Elastic vs Inelastic Electron-Proton Scattering

M&S 7.4&7.5

- Elastic collision: In the previous lecture we discuss the scattering reaction:
  $e^-p \rightarrow e^-p$
  - The reaction could be described by the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{4M_p E\sin^2(\theta/2)}\right)^2 \frac{E'}{E}\left[2K_1(q^2)\sin^2(\theta/2) + K_2(q^2)\cos^2(\theta/2)\right]$$

  - Form factors K_1 and K_2 contain information about the structure and size of the proton.
  - This process is an example of an elastic scattering:
    - same kind and number of particles in the initial and final state.
      - ☞ no new particles are created in the collision
      - ☞ satisfy the classical definition of an elastic collision:
        initial kinetic energy = final kinetic energy.
- Inelastic collision: "new" particles in the final state, e.g.:
  $e^-p \rightarrow e^-p\pi^o$
  $e^-p \rightarrow e^-\Delta^+$
  $e^-p \rightarrow e^-pK^+K^-$
  - These collisions are mediated by photons for center of mass energies well below the Z-boson mass.
    - ☞ Classified as electromagnetic interactions.
    - ☞ All quantum numbers respected by the electromagnetic interaction must be conserved.

Inelastic *ep* Scattering

- There are many final states in an inelastic scattering.
  - ☞ It is convenient to define a quantity called inclusive cross section.
  - Here we are interested in the reaction:

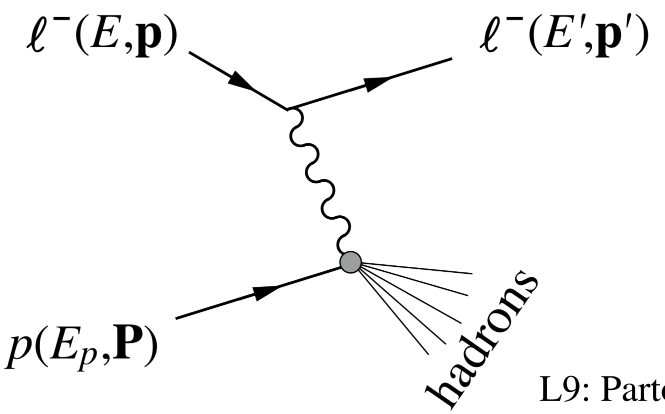
    $e^- p \to e^- X^+$

  - It called an inclusive reaction because we don't measure any of the properties of "X".
    - ☞ we include all available final states.
  - Experimentally: we only measure the 4-vector (energy and angle) of the final state electron.
  - The inclusive cross section (in the lab frame) for $e^- p \to e^- X^+$ can be written as:

$$\frac{d\sigma}{dE' d\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 \left[2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)\right]$$

Simplified version of M&S eq. 7.53

- This is similar in form to the Rosenbluth formula.
- There are some important differences:
  - This is the cross section for the scattered electron to have energy E' within a solid angle $d\Omega$.
  - The Rosenbluth formula does not contain this dependence.
  - Elastic scattering: energy of the scattered electron (E') is determined by the scattering angle θ.
  - Inelastic scattering: energy of the scattered electron (E') is not uniquely determined by θ.



Inelastic ep Scattering

- The scattered electron must be described (kinematically) by two variables.
  - A common set of (Lorentz invariant) kinematic variables are:

$$q^2 \quad \text{and} \quad x \equiv -\frac{q^2}{2qp}$$

    - $q^2 < 0$ and $0 \le x \le 1$
    - p: 4-vector of the target proton (e.g. $(M,0,0,0)$ in "lab" frame)
    - q: difference between the incoming electron (beam) and outgoing (scattered) electron:

$$q \equiv p_e - p_e' = p_X - p_T$$

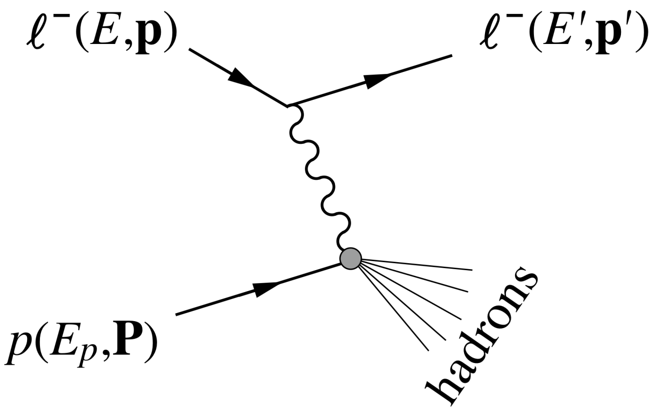
$$q^2 = (p_X - p_T)^2$$

$$= (E_X - M)^2 - \vec{p}_X^{\,2}$$

$$= M_X^2 + M^2 - 2ME_X$$

$$2pq = 2M(E_X - M) - 2\cdot 0\cdot(\vec{p}_X - \vec{p}_T)$$

$$= 2M(E_X - M)$$



    - It is convenient to use the positive variable Q^2:

$$Q^2 = -q^2 = (\vec{p}_e - \vec{p}_e')^2 - (E_e - E_e')^2$$

      - This is just the square of the invariant mass of the virtual photon.
      - In the limit E_e & $E_e' >> m_e$

$$Q^2 = 2E_eE_e' - 2\vec{p}_e\vec{p}_e'$$

$$= 2E_eE_e'(1 - \cos\theta)$$

  - For elastic scattering $e^-p \to e^-p$: $M_X = M$

$$x = -\frac{q^2}{2pq} = -\frac{2M_X^2 - 2ME_X}{2M(E_X - M_X)} = 1$$

Looking Inside a Nucleon

- What if spin ½ point-like objects are inside the proton?
  - As Q^2 increases, the wavelength of the virtual photon decreases.
  - At some point we should be able to see “inside” the proton.
  - This situation was analyzed by many people in the late 1960’s (Bjorken, Feynman, Callan and Gross).
  - Predictions were made for spin 1/2 point-like objects inside the proton (or neutron).
  - These theoretical predictions were quickly verified by a new generation of electron scattering experiments performed at SLAC and elsewhere (DESY, Cornell…)
  - Re-write W_1 and W_2 in terms of F_1 and F_2:

$$\frac{Q^2}{2Mx} W_2(Q^2,x) \to F_2(x)$$

$$MW_1(Q^2,x) \to F_1(x)$$

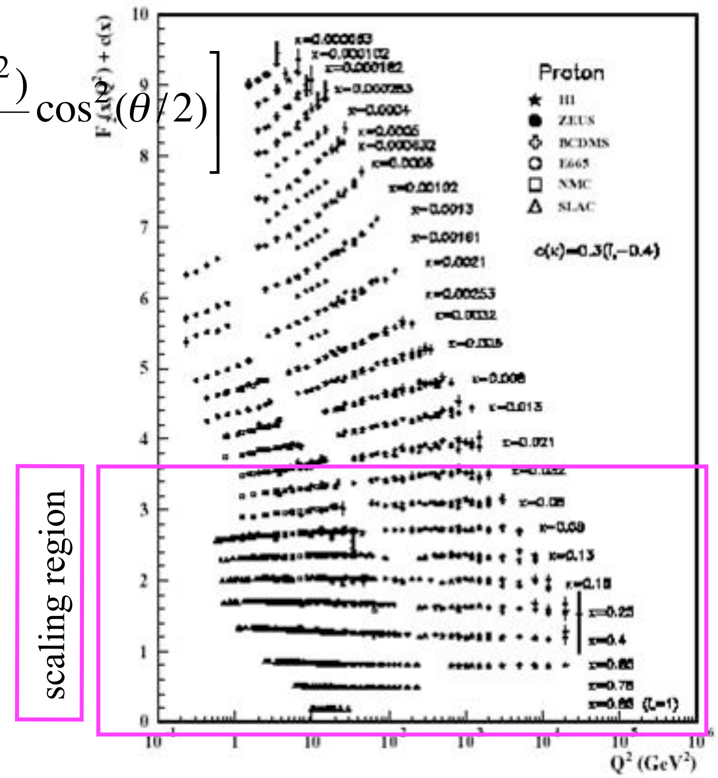
$$\frac{d\sigma}{dE'd\Omega} = \left(\frac{\alpha\hbar}{2E\sin^2(\theta/2)}\right)^2 \left[\frac{2F_1(x,Q^2)}{M}\sin^2(\theta/2) + \frac{2MxF_2(x,Q^2)}{Q^2}\cos^2(\theta/2)\right]$$

  - In the limit where Q^2 and $2pq$ are large,
    - Bjorken predicted that the form factors would only depend on the scaling variable x:

$$x = \frac{Q^2}{2pq}$$

Bjorken Scaling

- These relationships are a consequence of point-like objects (small compared to the wavelength of the virtual photon) being “inside” the proton or neutron.

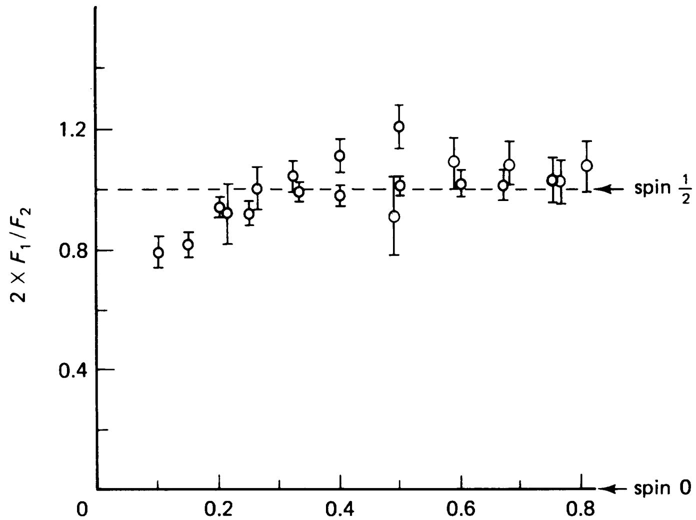


Looking Inside a Nucleon

- Callan-Gross Relationship:
  - If the point-like objects are spin ½
    - ☞ Callan and Gross predicted that the two form factors would be related:

      $2xF_1(x) = F_2(x)$
    - If however the objects had spin 0, then they predicted:

      $2xF_1(x)/F_2(x) = 0$



Good agreement with spin ½ point-like objects inside proton or neutron!

  - But are these objects quarks?
    - Do they have fractional charge?
    - In the late 1960's most theorists called these point-like spin ½ objects "partons".

Are Partons Quarks?

- Do these objects have fractional electric charge?

  $\frac{2}{3}|e|$ or $-\frac{1}{3}|e|$
- First we have to realize that x represents the fraction of the proton's momentum carried by struck quark.
- Next, we take into account how the quarks share the nucleon's momentum.
  - A quark inside a nucleon now have some probability (P) distribution to have momentum fraction x:

    $f(x) = dP/dx$ **$f(x)$ = pdf = probability distribution function**

    $=$ probability to find a quark with momentum fraction x to dx
  - Structure function for inelastic scattering of electron off a quark of momentum fraction x and charge e_i:

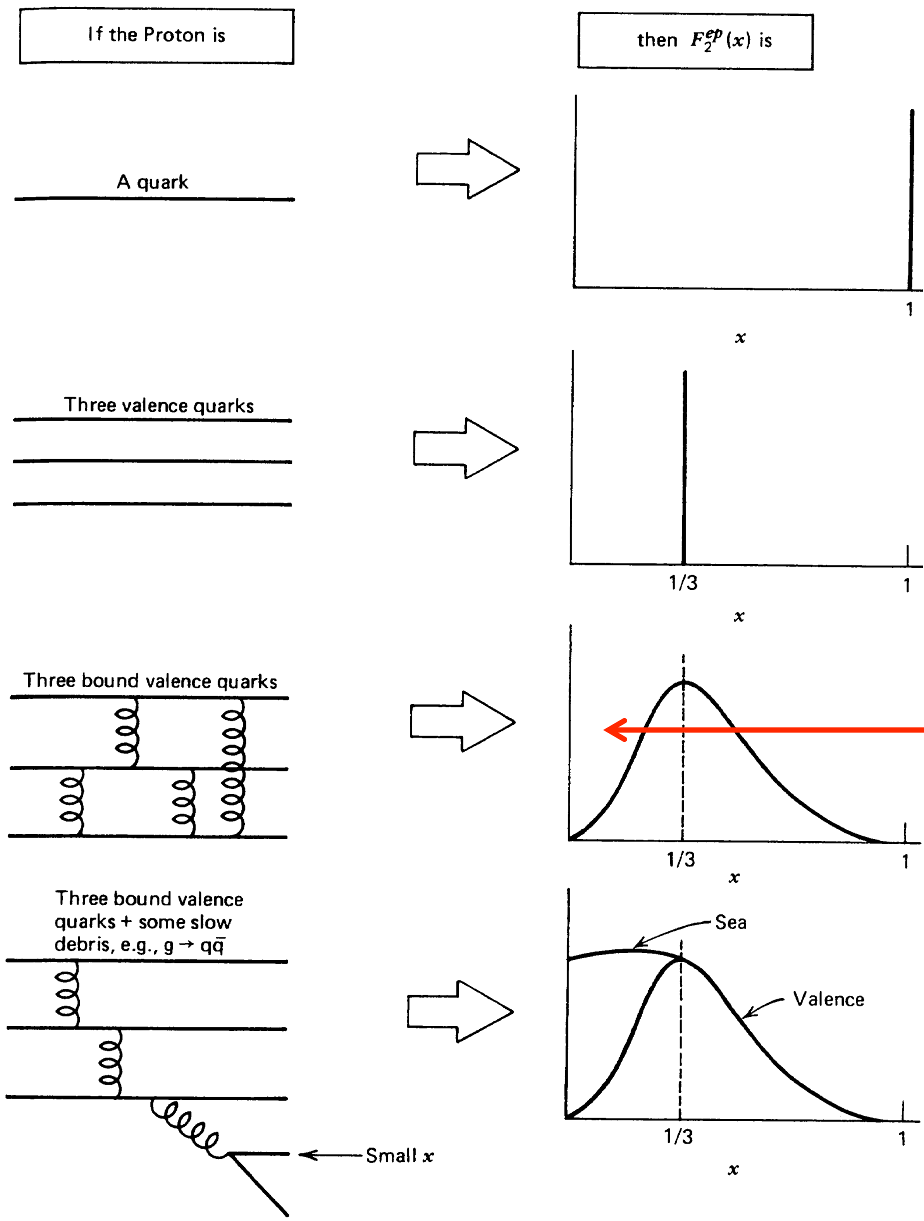
    $$F_1 = \frac{e_i^2 f(x)}{2} \quad \text{and} \quad F_2 = e_i^2 x f(x)$$
- To get the structure function of a proton (or neutron) we must add all the quark contributions.
  - Let $u(x)$ denote the x probability distribution for an up quark etc.:

    $u_p(x)$ = pdf for up quark in a proton
    $u_n(x)$ = pdf for up quark in a neutron
    $d_p(x)$ = pdf for down quark in a proton
    $d_n(x)$ = pdf for down quark in a neutron
- The structure functions:

$$F_{1p} = \frac{1}{2}\sum_{i=1} e_i^2 f_i(x) = \frac{1}{2}\left[\left(\frac{2}{3}\right)^2 u_p(x) + \left(\frac{-1}{3}\right)^2 d_p(x)\right] \qquad F_{2p} = x\sum_{i=1} e_i^2 f_i(x) = x\left[\left(\frac{2}{3}\right)^2 u_p(x) + \left(\frac{-1}{3}\right)^2 d_p(x)\right]$$

$$F_{1n} = \frac{1}{2}\sum_{i=1} e_i^2 f_i(x) = \frac{1}{2}\left[\left(\frac{-1}{3}\right)^2 d_n(x) + \left(\frac{2}{3}\right)^2 u_n(x)\right] \qquad F_{2n} = x\sum_{i=1} e_i^2 f_i(x) = x\left[\left(\frac{2}{3}\right)^2 u_n(x) + \left(\frac{-1}{3}\right)^2 d_n(x)\right]$$

Valence and Sea Quarks?



The simple model of having only three quarks in a proton does not describe the data!

☞ There must be a sea of quark-antiquark pairs.

Evidence for Sea Quarks

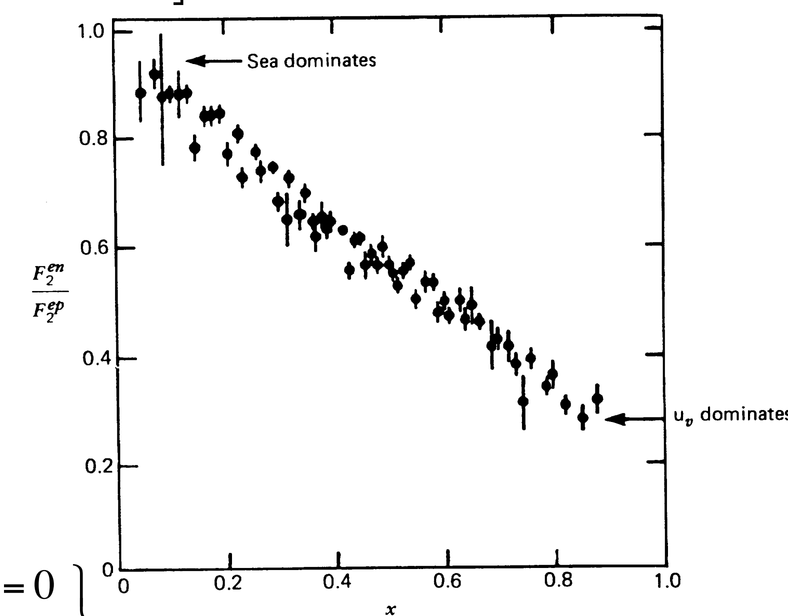
- We now talk about two types of quarks, valence and sea.
  - The valence quarks are the ones that we expect to be in the nucleon.
  - The sea quarks are the ones we get from the quark anti-quark pairs.
  - A proton has two valence u quarks and one valence d quark.
- Generalize structure functions to allow quark anti-quark pairs to also exist (for a short time) in a proton:

$$F_{1p} = \frac{1}{2}\left[\left(\frac{2}{3}\right)^2 \left[u_p(x) - \bar{u}_p(x)\right] + \left(\frac{-1}{3}\right)^2 \left[d_p(x) - \bar{d}_p(x) + s_p(x) - \bar{s}_p(x)\right]\right]$$

$$\int_0^1 \left[u_p(x) - \bar{u}_p(x)\right] dx = 2$$

$$\int_0^1 \left[d_p(x) - \bar{d}_p(x)\right] dx = 1$$

$$\int_0^1 \left[s_p(x) - \bar{s}_p(x)\right] dx = 0$$



- By isospin invariance (up quark = down quark):

$$u = u_p = d_n \qquad d = d_p = u_n$$

$$\frac{F_{2N}}{F_{2p}} = \frac{\left(\frac{2}{3}\right)^2 d(x) + \left(\frac{-1}{3}\right)^2 u(x)}{\left(\frac{2}{3}\right)^2 u(x) + \left(\frac{-1}{3}\right)^2 d(x)} = \frac{4d(x) + u(x)}{4u(x) + d(x)} = \begin{cases} 4 \text{ if } u(x) = 0 \\ 1/4 \text{ if } d(x) = 0 \end{cases}$$

  - If only "sea" quarks in proton and neutron:

$$F_{2N}/F_{2p} = 1$$

Electron vs Neutrino Scattering with Nucleons

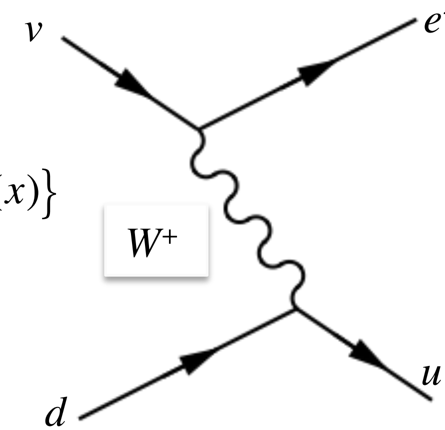
M&S section 7.5

- Compare electron-nucleon scattering with neutrino-nucleon scattering:
  - EM interaction (photon) sensitive to quark electric charge.
  - Neutrino scattering via W exchange is blind to quark electric charge.
    - W^+ interacts with the down quark only.
  - The structure functions for $e(p+n)$ and $\nu(p+n)$ scattering:

$$F_2^{ep+en}(x) = \left\{\left(\frac{2}{3}\right)^2 xu + \left(\frac{-1}{3}\right)^2 xd\right\} + \left\{\left(\frac{2}{3}\right)^2 xd + \left(\frac{-1}{3}\right)^2 xu\right\} = \frac{5}{9}\{xu(x) + xd(x)\}$$

$$F_2^{\nu p+\nu n}(x) = 2xd(x) + 2xu(x)$$

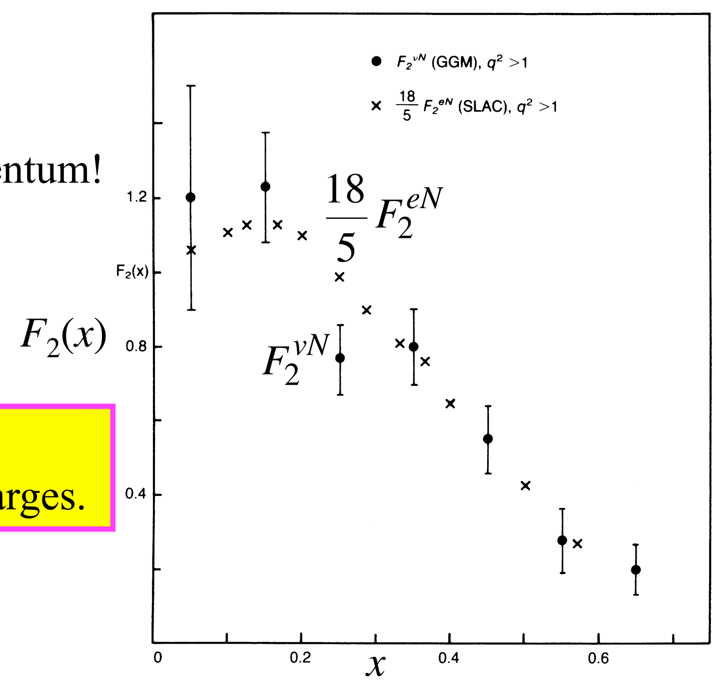
$$F_2^{\nu p+\nu n} = \frac{18}{5} F_2^{ep+en}$$



  - Can also measure the average fraction of the nucleon momentum carried by the quarks:
    - integrate the area under the measured νN curve:

$$\int_0^1 F_2^{\nu N} dx \approx 0.5$$

    - Something else in the nucleon is carrying half of its momentum!
    - Gluons!



Good Agreement with prediction!
The parton charges are the quark charges.