Solid-state nuclear-spin quantum computer based on magnetic resonance force microscopy

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We propose a nuclear-spin quantum computer based on magnetic resonance force microscopy (MRFM). It is shown that an MRFM single-electron spin measurement provides three essential requirements for quantum computation in solids: (a) preparation of the ground state, (b) one- and two-qubit quantum logic gates, and (c) a measurement of the final state. The proposed quantum computer can operate at temperatures up to 1 K.

I. INTRODUCTION

It is well known that a quantum computer can be implemented in a chain of two-level quantum atoms connected by weak interactions.[1] One-qubit rotations and two-qubit quantum logic gates can be implemented in this chain using resonant pulses which induce transitions between the energy levels of the system. The ''natural'' implementation of this idea is an Ising-spin quantum computer which contains a chain of 1/2 spins placed in a permanent magnetic field and interacting through a weak Ising interaction.[2] It has been realized that quantum computation is possible even for an ensemble of Ising-spin chains at temperatures which are much higher than the energy spacing between two stationary states of a spin.[3–5] High-resolution liquid-state NMR quantum computing has been extensively developed for a small number of spins.[6–8]

Magnetic resonance force microscopy (MRFM) has matured over the past few years. It promises atomic scale detection of both electron and nuclear spins.[9–12] MRFM could provide the crucial step from a liquid quantum computer to a solid-state quantum computer, which has the potential to incorporate a large number of qubits.

In this paper, we propose a nuclear-spin quantum computer based on the use of MRFM. In Sec. II, we describe standard MRFM techniques and the method of a single-spin measurement suggested in Ref. 13. This method employs a periodic sequence of short resonant pulses instead of the continuous excitation used in a standard MRFM. In Sec. III, we propose a strategy for detecting a nuclear-spin state via the electron-spin transition in a paramagnetic atom. This approach utilizes the hyperfine splitting of the electron-spin resonance (ESR). In Sec. IV, we describe an implementation of a MRFM nuclear-spin quantum computer in a chain of impurity paramagnetic atoms in a diamagnetic host. We have shown that this quantum computer can provide the three essential requirements for quantum computation: (a) preparation of the ground state (the initial polarization of nuclear spins), (b) quantum logic gates, and (c) a final measurement of the quantum states.

II. SINGLE-SPIN MEASUREMENT

First, we shall discuss an important issue: How can one detect a single spin using an MRFM? The main part of the MRFM is the cantilever. (See Fig. 1.) One end of the cantilever is free to vibrate. Its extremely weak mechanical oscillations can be detected by optical methods. The second important part of the MRFM is a small ferromagnetic particle F, which produces a nonuniform magnetic field B_F in the sample S. The remaining parts are standard for NMR, ESR,

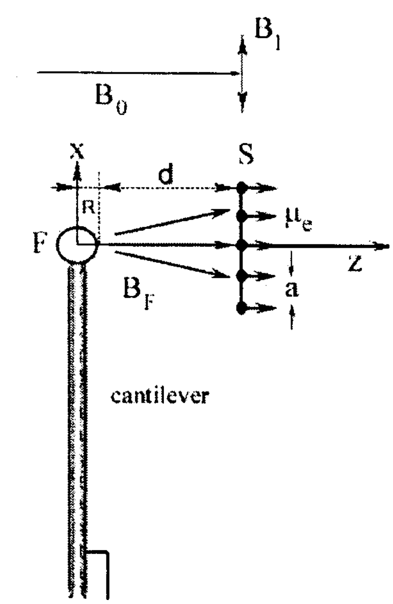

FIG. 1. A nuclear-spin quantum computer based on MRFM using single-spin measurement: B_0 is the permanent magnetic field, B_1 is the radio-frequency magnetic field, $\vec{B}_F$ is the nonuniform magnetic field produced by a ferromagnetic particle F in the sample S, R is the radius of the ferromagnetic particle, d is the distance between the ferromagnetic particle and the targeted atom, and a is the distance between neighboring impurity atoms. The origin is placed at the equilibrium position of the center of the ferromagnetic particle. Arrows on the sample S show the direction of electron magnetic moments μ_e.

and ferromagnetic resonance (FMR) techniques: a uniform magnetic field B_0 and a resonant radio-frequency (rf) field B_1. The nonuniform magnetic field B_F, produced by the ferromagnetic probe, changes the magnetic field in the sample in such a way that only selected spins are resonant with the rf field. Changes in the orientation of selected spins under the action of the resonant rf field influence the magnetic force between the ferromagnetic particle and the sample, causing oscillations of the cantilever. So far, experimenters usually have to put the sample on the cantilever. For MRFM imaging, however, it would be very important to place not a sample but a ferromagnetic particle on the cantilever. This is an experimental challenge. Significant achievements in resolving these problems were reported in Refs. 14 and 15. We will assume, for definiteness, that the ferromagnetic particle is placed on the cantilever, as shown in Fig. 1.

At present, MRFM utilizes the driven resonant oscillations of a cantilever. For example, in experiments with the ferromagnetic resonance[11] with yttrium garnet film, both the magnetic field B_0 and the rf field B_1 were modulated at two frequencies whose difference (or sum) equals the cantilever's frequency f_c (anharmonic modulation). In Ref. 11, $B_0 \sim 10^{-2}$ T, $B_1 \sim 10^{-4}$ T, the modulation frequency of the magnetic field, $f_{0m}=36.01$ kHz, the modulation frequency of the rf field was $f_{1m}=41.27$ kHz, and the resonant frequency of the cantilever was $f_c=5.26$ kHz.

In experiments with the proton magnetic resonance in ammonium nitrate,[10] the modulation scheme was based on cyclic adiabatic inversion. The frequency of the rf field was modulated, so that the effective magnetic field in the rotating frame adiabatically changed its direction. The nuclear magnetization followed the effective field. As a result, the longitudinal (z) component of the nuclear magnetic moment oscillated, producing a cyclic force on the cantilever. In this experiment, the permanent magnetic field B_0 was 1.95 T, the amplitude of the rf field B_1 was $\sim 10^{-3}$ T, the frequency of the rf field was ~ 100 MHz, and the modulation frequency equaled the cantilever's resonant frequency, $f_c = 1.4$ kHz.

In our proposed quantum computer discussed in Secs. III and IV, we use an electron single-spin measurement to determine a nuclear-spin state. We should note that a single-spin measurement is quite different from the measurement of a macroscopic (classical) magnetic moment. First, we shall discuss a single-electron-spin measurement using a static displacement of the cantilever. If the electron magnetic moment points in the positive z direction (state $|0_e\rangle$), the magnetic force on the electron, $F_z=\mu_B \partial B_z/\partial z$, is negative. In this case, the cantilever is attracted to the sample S. Suppose that this deflection from equilibrium of the cantilever's tip can be detected. Then the deflection of the cantilever's tip in the positive z direction will indicate that the electron spin is in the ground state $|0_e\rangle$. Similarly, a negative displacement of the cantilever indicates that the electron is in its excited state $|1_e\rangle$. Then, this cantilever is a classical measuring device: If the electron was initially in a superpositional state, it will jump to one of the states $|0_e\rangle$ or $|1_e\rangle$ due to the interaction with the measuring device. In any case, a cantilever measures one of the states $|0_e\rangle$ or $|1_e\rangle$. Certainly, the static displacement of the cantilever's tip is very small, and its measurement seems to be a complicated problem. That is why we would like to consider resonant methods of single-spin measurement using MRFM.

However, an obstacle appears when one tries to apply conventional resonant methods for single spin detection using MRFM. A single spin interacts with the cantilever as it does with any measuring device. A single spin interacting with any measuring device exhibits quantum jumps rather than smooth oscillations. Observation of resonant vibrations of a cantilever driven by harmonic oscillations of a single-spin z component contradicts the principles of quantum mechanics. To generate resonant vibrations of the cantilever, we proposed a modification of the conventional MRFM technique.[13] We use the fact that, under the action of a short rf pulse, a spin must change its direction in a time interval which is small in comparison with the cantilever's period of oscillations, $T_c=1/f_c$ (it can be a π pulse or a pulse used in adiabatic inversion). During this short time interval, the single spin evolves as a pure quantum system in an external field. For the measuring device (cantilever), this evolution looks like a quantum jump. If one applies a periodic sequence of these short rf pulses with a period equal to $T_c/2$, one will observe resonant vibrations of the cantilever driven by periodic spin inversion. The cantilever will behave as though it is driven by an external harmonic force.

If one applies a periodic sequence of π pulses, the electron Rabi frequency of a π pulse, f_{eR}, must be much larger than the resonant frequency of the cantilever: $f_{eR} \gg f_c$. If one applies a periodic sequence of adiabatic inversions, the following condition must be satisfied: $f_c \ll |d\vec{B}_{eff}/dt|/2\pi B_1 \ll f_{eR}$, where $\vec{B}_{eff}$ is the effective field in the rotating reference frame. (The second inequality is the standard adiabatic condition. The first inequality requires that the time of inversion must be much less than the cantilever's period, T_c.)

While a proton single-spin measurement is possible using MRFM,[13] it is much more difficult to detect a magnetic moment of a proton than the electron magnetic moment. That is why we consider below using ESR to detect a nuclear-spin state.

III. ELECTRON-SPIN RESONANCE STRATEGY FOR DETECTING A NUCLEAR-SPIN STATE

Suppose we want to detect the state of a nuclear spin which interacts with an electron spin via the hyperfine interaction. In a large external magnetic field B_0, the frequency of the electron-spin transition depends on the state of a nuclear spin due to the hyperfine interaction. This dependence was discussed in Ref. 16 in connection with quantum computation using nuclear spins of ^{31}P atoms implanted in silicon. (See also Refs. 17–19.) Suppose that the frequency of the electron-spin transition is f_{e0} for the ground state of the nuclear spin, $|0_n\rangle$, and f_{e1} for the excited state of the nuclear spin, $|1_n\rangle$. (See Fig. 2.) Then, in analogy with the scheme suggested in Ref. 13, if one applies a periodic sequence (with period $T=T_c/2$) of short rf pulses with frequency f_{e0}, one will observe resonant vibrations of the cantilever only if the nuclear spin is in the ground state. Thus, one can measure the state of the nuclear spin using the quantum transitions of the electron spin.

We shall describe here an example of a nuclear-spin quantum computer in which indirect single-spin measure-

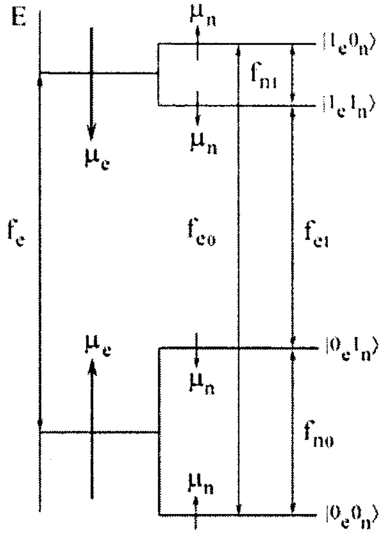

FIG. 2. Energy levels for electron and nuclear spins 1/2 of a paramagnetic atom in a high external magnetic field. The electron and nuclear magnetic moments are indicated by μ_e and μ_n. f_e and f_n are the frequencies of the electron-spin resonance (ESR) and nuclear magnetic resonance (NMR) in the external magnetic field without the hyperfine interaction. The frequencies $f_{e0}=f_e+f_{hf}$, $f_{e1}=f_e-f_{hf}$, $f_{n0}=f_n+f_{hf}$, and $f_{n1}=f_{hf}-f_n$. Here f_{hf} is the hyperfine frequency (half of the hyperfine splitting of the ESR). f_{eq} is the ESR frequency for the nuclear state $|q_n\rangle (q=0,1)$. f_{nq} is the NMR frequency for the electron state $|q_e\rangle$. We assume that the magnetic moment of the nucleus and the hyperfine constant are positive, and $f_{hf}>f_n$. In the external field, $B_0=10$ T, $f_e \approx 280$ GHz, and $f_n \approx 430$ MHz (for a proton).

ments could be made using a MRFM. Consider a chain of impurity paramagnetic atoms on the surface of a diamagnetic host. Assume that the nuclei of the host have zero magnetic moments. Suppose that the distance between the impurity atoms is $a=50$ Å, the distance between the ferromagnetic particle on the cantilever's tip (at equilibrium) and the target atom is $d=100$ Å, the radius of the ferromagnetic particle is, $R=50$ Å (see Fig. 1), and the energy levels for electron and nuclear spins of a paramagnetic atom are given in Fig. 2.

The magnetic field at the target atom, $\vec{B}_F$, caused by the ferromagnetic particle can be estimated as

$$\vec{B}_F=\frac{\mu_0}{4\pi}\frac{3\vec{n}(\vec{m}\cdot\vec{n})-\vec{m}}{r^3}, \tag{1}$$

where $\mu_0=4\pi\times10^{-7}$ H/m is the permeability of free space, $\vec{n}$ is the unit vector which points from the center of the ferromagnetic particle to the atom, $\vec{m}$ is the magnetic moment of the ferromagnetic particle, and r is the distance between the atom and the center of the ferromagnetic particle. (The target atom is the atom in the sample which is closest to the ferromagnetic particle.) Substituting

$$m=\frac{4}{3}\pi R^3 M, \tag{2}$$

where M is the magnetization of the ferromagnetic particle, we obtain the magnetic field at the target atom produced by the ferromagnetic particle,

$$B_{Fz}=\frac{2}{3}\mu_0 M\left(\frac{R}{z}\right)^3, \quad B_{Fx}=B_{Fy}=0. \tag{3}$$

In Eq. (3), $z=R+d$ is the coordinate of the target atom. (We place the coordinate origin at the equilibrium position of the center of the ferromagnetic particle, neglecting its static displacement.) Choosing $\mu_0 M=2.2$ T (which corresponds to an iron ferromagnetic particle), we get $B_{Fz}=5.4\times10^{-2}$ T. The corresponding shift of the ESR frequency is $(\gamma_e/2\pi)B_{Fz}\approx1.5$ GHz. Here γ_e is the electron gyromagnetic ratio ($\gamma_e/2\pi=2.8\times10^{10}$ Hz/T). Now, we estimate the z component of the magnetic field B'_{Fz} at the neighboring impurity atom,

$$B'_{Fz}=\frac{1}{3}\mu_0 M\left(\frac{R}{r}\right)^3\left[3\left(\frac{z}{r}\right)^2-1\right]. \tag{4}$$

Putting $r=\sqrt{z^2+a^2}$, we obtain $B'_{fz}=3.6\times10^{-2}$ T. We assume here that the external magnetic field B_0 is much greater than the field produced by the ferromagnetic particle. (Below we choose $B_0=10$ T.) Thus, only the z component B'_{Fz} is required to estimate the ESR frequency. The difference between the ESR frequencies for two neighboring atoms is

$$\Delta f'_e=(\gamma_e/2\pi)(B_{Fz}-B'_{Fz})\approx500 \text{ MHz}. \tag{5}$$

The electron Rabi frequency f_{eR}, which will drive a target electron spin, must be smaller than $\Delta f'_e$, to provide a selective measurement of the electron spin on the target atom.

Consider the cantilever's vibrations driven by the target electron spin. We take parameters of the cantilever from Ref. 10: the spring constant of the cantilever is $k_c=10^{-3}$ N/m, the frequency of the cantilever is $f_c=1.4$ kHz, the effective quality factor is $Q=10^3$, and the amplitude of thermal vibrations at room temperature ($T\approx300$ K) is $z_{rms}=5$ Å (z_{rms} is the root-mean-square amplitude of thermal vibrations). Assume that the temperature is 1 K. Scaling the square root dependence of z_{rms} on T,[13] we obtain the characteristic amplitude of thermal vibrations of the cantilever[10] at $T=1$ K: $z_{rms}\approx0.3$ Å.

The magnetostatic force on a target electron created by the ferromagnetic particle can be estimated as

$$F_z=\pm\mu_B\frac{\partial B_z}{\partial z}, \tag{6}$$

for the positive and negative directions of the electron magnetic moment, respectively. Differentiating Eq. (3), we obtain the value of F_z,

$$F_z=\mp2\mu_B\frac{\mu_0 M}{z}\left(\frac{R}{z}\right)^3. \tag{7}$$

For $z=R+d$, the value of F_z is $F_z\approx\mp10^{-16}$ N. The magnetostatic force on the ferromagnetic particle is of the same magnitude, but it points in the opposite direction of the force on the target electron. When the magnetostatic force on the cantilever, $F_z(t)$, takes the values $\pm F$ and the period of the function $F_z(t)$ is equal to the cantilever period T_c, the stationary amplitude of the cantilever vibrations is[13]

$$z_c=4FQ/\pi k_c\approx1.2 \text{ Å}. \tag{8}$$

This amplitude of the stationary oscillations can be achieved after a transient process, whose duration, τ_c, can be estimated as

$$\tau_c = Q/\pi f_c \approx 0.2 \text{ s}. \quad (9)$$

Note that one does not have to ''wait'' until the stationary amplitude is reached. In fact, the experiment can be stopped when the rms amplitude of the driven cantilever's vibrations exceeds the thermal vibration amplitude, $z_{rms} \approx 0.3$ Å. This occurs when the value of the driven amplitude, $z_c' = \sqrt{2} z_{rms} \approx 0.4$ Å. Assuming the cantilever's amplitude increases as $z_c[1-\exp(-t/\tau_c)]$, we obtain the time of a single-spin measurement:

$$\tau_m = -\tau_c \ln(2/3) \approx 80 \text{ ms}. \quad (10)$$

The lifetime of the electron excited state must be greater than τ_m. (Otherwise a spontaneous electron transition can destroy the process of measurement.)

Next, we shall estimate the deviation of the magnetic field at the target atom due to the cantilever's vibrations. For the value of $z_c' \approx 0.4$ Å, the deviation of the magnetic field, ΔB_z, is

$$\Delta B_z \approx \left| \frac{\partial B_z}{\partial z} \right| z_c' \approx 4 \times 10^{-4} \text{ T}. \quad (11)$$

The corresponding deviation of the ESR frequency is

$$\Delta f_e = (\gamma_e/2\pi) \Delta B_z \approx 10 \text{ MHz}. \quad (12)$$

To provide the ESR, the electron Rabi frequency f_{eR} must be greater than Δf_e. Thus, the requirement for the electron Rabi frequency f_{eR} can be written as

$$\Delta f_e \ll f_{eR} \ll \Delta f_e', \quad (13)$$

or 10 MHz$\ll f_{eR} \ll$500 MHz. Taking, for example, $f_{eR} = 100$ MHz we obtain the duration of an ''electron'' π pulse: $\tau = 1/2 f_{eR} = 5$ ns. (We do not mention the condition $f_c \ll f_{eR}$, because in our case $f_c \ll \Delta f_e$.) Certainly, to measure a nuclear state, the value of f_{eR} must be less than the ESR hyperfine splitting $2 f_{hf}$. This imposes the requirement $f_{hf} \gg \Delta f_e$. Note that the value of the electron Rabi frequency f_{eR} can be close to $\Delta f_e'$ (or $2 f_{hf}$ if $2 f_{hf} < \Delta f_e'$) if one uses the $2\pi k$ method. (See Refs. 20–22.)

Finally, we estimate the dipole field produced by paramagnetic atoms. The main contribution to the dipole field is associated with the neighboring atoms. For any inner atom in the chain, two neighboring electron magnetic moments which point in the positive z direction produce the magnetic field

$$B_{dz} = -2 \mu_0 \mu_B / 4 \pi a^3 \approx -1.5 \times 10^{-5} \text{ T}. \quad (14)$$

The maximal contribution from all other paramagnetic atoms to the dipole field B_{dz}' does not exceed -3×10^{-6} T. (For a chain of 1000 paramagnetic atoms with electron spins in the ground state, the value of B_{dz}' at the center of the chain is $B_{dz}' \approx 0.202 B_{dz}$.) The corresponding frequency shift of the ESR is

$$(\gamma_e/2\pi) |B_{dz} + B_{dz}'| \leqslant 500 \text{ kHz}. \quad (15)$$

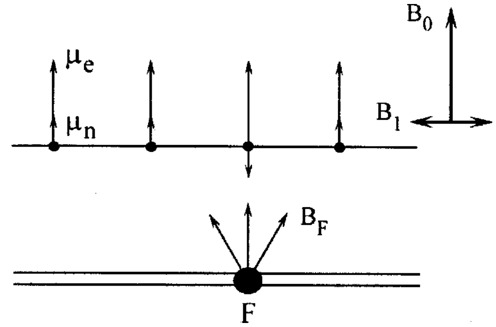

FIG. 3. The polarization of nuclear spins: The nonvibrating ferromagnetic particle targets nuclear spins which are initially in the excited state.

This frequency shift is negligible compared to the estimated electron Rabi frequency $f_{eR} \geqslant 100$ MHz. Thus, to measure the nuclear-spin state of the target atom, one can use a periodic sequence of ''electron'' π pulses with frequency

$$f \approx f_{e0} + (\gamma_e/2\pi) B_{Fz}. \quad (16)$$

IV. NUCLEAR-SPIN QUANTUM COMPUTER USING SINGLE-SPIN MEASUREMENTS

In this section, we assume that the MRFM can provide a single-electron-spin measurement. We shall now discuss a nuclear-spin quantum computer using this MRFM single-spin detection.

First, the single-spin MRFM can be used to create 100% polarization in a nuclear-spin chain. To do this, one should determine the initial state of each nuclear spin in the chain. [Note that we assume 100% polarization of electron spins. In the external magnetic field $B_0 \approx 10$ T, at the temperature $T \approx 1$ K, the probability for an electron spin to occupy the upper energy level is approximately $\exp(-2\mu_B B_0/k_B T) \approx 1.4 \times 10^{-6}$. In fact, we can relax these restrictions on the magnetic field B_0 or the temperature.] During the measurement process, one should use an even number of pulses to return the electron spin to the ground state.

To create 100% polarization of the nuclear spins or to carry out a quantum computation, one must fix the z coordinate of the ferromagnetic particle. This nonvibrating particle is not a classical measuring device. It is only a static source of the inhomogeneous magnetic field. One can imagine that the ferromagnetic particle could move along the spin chain. (See Fig. 3.) It can be used to target each nuclear spin which is in the excited state.

According to Eqs. (3) and (4), the target nuclear spin experiences an additional magnetic field $B_{Fz} \approx 5.4 \times 10^{-2}$ T. The neighboring nuclear spin experiences an additional magnetic field $B_{Fz}' \approx 3.6 \times 10^{-2}$ T. The corresponding shifts of the NMR frequencies are $(\gamma_n/2\pi) B_{Fz} \approx 2.3$ MHz and $(\gamma_n/2\pi) B_{Fz}' \approx 1.5$ MHz, where γ_n is the nuclear gyromagnetic ratio. (Here and below we present estimations for a proton, $\gamma_n/2\pi \approx 4.3 \times 10^7$ Hz/T.) The frequency difference between the target nuclear spin and its neighbor is, $\Delta f_n' \approx 800$ kHz.

The Rabi frequency f_{nR} must be less than $\Delta f_n' \approx 800$ kHz to provide a selective action of a π pulse. Again, using the $2\pi k$ method,[20–22] one can choose a value of f_{nR}

close to $\Delta f_n'$. The corresponding duration of the π pulse, $\tau = 1/2f_{nR}$, must be greater than 630 ns.

The dipole field B_{dz} produced by two neighboring paramagnetic atoms (for inner nuclear spins in the chain) is given by Eq. (14). The maximal additional contribution of all other paramagnetic atoms is estimated to be $B_{dz}' \approx -3\times10^{-6}$ T. The corresponding shift of the NMR frequency is approximately 780 Hz, much less than the assumed value of the Rabi frequency, $f_{nR}\sim100$ kHz. Thus, to drive the target nuclear spin into its ground state, one can apply a ''nuclear'' π pulse with frequency $f\approx f_{n0}+(\gamma_n/2\pi)B_{Fz}$, where f_{n0} is the NMR frequency of a paramagnetic atom with a ground state of the electron spin. (See Fig. 2.) In this way, the whole chain of the nuclear spins can be initialized in its the ground state. In the same way, one can provide a one-qubit rotation for any selected nuclear spin.

Now we consider the possibility of implementing conditional logic in a chain of nuclear spins. The direct interaction between nuclear spins for interatomic distance $a=50$ Å is negligible. That is why, to provide conditional logic in a system of nuclear spins, we propose using an electron dipole field. Suppose that we want to implement a two-qubit quantum CONTROL-NOT (CN) logic gate. Because for our system, the ESR frequency $f_{e0}+(\gamma_e/2\pi)B_{Fz}$ is the highest and the unique frequency in the chain, we will consider the ''inverse'' CN gate: the target qubit changes its state if the control qubit is in the ground state. (Here the target qubit is a nuclear spin which can change its state during the CN operation, but not necessarily the nuclear spin closest to the ferromagnetic particle.)

Assume that the target qubit is any inner nuclear spin in the chain, and the control qubit is one of its neighboring nuclear spin. (See Fig. 4, where A and B are the control and target qubits.) We want to implement an ''inverse'' CN gate in three steps.

(1) One sets the ferromagnetic particle near the control qubit [Fig. 4(a)], and applies an ''electron'' π pulse with frequency given by Eq. (16). The electron Rabi frequency f_{eR} satisfies the inequalities $f_{eR}<\Delta f_e', 2f_{hf}$. So the electron magnetic moment of the left paramagnetic atom in Fig. 4 changes its direction only if the control qubit is in the ground state $|0_n\rangle$.

(2) The ferromagnetic particle moves to the target qubit. [See Fig. 4(b).] If the control qubit is in the excited state, then the electron magnetic moment of the left atom did not change its direction during the first step. In this case, the NMR frequency for the target qubit is

$$f_{n0}=(\gamma_n/2\pi)(B_{Fz}+B_{dz}+B_{dz}'). \tag{17}$$

The second term in the summ is important for us: $(\gamma_n/2\pi)B_{dz}\approx650$ Hz. [The value $(\gamma_n/2\pi)B_{dz}'$ depends on the position of the nuclear spin in the chain. We estimated that the range of variation for this term is approximately between 70 Hz and 130 Hz. The exact value of this term can be calculated or measured experimentally for each nuclear spin in the chain.] If the control qubit is in the ground state (as in Fig. 4), then the electron magnetic moment of the left atom changed its direction during the first step. In this case, the NMR frequency of the target qubit is

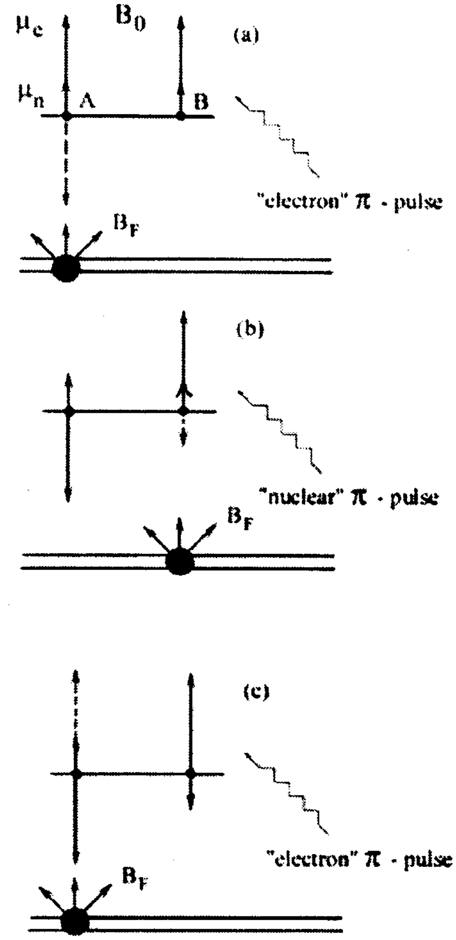

FIG. 4. Implementation of a quantum CN gate: (a) an ''electron'' π pulse drives the electron magnetic moment of the control qubit (nuclear spin A) if A is in the ground state; (b) a ''nuclear'' π pulse causes a transition in the target qubit (nuclear spin B) if the control qubit A is in the ground state; (c) ''electron'' π pulse drives the electron magnetic moment back into the ground state.

$$f_{n0}=(\gamma_n/2\pi)(B_{Fz}+B_{dz}'), \tag{18}$$

because the dipole field produced by neighboring paramagnetic atoms cancels out: $B_{dz}=0$.

Next, one applies a ''nuclear'' π pulse with frequency (18). The difference between the frequencies in Eqs. (17) and (18) is approximately 650 Hz. Thus, the nuclear Rabi frequency f_{nR} must be less that 650 Hz. The corresponding duration of the ''nuclear'' π pulse is $\tau>770$ μs. [Using the $2\pi k$ method,[20–22] one can choose the nuclear Rabi frequency f_{nR} close to 650 Hz. In this case, one can neglect the last term $(\gamma_n/2\pi)B_{dz}'$ in the expressions (17) and (18) because $(\gamma_n/2\pi)B_{dz}'\leqslant130$ Hz.] Under the action of a ''nuclear'' π pulse the target qubit changes its state if the control qubit is in its ground state.

(3) To complete the CN gate, the ferromagnetic particle moves back to the control qubit. [See Fig. 4(c).] Then one should again apply the ''electron'' π pulse with frequency (16). This pulse drives the electron magnetic moment back to its ground state. A similar procedure can be applied if the target qubit is at either end of the chain.

It is well known that any quantum algorithm can be implemented with one-qubit rotations and two-qubit CN gates.[23] So the MRFM designed for single-spin measurement could accomplish three tasks: preparation of the spin system in the ground state, implementation of quantum computation, and measurement of the final state. Certainly, we require a long relaxation time for the nuclear spins in order to perform a considered quantum computation.

Finally, we discuss here two possible unwanted effects related to the influence of the ferromagnetic particle on the process of quantum computation.

(1) The influence of the ferromagnetic particle on the precession frequency of the neighboring spins in the process of quantum computation. This is a controllable effect because in the process of quantum computation the ferromagnetic particle is fixed. The additional phase change caused by the ferromagnetic particle can be easily calculated for each nuclear spin using a conventional digital computer. The phases of the rf pulses acting on any particular spin can be adjusted to this additional phase advance.

(2) Possible increased decoherence due to the ferromagnetic particle. We believe that the influence of nonvibrating ferromagnetic particle on the decoherence will be not significant. (At least, it could be less significant than the influence of voltage gates used in other solid-state quantum computer proposals.)

V. CONCLUSION

In this paper, we proposed an MRFM nuclear-spin quantum computer which can operate at the temperature $T \approx 1$ K. We discussed procedures required for a single-spin measurement using MRFM. We pointed out an obstacle which impedes the application of a conventional MRFM to a single-spin measurement and described a way to overcome this obstacle. We also suggested a way to make an indirect measurement of the nuclear-spin state by measuring an associated electron spin and we presented related estimates.

We have shown that a single-spin MRFM can provide the required operations for a nuclear-spin quantum computer: initial polarization of nuclear spins, the necessary quantum transformations, and final spin measurements.

ACKNOWLEDGMENTS

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