1960's Quark Model

- Quark model was developed by: Gell-Mann, Zweig, Okubo, and Ne'eman (Salam)
  - There are three fundamental building blocks: u, d, s.
    - point like
    - spin 1/2 fermions
    - parity = +1 (-1 for anti-quarks)
    - two quarks are in isospin doublet (u and d, $I = 1/2$, $B = 1/3$)
    - one quark is in iso-singlet (s, $I = 0$, $B = 1/3$)
    - obey $Q = I_3 + (S + B)/2 = I_3 + Y/2$
      - Y is the hypercharge
    - group structure is SU(3)

"Three Quarks for Muster Mark", J. Joyce, Finnegan's Wake

- SU(n): nxn Unitary matrices ($M^{T*}M = 1$) with determinant = 1 (i.e. Special)
  - Example: With 2 fundamental objects obeying SU(2) (e.g. u and d)
    - We can combine these objects using 1 quantum number (e.g. isospin)
      - Get three $I = 1$ states that are symmetric under interchange of u and d:

        $|11\rangle = |1/2\ 1/2\rangle\,|1/2\ 1/2\rangle$

        $|1\text{-}1\rangle = |1/2\ \text{-}1/2\rangle\,|1/2\ \text{-}1/2\rangle$

        $|10\rangle = [1/\sqrt{2}][|1/2\ 1/2\rangle\,|1/2\ \text{-}1/2\rangle + |1/2\ \text{-}1/2\rangle\,|1/2\ 1/2\rangle]$
      - Get one $I = 0$ state that is anti-symmetric under interchange of u and d:

        $|00\rangle = [1/\sqrt{2}][|1/2\ 1/2\rangle\,|1/2\ \text{-}1/2\rangle - |1/2\ \text{-}1/2\rangle\,|1/2\ 1/2\rangle]$
    - In group theory we have 2 multiplets, a 3 and a 1:

      $2 \otimes 2 = 3 \oplus 1$
  - For SU(3) there are 2 quantum numbers and the group structure is more complicated:

    $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
    - Expect 4 multiplets (groups of similar particles) with either 1, 8, or 10 members.

Successes/Failures of 1960's Quark Model

- Successes:
  - Classify all known particles in terms of 3 building blocks
  - Predict new particles (e.g. Ω^-)
  - Explain why certain particles don't exist (e.g. baryons with $S = +1$)
  - Explain mass splitting between mesons and baryons
  - Explain/predict magnetic moments of mesons and baryons
  - Explain/predict scattering cross sections (e.g. $\sigma_{\pi p}/\sigma_{pp} = 2/3$)
- Failures:
  - No evidence for free quarks (fixed up by QCD)
  - Pauli principle violated ($\Delta^{++} = uuu$ wavefunction is totally symmetric) (fixed up by color)
  - What holds quarks together in a proton? (gluons!)
  - How many different types of quarks exist? (6?)

Dynamic Quarks

- Dynamic Quark Model (mid 70's to now!)
  - Theory of quark-quark interaction: QCD
- Successes of QCD:
  - "Real" Field Theory:
    - Gluons instead of photons
    - Color instead of electric charge
  - Explains why no free quarks: confinement of quarks
  - Calculate cross section, e.g. $e^+e^- \to q\bar{q}$
  - Calculate lifetimes of baryons, mesons
- Failures/problems of QCD:
  - Hard to do calculations in QCD (non-perturbative)
  - Polarization of hadrons (e.g. Λ's) in high energy collisions
  - How many quarks are there
- How to "construct" baryons and mesons from quarks?
  - Use SU(3) as the group.
  - This group has 8 generators ($n^2 - 1$, $n = 3$).
  - Each generator is a 3x3 linearly independent traceless hermitian matrix.
  - Only 2 of the generators are diagonal
    - ☞ 2 quantum numbers: Y and I_3
    - Quarks are the eigenvectors of the two diagonal generators
  - Quarks are added together to form mesons and baryons using the rules of SU(3).

Making Mesons with Quarks

- Making mesons with (orbital angular momentum $L = 0$)
  - The properties of SU(3) tell us how many mesons to expect:

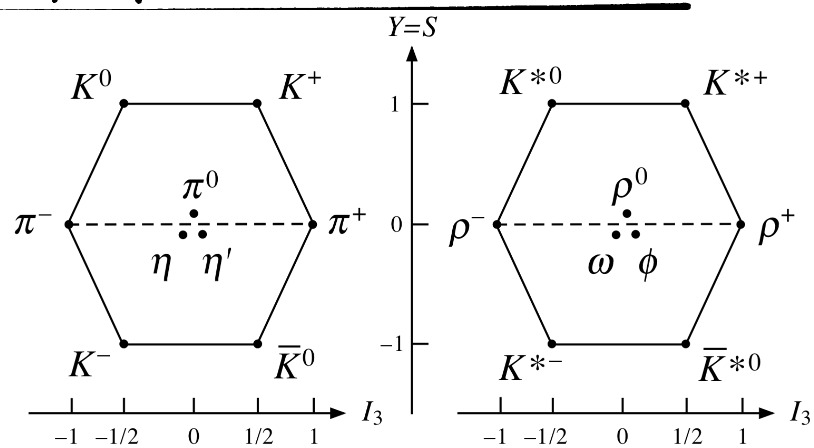
    $$3 \otimes \bar{3} = 1 \oplus 8$$

    - ☞ Expect an octet with 8 particles and a singlet with 1 particle.

Table 14-2 The quark model content of the octet and singlet multiplets of the pseudoscalar and vector mesons

SU(3) Multiplet	Quark Content	I	I_3	S	0^- Pseudoscalar Name	Mesons Mass (MeV/c²)	1^- Vector Name	Meson Mass (MeV/c²)
8	$\bar{n}p$	1	1	0	π^+	140	ρ^+	765
8	$(\bar{p}p-\bar{n}n)/\sqrt{2}$	1	0	0	π^0	135	ρ^0	765
8	$n\bar{p}$	1	−1	0	π^-	140	ρ^-	765
8	$\bar{\lambda}p$	$\frac{1}{2}$	$+\frac{1}{2}$	+1	K^+	494	$K^{*+}(890)$	892
8	$\bar{\lambda}n$	$\frac{1}{2}$	$-\frac{1}{2}$	+1	K^0	498	$K^{*0}(890)$	892
8	$\bar{n}\lambda$	$\frac{1}{2}$	$+\frac{1}{2}$	−1	$\bar{K}^0$	498	$\bar{K}^{*0}(890)$	892
8	$\bar{p}\lambda$	$\frac{1}{2}$	$-\frac{1}{2}$	−1	K^-	494	$K^{*-}(890)$	892
8	$(\bar{p}p+\bar{n}n-2\bar{\lambda}\lambda)/\sqrt{6}$	0	0	0	Mixture of $\eta(549)$		Mixture of $\omega(784)$	
1	$(\bar{p}p+\bar{n}n+\bar{\lambda}\lambda)/\sqrt{3}$	0	0	0	and perhaps $\eta'(958)$		and $\phi(1019)$	

If SU(3) were a perfect symmetry then all particles in a multiplet would have the same mass.



Baryon Octet

- Making Baryons (orbital angular momentum $L = 0$).
  - Combine 3 quarks together:

    $$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

    - Expect a singlet, 2 octets, and a decuplet (10 particles).
      - 27 objects total.

Octet with $J = ½$

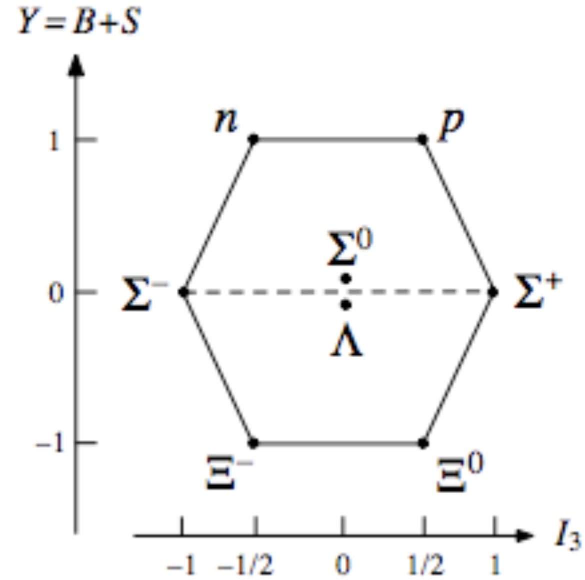


Table 14-3 The quark model content of the baryon octet. The terms symmetric (s) and antisymmetric (a) refer to the symmetry of the first two quarks in the state function. To emphasize the symmetries, we use the notation $q_1 = p$, $q_2 = n$, $q_3 = \lambda$. (Taken from Ref. 14LIC)

Baryon	I	I_z	S	Mass (MeV/c^2)	Symmetric (s) Quark State Function	Antisymmetric (a) Quark State Function
p	$\frac{1}{2}$	$\frac{1}{2}$	0	938	$(2q_1q_1q_2 - q_1q_2q_1 - q_2q_1q_1)/\sqrt{6}$	$(q_1q_2q_1 - q_2q_1q_1)/\sqrt{2}$
n	$\frac{1}{2}$	$-\frac{1}{2}$	0	940	$(q_1q_2q_2 + q_2q_1q_2 - 2q_2q_2q_1)/\sqrt{6}$	$(q_1q_2q_2 - q_2q_1q_2)/\sqrt{2}$
Σ^+	1	1	−1	1189	$(q_1q_3q_1 + q_3q_1q_1 - 2q_1q_1q_3)/\sqrt{6}$	$(q_3q_1q_1 - q_1q_3q_1)/\sqrt{2}$
Σ^0	1	0	−1	1192	$(q_1q_3q_2 + q_3q_1q_2 + q_2q_3q_1 + q_3q_2q_1 - 2q_1q_2q_3 - 2q_2q_1q_3)/\sqrt{12}$	$(q_3q_1q_2 - q_1q_3q_2 + q_3q_2q_1 - q_2q_3q_1)/2$
Σ^-	1	−1	−1	1197	$(q_2q_3q_2 + q_3q_2q_2 - 2q_2q_2q_3)/\sqrt{6}$	$(q_3q_2q_2 - q_2q_3q_2)/\sqrt{2}$
Λ^0	0	0	−1	1116	$(q_1q_3q_2 + q_3q_1q_2 - q_2q_3q_1 - q_3q_2q_1)/2$	$(2q_1q_2q_3 - 2q_2q_1q_3 + q_3q_2q_1 - q_2q_3q_1 + q_1q_3q_2 - q_3q_1q_2)/\sqrt{12}$
Ξ^0	$\frac{1}{2}$	$\frac{1}{2}$	−2	1314	$(2q_3q_3q_1 - q_1q_3q_3 - q_3q_1q_3)/\sqrt{6}$	$(q_3q_1q_3 - q_1q_3q_3)/\sqrt{2}$
Ξ^-	$\frac{1}{2}$	$-\frac{1}{2}$	−2	1321	$(2q_3q_3q_2 - q_2q_3q_3 - q_3q_2q_3)/\sqrt{6}$	$(q_3q_2q_3 - q_2q_3q_3)/\sqrt{2}$

Baryon Decuplet

- Baryon decuplet ($I = 3/2$)
  - Expect 10 states.
  - Prediction of the Ω^- (mass =1672 MeV/c^2, S = -3)
  - Use bubble chamber to find the event.
    - 1969 Nobel Prize to Gell-Mann

Table 14-4 The quark model content of the baryon decuplet: $q_1 = p$, $q_2 = n$, $q_3 = \lambda$ (Taken from Ref. 14LIC)

Baryon	I	I_z	S	Mass (MeV/c^2)	Quark State Function
Δ^{++}	$\frac{3}{2}$	$\frac{3}{2}$	0	1236	$q_1q_1q_1$
Δ^{+}	$\frac{3}{2}$	$\frac{1}{2}$	0	1236	$(q_1q_1q_3 + q_1q_2q_1 + q_2q_1q_1)/\sqrt{3}$
Δ^{0}	$\frac{3}{2}$	$-\frac{1}{2}$	0	1236	$(q_1q_2q_2 + q_2q_1q_2 + q_2q_2q_1)/\sqrt{3}$
Δ^{-}	$\frac{3}{2}$	$-\frac{3}{2}$	0	1236	$q_2q_2q_2$
Y^{*+}	1	1	-1	1385	$(q_1q_1q_3 + q_1q_3q_1 + q_3q_1q_1)/\sqrt{3}$
Y^{*0}	1	0	-1	1385	$(q_1q_2q_3 + q_1q_3q_2 + q_2q_1q_3 + q_2q_3q_1 + q_3q_1q_2 + q_3q_2q_1)/\sqrt{6}$
Y^{*-}	1	-1	-1	1385	$(q_2q_2q_3 + q_2q_3q_2 + q_3q_2q_2)/\sqrt{3}$
Ξ^{*0}	$\frac{1}{2}$	$\frac{1}{2}$	-2	1530	$(q_1q_3q_3 + q_3q_1q_3 + q_3q_3q_1)/\sqrt{3}$
Ξ^{*-}	$\frac{1}{2}$	$-\frac{1}{2}$	-2	1530	$(q_2q_3q_3 + q_3q_2q_3 + q_3q_3q_2)/\sqrt{3}$
Ω^{-}	0	0	-3	1672	$q_3q_3q_3$

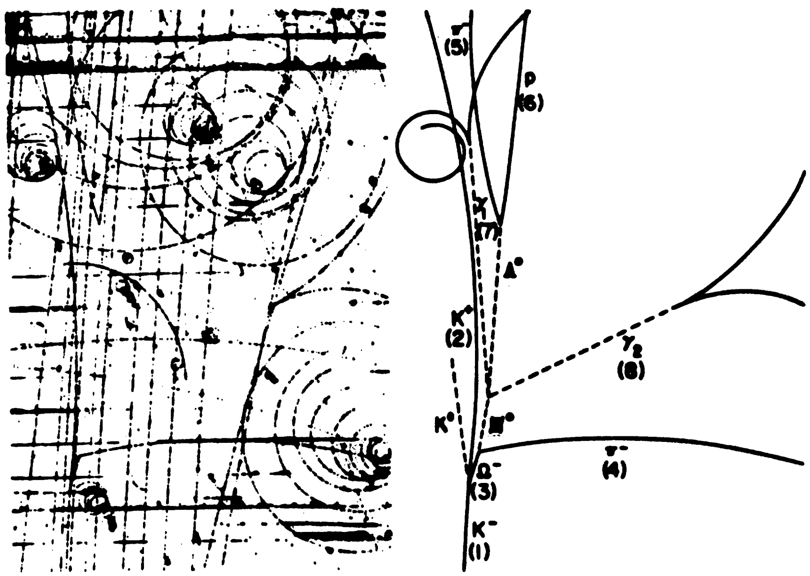
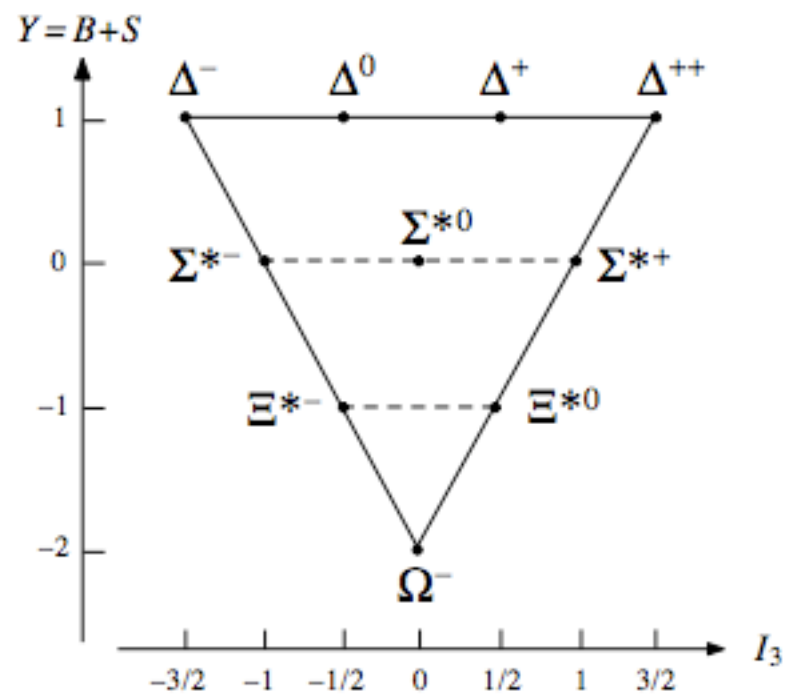


FIG. 2. Photograph and line diagram of event showing decay of Ω^-.



“Observation of a hyperon with strangeness minus 3”
PRL V12, 1964.

Evidence for Quarks with Fractional Charge

- What is the experimental evidence that quarks have non-integer charge?
  - Some models with integer charge quarks were also successful in explaining meson/baryon mass pattern
    - ☞ Need a quantity that can be measured that depends only on electric charge!
- The decay rate (or partial width) for a vector meson to decay to leptons is:

$$\Gamma_{ll} = \frac{16\pi\alpha_{em}^2}{M_V^2}\left|\sum_i a_i Q_i\right|^2 |\psi(0)|^2$$

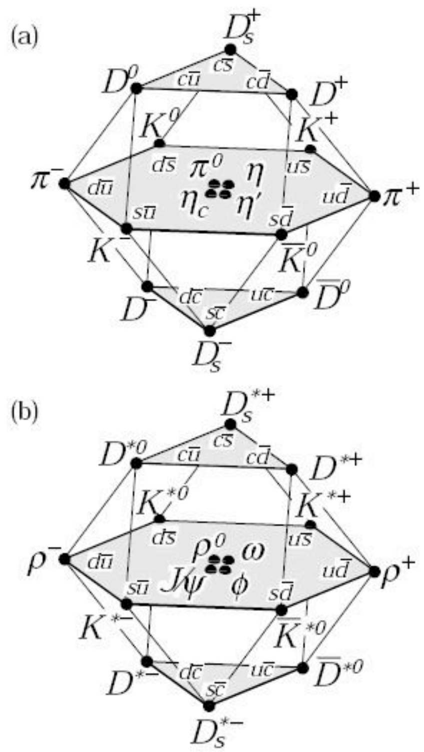
  - ◆ M_V is the mass of the vector meson.
  - ◆ the sum is over the amplitudes that make up the meson.
  - ◆ Q is the charge of the quarks.
  - ◆ $\psi(0)$ is the wavefunction for the two quarks to overlap each other.

Meson	Quarks	$\lvert\sum a_i Q_i\rvert^2$	Γ_{ll}(exp) (KeV)
ρ	$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	$\left[\frac{1}{\sqrt{2}}\left(\frac{2}{3}-\left(-\frac{1}{3}\right)\right)\right]^2 = \frac{1}{2}$	7
ω	$\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$	$\left[\frac{1}{\sqrt{2}}\left(\frac{2}{3}+\left(-\frac{1}{3}\right)\right)\right]^2 = \frac{1}{18}$	0.8
ϕ	$s\bar{s}$	$\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$	1.3

  - ◆ Since the mesons $\rho(770)$, $\omega(780)$ and $\phi(1020)$ have about the same mass
    - ☞ we can assume that $|\psi(0)|^2/M^2$ is the same for the three mesons.
    - ☞ $\Gamma_{ll}(\rho) : \Gamma_{ll}(\omega) : \Gamma_{ll}(\phi) = 9 : 1 : 2$
    - ★ measure: $(8.8 \pm 2.6) : 1 : (1.7 \pm 0.4)$
    - ☞ Good agreement!

Four Quarks

- Once the charm quark was discovered SU(3) was extended to SU(4)!



SU(4) 16-plets for the (a) pseudoscalar and (b) vector mesons made of u, d, s, and c quarks. The nonets of light mesons occupy the central planes, to which the $c\bar{c}$ states have been added. The neutral mesons at the centers of these planes are mixtures of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ states.

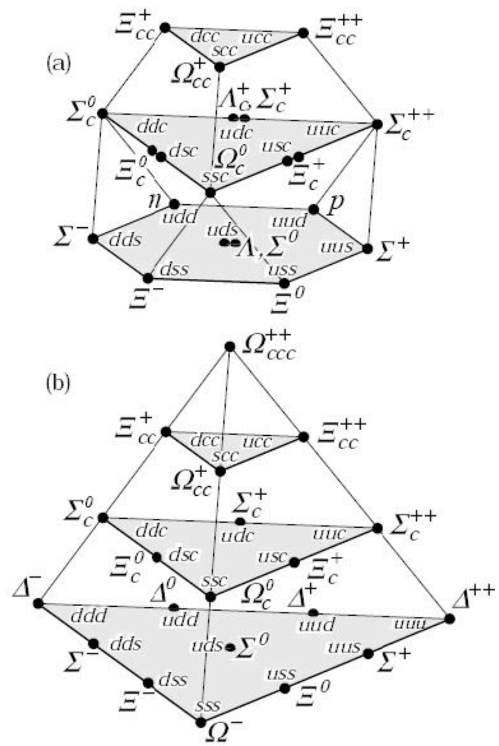


Figure 13.2: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

More Quarks

Table 13.2: Suggested $q\bar{q}$ quark-model assignments for most of the known mesons. Some assignments, especially for the 0^{++} multiplet and for some of the higher multiplets, are controversial. Mesons in bold face are included in the Meson Summary Table. Of the light mesons in the Summary Table, the $f_0(1500)$, $f_1(1510)$, $f_2(1950)$, $f_2(2300)$, $f_2(2340)$, and one of the two peaks in the $\eta(1440)$ entry are not in this table. Within the $q\bar{q}$ model, it is especially hard to find a place for the first two of these f mesons and for one of the $\eta(1440)$ peaks. See the "Note on Non-$q\bar{q}$ Mesons" at the end of the Meson Listings.

$N\ ^{2S+1}L_J$	J^{PC}	$u\bar{d}$, $u\bar{u}$, $d\bar{d}$ $I=1$	$u\bar{u}$, $d\bar{d}$, $s\bar{s}$ $I=0$	$c\bar{c}$ $I=0$	$b\bar{b}$ $I=0$	$\bar{s}u$, $\bar{s}d$ $I=1/2$	$c\bar{u}$, $c\bar{d}$ $I=1/2$	$c\bar{s}$ $I=0$	$\bar{b}u$, $\bar{b}d$ $I=1/2$	$\bar{b}s$ $I=0$	$\bar{b}c$ $I=0$
$1\ ^1S_0$	0^{-+}	π	η, η'	η_c		K	D	D_s	B	B_s	B_c
$1\ ^3S_1$	1^{--}	ρ	ω, ϕ	$J/\psi(1S)$	$\Upsilon(1S)$	$K^*(892)$	$D^*(2010)$	D_s^*	B^*	B_s^*	
$1\ ^1P_1$	1^{+-}	$b_1(1235)$	$h_1(1170)$, $h_1(1380)$	$h_c(1P)$		K_{1B}†	$D_1(2420)$	$D_{s1}(2536)$			
$1\ ^3P_0$	0^{++}	$a_0(1450)$*	$f_0(1370)$*, $f_0(1710)$*	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$					
$1\ ^3P_1$	1^{++}	$a_1(1260)$	$f_1(1285)$, $f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	K_{1A}†					
$1\ ^3P_2$	2^{++}	$a_2(1320)$	$f_2(1270)$, $f_2'(1525)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	$D_2^*(2460)$				
$1\ ^1D_2$	2^{-+}	$\pi_2(1670)$	$\eta_2(1645)$, $\eta_2(1870)$			$K_2(1770)$					
$1\ ^3D_1$	1^{--}	$\rho(1700)$	$\omega(1650)$	$\psi(3770)$		$K^*(1680)$‡					
$1\ ^3D_2$	2^{--}					$K_2(1820)$					
$1\ ^3D_3$	3^{--}	$\rho_3(1690)$	$\omega_3(1670)$, $\phi_3(1850)$			$K_3^*(1780)$					
$1\ ^3F_4$	4^{++}	$a_4(2040)$	$f_4(2050)$, $f_4(2220)$			$K_4^*(2045)$					
$2\ ^1S_0$	0^{-+}	$\pi(1300)$	$\eta(1295)$, $\eta(1440)$	$\eta_c(2S)$		$K(1460)$					
$2\ ^3S_1$	1^{--}	$\rho(1450)$	$\omega(1420)$, $\phi(1680)$	$\psi(2S)$	$\Upsilon(2S)$	$K^*(1410)$‡					
$2\ ^3P_2$	2^{++}		$f_2(1810)$, $f_2(2010)$		$\chi_{b2}(2P)$	$K_2^*(1980)$					
$3\ ^1S_0$	0^{-+}	$\pi(1800)$	$\eta(1760)$			$K(1830)$					

* See our scalar minireview in the Particle Listings. The candidates for the $I=1$ states are $a_0(980)$ and $a_0(1450)$, while for $I=0$ they are: $f_0(400\text{–}1200)$, $f_0(980)$, $f_0(1370)$, and $f_0(1710)$. The light scalars are problematic, since there may be two poles for one $q\bar{q}$ state and $a_0(980)$, $f_0(980)$ may be $K\bar{K}$ bound states.

† The K_{1A} and K_{1B} are nearly equal (45°) mixes of the $K_1(1270)$ and $K_1(1400)$.

‡ The $K^*(1410)$ could be replaced by the $K^*(1680)$ as the $2\ ^3S_1$ state.

PDG listing of the known mesons.

All ground state mesons ($L = 0$) have been observed and are in good agreement with the quark model.

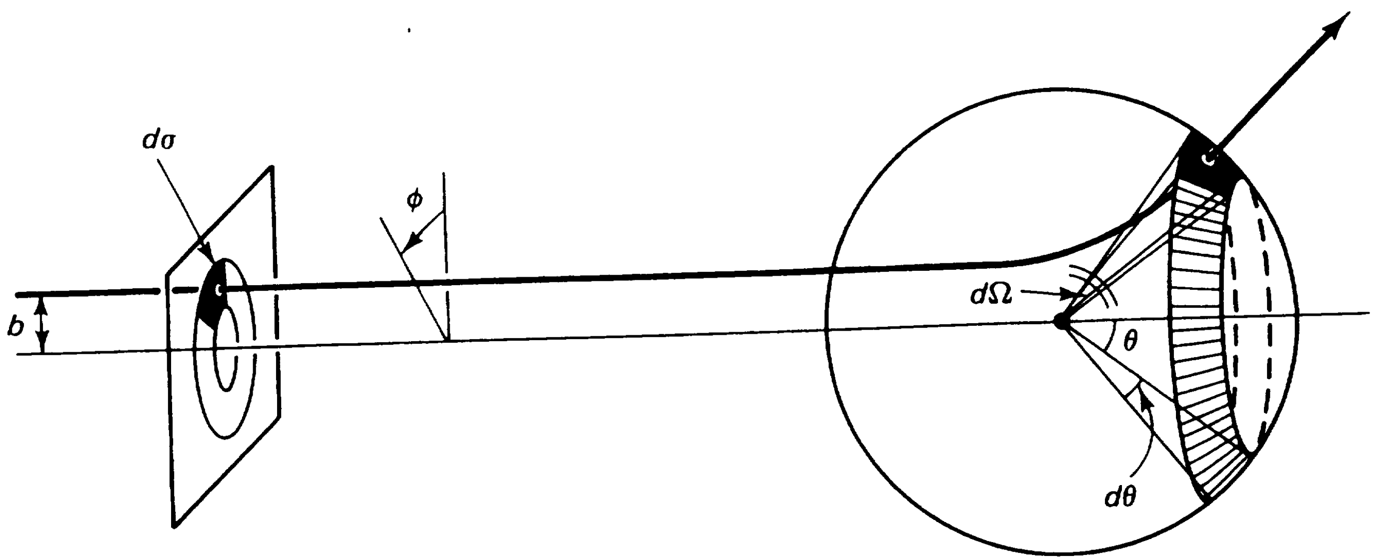
Are Quarks Really Inside the Proton?

- Try to look inside a proton (neutron) by shooting high energy electrons and muons at it and see how they scatter.
- Review of scatterings and differential cross section:
  - The cross section (σ) gives the probability for a scattering to occur:
    - Unit of cross section is area (barn = 10^{-24} cm^2).
    - Differential cross section = $d\sigma/d\Omega$
      - Number of scatters into a given amount of solid angle: $d\Omega = d\phi d\cos\theta = \sin\theta d\phi d\theta$
      - Total amount of solid angle (Ω):

$$\int_{-1}^{+1}\int_{0}^{2\pi} d\Omega = \int_{-1}^{+1} d\cos\theta \int_{0}^{2\pi} d\phi = 4\pi$$

- Relationship between cross section (σ) and impact parameter (b):

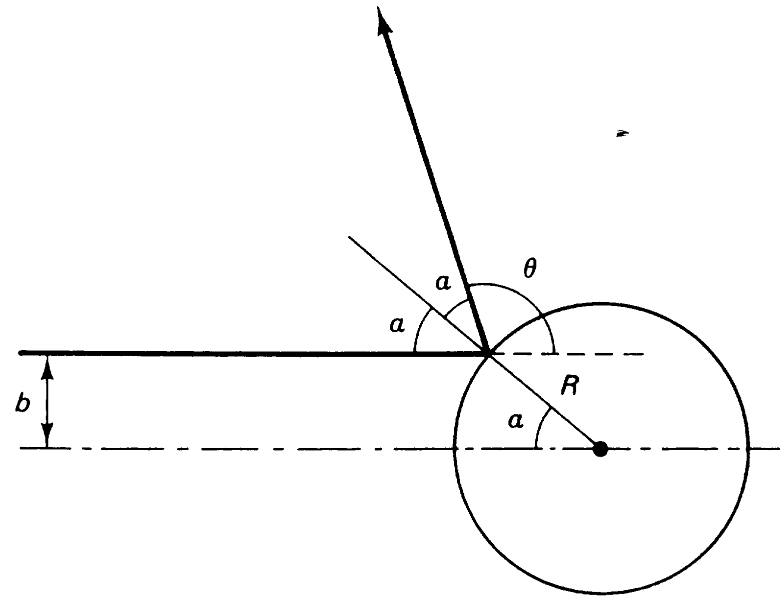
  $d\sigma = |bd\phi db|$



Scattering cross section of a Hard Sphere

- Consider two marbles of radius r and R with $R >> r$:

$$b = R\sin\alpha$$
$$= R\cos(\theta/2)$$
$$db = -\tfrac{1}{2}R\sin(\theta/2)d\theta$$
$$d\sigma = |bdbd\phi|$$
$$= R\cos(\theta/2)\cdot\tfrac{1}{2}R\sin(\theta/2)d\theta\cdot d\phi$$
$$= \tfrac{1}{4}R^2\sin\theta d\theta d\phi$$



- The differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{bdbd\phi}{\sin\theta d\theta d\phi} = \frac{(R^2/4)\sin\theta d\theta d\phi}{\sin\theta d\theta d\phi} = \frac{R^2}{4}$$

- The total cross section is:

$$\sigma = \iint\frac{d\sigma}{d\Omega} = \frac{R^2}{4}\iint\sin\theta d\theta d\phi = \pi R^2$$

  - This result should not be too surprising:
    - Any "small" marble within this area will scatter.
    - Any marble at larger radius will not.

Rutherford/Mott Scatterings

- Rutherford Scattering: a spin-less, point particle with kinetic energy E and electric charge e scatters off a stationary point-like target also of electric charge e:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E\sin^2(\theta/2)}\right)^2$$

  - $\sigma = \infty$ which is not too surprising since the Coloumb force is long range.
  - This formula can be derived using either classical mechanics or non-relativistic QM.
  - The quantum mechanics treatment usually uses the Born Approximation:

$$\frac{d\sigma}{d\Omega} \propto \left|f(q^2)\right|^2$$

    - $f(q^2)$: Fourier transform of the scattering potential V:

$$f(q^2) = \int e^{i\vec{q}\bullet\vec{r}} V(\vec{r})d\vec{r}$$

- Mott Scattering: A relativistic spin 1/2 point particle with mass m, initial momentum p, and electric charge e scatters off a stationary point-like target with electric charge e:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{2p^2\sin^2(\theta/2)}\right)^2\left[(mc)^2 + p^2\cos^2\frac{\theta}{2}\right]$$

  - In the low energy limit, $p << mc^2$, this reduces to the Rutherford cross section.
    Kinetic Energy = $E = p^2/2m$
  - In the high energy limit, $p >> mc^2$ and $E \approx p$ we have

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar\cos(\theta/2)}{2E\sin^2(\theta/2)}\right)^2$$

stationary target has $M >> m$

Scattering off a "Dirac" Proton

- "Dirac" proton: The scattering of a relativistic electron with initial energy E and final energy E' by a heavy point-like spin 1/2 particle with finite mass M and electric charge e is:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar\cos(\theta/2)}{2E\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left[1 - \frac{q^2}{2M^2}\tan^2(\theta/2)\right]$$

Scattering with recoil, neglect mass of electron, $E >> m_e$.

  - q^2 is the electron four-momentum transfer:
    $$(p' - p)^2 = -4EE'\sin^2(q/2)$$
  - The final electron energy E' depends on the scattering angle θ:
    $$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

Form Factor

- What happens if we don't have a point-like target, i.e. there is some structure inside the target?
  - The most common example is when electric charge is spread out over space, i.e. not a "point" charge.
- Example: Scattering off of a charge distribution.
  - The Rutherford cross section is modified to:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E\sin^2(\theta/2)}\right)^2 \left|F(q^2)\right|^2$$

    - $E = E'$
    - $q^2 = -4E\sin^2(\theta/2)$
    - The new term $F(q^2)$ is often called the form factor.
      - The form factor is the Fourier transform of the charge distribution $\rho(r)$:

$$F(q^2) = \int \rho(r) e^{i\vec{q}\bullet\vec{r}} d^3r \qquad \text{with } \int \rho(r) d^3r = 1$$

    - In this simple model we could learn about an unknown charge distribution (structure) by measuring how many scatters occur in an angular region and comparing this measurement with what is expected for a "point charge", $|F(q^2)| = 1$ and our favorite theoretical mode of the charge distribution.

Elastic Electron Proton Scattering (1950's)

- We assume that the electron is a point particle.
- The "target" is a proton which is assumed to have some "size" (structure).
- Consider the case where the scattering does not break the proton apart (elastic scattering).
- Everything is "known" about electron and photon part of the scattering process since we are using QED.
- As shown in Griffiths (8.3) and many other textbooks
  - describe proton in terms of two (theoretically) unknown (but measurable) functions or "form factors":

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4M_p E\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left[2K_1(q^2)\sin^2(\theta/2) + K_2(q^2)\cos^2(\theta/2)\right]$$

  - This is known as the Rosenbluth formula (1950).
  - Assumes scattering is due to interactions with both electric charge and the magnetic moment of proton.
    - ☞ shoot electrons at protons at various energies
    - ☞ count the number of electrons scattered into a given solid angle, $d\Omega = \sin\theta d\phi d\theta$
    - ☞ measure K_1 and K_2
    - ◆ q^2 is the electron four-momentum transfer:

$$(p' - p)^2 = -4EE'\sin^2(\theta/2)$$

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\theta/2)}$$

Elastic Electron Proton Scattering (1950's)

- An extensive experimental program of electron nucleon (e.g. proton, neutron) scattering was carried out by Hofstadter (Nobel Prize 1961) and collaborators at Stanford.
  - They measured the "size" of the proton by measuring the form factors.
  - We can get information concerning the "size" of the charge distribution by noting that:

$$F(q^2) = \int \rho(r) e^{i\vec{q}\bullet\vec{r}} d^3r \approx \int \rho(r)\left\{1 + i\vec{q}\bullet\vec{r} - \frac{(\vec{q}\bullet\vec{r})^2}{2!} + + +\right\} d^3r$$

  - For a spherically symmetric charge distribution we have:

$$F(q^2) \approx 1 - \frac{q^2}{6}\int r^2 \rho(r) dr = 1 - \frac{q^2}{6}\left\langle r^2 \right\rangle$$

  - The measured root mean square radii of the proton charge:

$$\left\langle r^2 \right\rangle^{1/2}_{\text{charge}} = (0.74 \pm 0.24) \times 10^{-13}\ cm$$

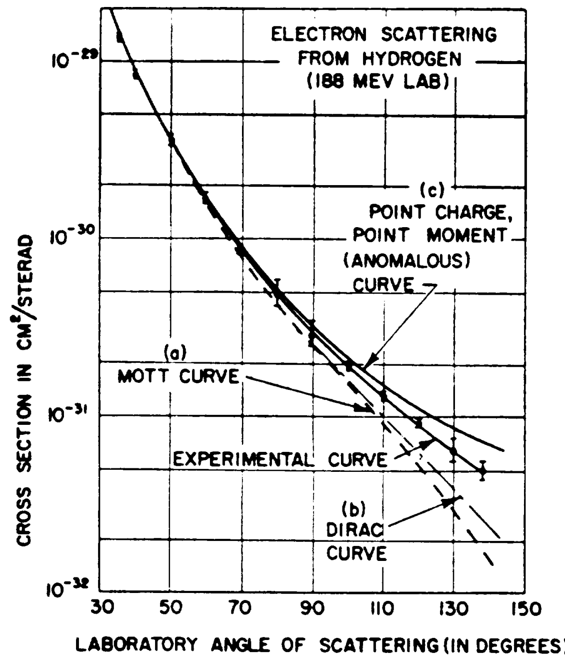


FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of 0.70×10^{-13} cm.

McAllister and Hofstadter, "Scattering of 188 MeV electrons from protons and helium," Phys. Rev., V102, May 1, 1956.