pi\pi$ because the amplitude for K_2 allows $K_2 \to \pi\pi$
  - Experimental observation: indirect >> direct!
- The strong interaction eigenstates with definite strangeness are:

  $K^0 = |\bar{s}d\rangle$ and $\overline{K}^0 = |s\bar{d}\rangle$
  - Consider the strangeness operator:

    $S|K^0\rangle = |K^0\rangle$ and $S|\overline{K}^0\rangle = -|\overline{K}^0\rangle$
    - They are particle and anti-particle and by the *CPT* theorem have the same mass.
      - Experimentally we find:

        $$(m_{K^0} - m_{\overline{K}^0})/m < 9\times 10^{-19}$$

Indirect CP violation

- The K_L and K_S states are not CP (or S) eigenstates:
$$CP|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(CP|K_1\rangle + \varepsilon CP|K_2\rangle\right) = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1\rangle - \varepsilon|K_2\rangle\right) \neq |K_S\rangle$$
$$CP|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(CP|K_2\rangle + \varepsilon CP|K_1\rangle\right) = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(-|K_2\rangle + \varepsilon|K_1\rangle\right) \neq |K_L\rangle$$
  - In fact these states are not orthogonal:
  $$\langle K_S|K_L\rangle = \frac{2\,\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2} \neq 0$$
  - If CP violation is due to mixing ("indirect" only):
    - The amplitude for $K_L \to \pi\pi$:
    $$\langle K_1|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\langle K_1|K_2\rangle + \varepsilon\langle K_1|K_1\rangle\right) = \frac{\varepsilon}{\sqrt{1+|\varepsilon|^2}}$$
    - The amplitude for $K_L \to \pi\pi\pi$:
    $$\langle K_2|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\langle K_2|K_2\rangle + \varepsilon\langle K_2|K_1\rangle\right) = \frac{1}{\sqrt{1+|\varepsilon|^2}}$$
    - Experimental measurement: $|\varepsilon| = 2.3\mathrm{x}10^{-3}$.
    - The standard model predicts a small amount of direct CP violation too!

Direct *CP* violation

- Standard model predicts that quantities η_{+-} and η_{00} should differ very slightly due to direct *CP* violation:

$$\eta_{+-} \equiv \frac{Amp(K_L \to \pi^+\pi^-)}{Amp(K_s \to \pi^+\pi^-)} \qquad \eta_{oo} \equiv \frac{Amp(K_L \to \pi^o\pi^o)}{Amp(K_s \to \pi^o\pi^o)}$$

  - This is *CP* violation in the amplitude.
- *CP* violation is now described by two complex parameters, ε and ε':
  - ε' is related to direct *CP* violation.
  - The standard model estimates:

    $\mathrm{Re}(\varepsilon'/\varepsilon) \sim (4\text{-}30)\times10^{-4}$!
  - Experimentally what is measured is the ratio of branching ratios:

$$\frac{\dfrac{BR(K_L \to \pi^+\pi^-)}{BR(K_s \to \pi^+\pi^-)}}{\dfrac{BR(K_L \to \pi^o\pi^o)}{BR(K_s \to \pi^o\pi^o)}} = \left|\frac{\eta_{+-}}{\eta_{oo}}\right|^2 = \frac{|\varepsilon+\varepsilon'|^2}{|\varepsilon-2\varepsilon'|^2} \approx 1+6\,\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)$$

  - There were multiple attempts to measure the ratio since 1970's with some controversial results.
    - A non-zero value has recently been measured by 2 different experiments:

      $\mathrm{Re}(\varepsilon'/\varepsilon) = (17.2 \pm 1.8)\times10^{-4}$
    - Currently the measurement is more precise than the theoretical calculation!
    - Calculating $\mathrm{Re}(\varepsilon'/\varepsilon)$ is presently one of the most challenging HEP theory projects.

Neutral Kaons and Strangeness Oscillations

- We now consider how the neutral kaon state evolves with time:
  - ☞ allow us to measure K_L, K_S mass difference and phases of the *CP* violation parameters, η_{+-} and η_{00}.
  - How a two-particle quantum system evolves in time is covered in many texts including:

    Particle Physics, Martin and Shaw, section 10.3

    Introduction to High Energy Physics, Perkins

    Introduction to Nuclear and Particle Physics, Das and Ferbel

    Lectures on Quantum Mechanics, Baym, Ch 2.

    The Feynman Lectures, Vol III, section 11-5.
    - Written in about 1963, before there was good experimental data on this topic!
      - *CP* violation was discovered in 1964.
  - The following derivation is for the neutral kaon system.
    - It is also applicable to *B*-meson and neutrino oscillations.

Kaon Oscillations

- First, consider the case where *CP* is conserved.
  - *CP* eigenstates $|K_1>$ and $|K_2>$ are solutions to the time dependent Schrodinger equation:

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H_{eff}|\psi(t)\rangle$$

    - H_{eff} is a phenomenological Hamiltonian that describes the system.
    - Since our particles can decay, H_{eff} is not a Hermitian operator!
    - Since there are two states in this problem it is customary:
      - Describe H_{eff} by a 2x2 matrix (e.g. see Das and Ferbel Chapter XII).
      - Describe $|K_1>$ and $|K_2>$ by column vectors.
    - H_{eff} matrix is written in terms of the masses (m_1, m_2) and lifetimes (τ_1, τ_2) of the two states:

$$H_{eff} = \begin{pmatrix} \frac{1}{2}(m_1+m_2)-\frac{1}{4}i\hbar\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right) & \frac{1}{2}(m_2-m_1)-\frac{1}{4}i\hbar\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right) \\ \frac{1}{2}(m_2-m_1)-\frac{1}{4}i\hbar\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right) & \frac{1}{2}(m_1+m_2)-\frac{1}{4}i\hbar\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right) \end{pmatrix}$$

      - The eigenvalues and eigenvectors of H_{eff} are:

$$H_{eff}|K_1(t)\rangle = (m_1 - \frac{i\hbar}{2\tau_1})|K_1(t)\rangle \qquad \text{with } |K_1(t)\rangle = \begin{pmatrix} +f_1(t) \\ -f_1(t) \end{pmatrix}$$

$$H_{eff}|K_2(t)\rangle = (m_2 - \frac{i\hbar}{2\tau_2})|K_2(t)\rangle \qquad \text{with } |K_2(t)\rangle = \begin{pmatrix} +f_2(t) \\ -f_2(t) \end{pmatrix}$$

f(*t*) contains the time dependence.

        - The states are orthogonal to each other: $<K_1(t)|K_2(t)> = <K_2(t)|K_1(t)> = 0$.
      - The solutions to the Hamiltonian:

$$|K_1(t)\rangle = e^{-\frac{i}{\hbar}(m_1-\frac{i\hbar}{2\tau_1})t}|K_1\rangle \qquad |K_2(t)\rangle = e^{-\frac{i}{\hbar}(m_2-\frac{i\hbar}{2\tau_2})t}|K_2\rangle$$

Neutral Kaons and Strangeness Oscillations

- Consider an experiment which produces a beam of pure K^0's (at $t = 0$) using the strong interaction:
  $\pi p \to \Lambda K^0$
  - As in the previous lecture, we can express a K^0 as of a mixture of $|K_1>$ and $|K_2>$ (and visa versa):

$$\left|K_1\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\bar{K}^0\right\rangle\right) \qquad \left|K_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\bar{K}^0\right\rangle\right)$$

$$\left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1\right\rangle + \left|K_2\right\rangle\right) \qquad \left|\bar{K}^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_1\right\rangle - \left|K_2\right\rangle\right)$$

  - Using the time dependent solutions for K_1 and K_2 we can find the time dependent solution for K^0:

$$\left|K^0(t)\right\rangle = \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\left(m_1 - \frac{i\hbar}{2\tau_1}\right)t}\left|K_1\right\rangle + e^{-\frac{i}{\hbar}\left(m_2 - \frac{i\hbar}{2\tau_2}\right)t}\left|K_2\right\rangle\right)$$

  - The amplitude for finding a K^0 in the beam at a later time (t) is given by:

$$\left\langle K^0 \middle| K^0(t)\right\rangle = \frac{1}{2}\left(\left\langle K_1\right| + \left\langle K_2\right|\right)\left(e^{-\frac{i}{\hbar}\left(m_1 - \frac{i\hbar}{2\tau_1}\right)t}\left|K_1\right\rangle + e^{-\frac{i}{\hbar}\left(m_2 - \frac{i\hbar}{2\tau_2}\right)t}\left|K_2\right\rangle\right)$$

$$= \frac{1}{2}\left(e^{-\frac{i}{\hbar}\left(m_1 - \frac{i\hbar}{2\tau_1}\right)t} + e^{-\frac{i}{\hbar}\left(m_2 - \frac{i\hbar}{2\tau_2}\right)t}\right)$$

Neutral Kaons and Strangeness Oscillations

- The probability to find a K^0 in the beam at a later time is given by:

$$\left|\left\langle K^0 \middle| K^0(t)\right\rangle\right|^2 = \frac{1}{4}\left(e^{+\frac{i}{\hbar}\left(m_1+\frac{i\hbar}{2\tau_1}\right)t}+e^{+\frac{i}{\hbar}\left(m_2+\frac{i\hbar}{2\tau_2}\right)t}\right)\left(e^{-\frac{i}{\hbar}\left(m_1-\frac{i\hbar}{2\tau_1}\right)t}+e^{-\frac{i}{\hbar}\left(m_2-\frac{i\hbar}{2\tau_2}\right)t}\right)$$

$$=\frac{1}{4}\left(e^{-\frac{t}{\tau_1}}+e^{-\frac{t}{\tau_2}}+e^{-\frac{i}{\hbar}\left(m_1-\frac{i\hbar}{2\tau_1}\right)t}e^{+\frac{i}{\hbar}\left(m_2+\frac{i\hbar}{2\tau_2}\right)t}+e^{+\frac{i}{\hbar}\left(m_1+\frac{i\hbar}{2\tau_1}\right)t}e^{-\frac{i}{\hbar}\left(m_2-\frac{i\hbar}{2\tau_2}\right)t}\right)$$

$$=\frac{1}{4}\left(e^{-\frac{t}{\tau_1}}+e^{-\frac{t}{\tau_2}}+e^{\frac{i}{\hbar}\left(m_2-m_1+\frac{i\hbar}{2\tau_1}+\frac{i\hbar}{2\tau_2}\right)t}+e^{+\frac{i}{\hbar}\left(m_1-m_2+\frac{i\hbar}{2\tau_1}+\frac{i\hbar}{2\tau_2}\right)t}\right)$$

$$=\frac{1}{4}\left(e^{-\frac{t}{\tau_1}}+e^{-\frac{t}{\tau_2}}+e^{-\frac{t}{2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)}\left(e^{\frac{i}{\hbar}(m_2-m_1)t}+e^{+\frac{i}{\hbar}(m_1-m_2)t}\right)\right)$$

$$=\frac{1}{4}\left(e^{-\frac{t}{\tau_1}}+e^{-\frac{t}{\tau_2}}+2e^{-\frac{t}{2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)}\cos\left(\frac{t}{\hbar}(m_2-m_1)\right)\right)$$

To make the units come out right in the cos term substitute $(m_2-m_1)c^2$ for (m_2-m_1).

- The third term in the above equation is an interference term.
  - It causes an oscillation that depends on the mass difference between $|K_1>$ and $|K_2>$.
  - Observation of neutrino oscillation implies that neutrinos must have mass.

Flavor Oscillations

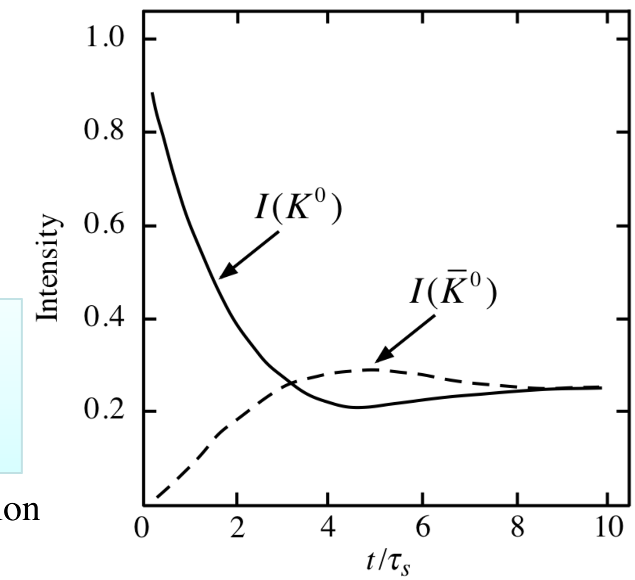
- Using the same procedure,
  - we can calculate the probability that a beam initially consisting of K^0's contains $\overline{K}^0$'s at a later time:

$$\left|\left\langle \overline{K}^0 \middle| K^0(t)\right\rangle\right|^2 = \frac{1}{4}\left(e^{-\frac{t}{\tau_1}} + e^{-\frac{t}{\tau_2}} - 2e^{-\frac{t}{2}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)}\cos\left(\frac{t}{\hbar}(m_2 - m_1)\right)\right)$$

  - The sign of the interference term is now "-".
  - Since the strangeness of a K^0 differs from a $\overline{K}^0$'s
    - the strangeness content of the beam is changing as a function of time.
      - This phenomena is called strangeness oscillations.
      - More generally we call this phenomena flavor oscillations:
        - It also occurs with *B*-mesons (*b* quark oscillations) and neutrinos (e.g. $\nu_e \Leftrightarrow \nu_\mu$).
- We can measure the strangeness content of a beam as a function of time (distance):
  - put some material in the beam
  - counting the number of strong interactions with $S = +1$ in the final state vs. number with $S = -1$:

$$K^0 p \to K^+ n \qquad S = +1$$

$$\overline{K}^0 p \to \Lambda p \qquad S = -1$$



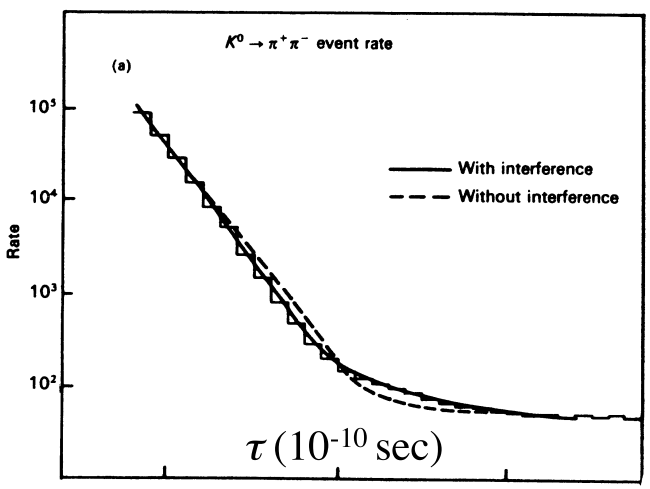
Strangeness oscillations for an initially pure K^0 beam. A value of $(m_2 - m_1)\tau_1 = 0.5$ is used.

CP Violation

- *CP* violation requires modifying H_{eff} to include additional complex parameters.
  - The details of the derivation are given in many texts.
    - e.g. *Weak Interactions of Leptons and Quarks* by Commins and Bucksbaum.
- *CP* violation can be measured using the following procedure:
  - produce a beam that is K^0 initially
  - measure the yield of $\pi^+\pi^-$ decays as a function of proper time, time measured in the rest frame of K^0.
    - This is a measure of the sum of the square of the amplitude $|K_L \to \pi^+\pi^->$ and $|K_S \to \pi^+\pi^->$.
      - There will be an interference term in the number of $\pi^+\pi^-$ decays per unit time ($\equiv I(t)$).
      - The yield of $\pi^+\pi^-$ decays is given by:

$$I_{\pi^+\pi^-}(t) = I_{\pi^+\pi^-}(0)\left(e^{-\frac{t}{\tau_s}} + |\eta_{+-}|^2 e^{-\frac{t}{\tau_L}} + 2|\eta_{+-}| e^{-\frac{t}{2}\left(\frac{1}{\tau_s}+\frac{1}{\tau_L}\right)} \cos\left(\frac{t}{\hbar}(m_2 - m_1) + \phi_{+-}\right)\right)$$

      - measuring this yield provides information on:
        - the mass difference and the *CP* violation parameters η^{+-} and ϕ^{+-}.



Event rate for $\pi^+\pi^-$ decays as a function of proper time. The best fit requires interference between the K_L and K_S amplitudes:

m_2-m_1 = (3.491 ± 0.009)x10^{-6} eV

$|\eta_{+-}|$ = (2.29 ± 0.01)x10^{-3}

ϕ_{+-} = (43.7 ± 0.6)^0

τ_S = 0.893x10^{-10} sec

τ_L = 0.517x10^{-7} sec