Static Stern-Gerlach effect in magnetic force microscopy

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(Received 23 June 2000; published 20 February 2002)

We examine static single-spin measurements using magnetic-force microscopy methods. We show that these measurements could be carried out at millikelvin temperatures. A simple estimate of the decoherence time is presented. Possible experiments for measuring single-spin relaxation processes and the use of these measurements in quantum computations are discussed.

DOI: 10.1103/PhysRevA.65.032311 PACS number(s): 03.65.Ta, 07.79.Pk

I. INTRODUCTION

The Stern-Gerlach (SG) effect, one of the cornerstones of a quantum mechanics, still attracts much attention. (See, for example, Refs. [1–7].) The SG apparatus clearly demonstrates the quantization of angular momentum and it can also demonstrate the mysterious puzzles of quantum measurement, macroscopic quantum superposition (Schrödinger-cat) states, and decoherence. The SG effect was used to measure the spin state of a single electron in a Penning trap [8] and the spin state of a single hydrogen-like ion in a Penning trap [9]. So far, all experimental and theoretical versions of the SG apparatus exploit the influence of the spin state on the characteristics of the classical motion of a particle.

In this paper, we discuss the static SG effect, in which the spin state influences the equilibrium position of a classical system. Implementation of the static SG apparatus can be used to understand the puzzles of a quantum measurement, and it also can find applications in quantum computation.

Our idea is similar to Ref. [1] in which the magnetic-resonance force microscopy (MRFM) was proposed as a new method for performing a SG measurement. During the last decade remarkable progress has been achieved in building ultra-thin cantilevers. (See, for example, Refs. [10–12].) In this connection, we explore the possibility of detecting static SG effect using the latest ultra-thin cantilevers.

II. DETECTION OF A SINGLE SPIN AND SINGLE-SPIN FLIPS

Consider a "gedanken" experiment. We will assume that a single paramagnetic atom with spin 1/2 is placed on a tip of the diamagnetic cantilever. (See Fig. 1.) A spherical ferromagnetic particle is placed near the cantilever tip. The whole system is placed in a uniform external magnetic field, $\vec{B}_0$. We assume that $\vec{B}_0$ is large enough, $k_B T \ll 2\mu_B B_0$ (where μ_B is the Bohr magneton). Thus, the magnetic moment of the paramagnetic atom $\vec{m}_p$, points in the direction of $\vec{B}_0$ (the positive z-direction in Fig. 1). The magnetic field, $\vec{B}_0$, also induces a magnetic moment, $\vec{m}_d$, in the diamagnetic atoms of the cantilever which points in the negative z-direction. As a result, the ferromagnetic particle attracts the paramagnetic atom and repels the diamagnetic atoms near the cantilever tip.

Now, we will make some estimates. Assume that the ferromagnetic particle has a spherical shape with the radius, $R = 15$ nm. Suppose that the distance, d, between the particle and the cantilever surface is $d = 5$ nm. (For a fixed value of d, the ratio $R:d = 3:1$ corresponds to the maximum attraction between the ferromagnetic particle and the paramagnetic atom.) The magnetic field produced by the ferromagnetic particle on the cantilever surface is

$$B_F = \frac{2}{3}\mu_0 M \left(\frac{R}{R+d}\right)^3, \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the permeability of the free space, and M is the magnetization of the ferromagnetic particle. Setting $\mu_0 M = 1$ T, we obtain $B_F \approx 0.28$ T. To estimate the magnetic moment of the diamagnetic host effectively interacting with the ferromagnetic particle, we consider a rectangular solid with an area $(2R)^2$ and a depth 5.2 nm. (The magnetic field, B_F, halves at this depth.) Assuming that the magnetic susceptibility of the cantilever is

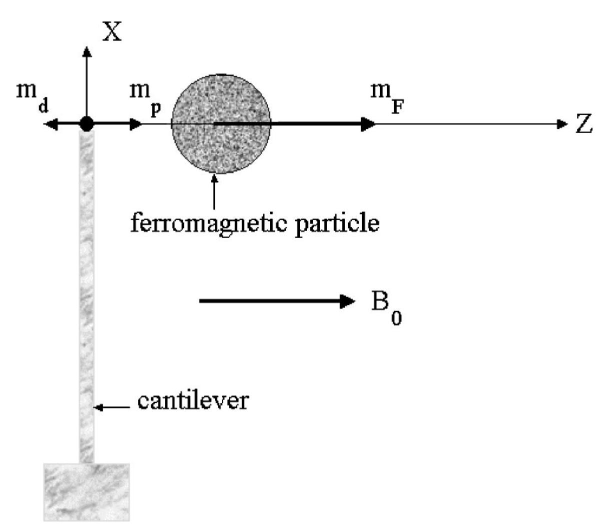

FIG. 1. Geometry of the proposed experiment. m_d is the magnetic moment of the diamagnetic host; m_p is the magnetic moment of a paramagnetic atom, and m_F is the magnetic moment of a ferromagnetic particle.

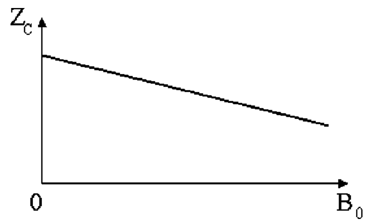

FIG. 2. A dependence of the cantilever position, z_c, on the magnitude of the external magnetic field, B_0.

-3×10^{-7} (as in silicon), we have for the magnetic moment of the cantilever that effectively interacts with the ferromagnetic particle,

$$m_d \approx 3.6\times10^{-25}\ \text{J/T}, \quad \text{for} \quad B_0=0, \quad B_F=0.28\ \text{T}, \tag{2}$$

$$m_d \approx 6\times10^{-24}\ \text{J/T}, \quad \text{for} \quad B_0+B_F=5\ \text{T}.$$

Thus, by changing the external magnetic field, B_0, one can vary the magnetic moment, m_d, from $m_d \ll m_p$ (where $m_p = \mu_B = 9.3\times10^{-24}$ J/T) up to a value comparable with m_p. Figure 2 demonstrates the expected dependence of the cantilever equilibrium position, z_c, on the magnitude of the external magnetic field, B_0. (The origin is placed at the equilibrium position of the cantilever in the absence of the magnetic field, B_F.) The polarization of the diamagnetic atoms weakens the attraction between the cantilever and the ferromagnetic particle and decreases the cantilever displacement, z_c.

The objective of this *gedanken* experiment is the estimation of the static cantilever displacement, z_c, due to its interaction with a single paramagnetic atom. The static displacement of the cantilever tip under the action of the constant force, $\vec{F}$, acting in the positive z-direction is given by (see, for example, [14])

$$z_c = \frac{4F}{m_c}\sum_{n=1}^{\infty}\omega_n^{-2}, \tag{3}$$

where m_c is the mass of the cantilever, and ω_n are its eigenfrequencies. It was shown elsewhere (see, for example, [1]) that Eq. (3) can be rewritten as $z_c = F/k_c$, where k_c is the effective spring constant of the cantilever,

$$k_c \approx \frac{m_c\omega_1^2}{4}. \tag{4}$$

The expression (4) is the same as for the classical massless spring with an effective mass, $m_c/4$, attached to its end.

For our estimations we consider the ultra-thin cantilever reported in Ref. [12] with a spring constant, $k_c = 6.5\times10^{-6}$ N/m. If a ferromagnetic sphere with radius, $R = 15$ nm, is placed at the distance $d=5$ nm from the paramagnetic atom, the magneto-static attractive force between the ferromagnetic particle and the paramagnetic atom can be estimated to be

$$F = \mu_B\left|\frac{\partial B_z}{\partial z}\right| = 2\mu_B\frac{\mu_0 M}{R+d}\left(\frac{R}{R+d}\right)^2 \approx 3.9\times10^{-16}\ \text{N}. \tag{5}$$

The corresponding displacement of the cantilever tip is

$$z_c = F/k_c \approx 6\times10^{-11}\ \text{m}. \tag{6}$$

Next, we compare this value with the thermal vibrations of the cantilever. The root-mean-square vibration amplitude at temperature, T, can be estimated as

$$z_{rms} \approx (k_B T/k_c)^{1/2}.$$

This is smaller than the displacement of the cantilever, z_c, in Eq. (6) at temperatures

$$T < F^2/k_B k_c \approx 1.7\ \text{mK}. \tag{7}$$

When estimating z_{rms} we assume that the bandwidth of the measuring device, ω_b, is larger than the cantilever frequency, ω_1, as the noise spectral density has a maximum at $\omega=\omega_1$.

A more serious assumption is that the system is in thermodynamic equilibrium. This assumes that we neglect slow relaxation processes that cause $1/f$ noise [13]. Generally, $1/f$ noise originated, for example, from the tip-sample electrostatic interaction can be more important than the thermodynamic noise considered above. To reduce noise, we can consider the opportunity of decreasing the spectral bandwidth, ω_b, of the measuring device. The price of reducing ω_b is the increase in measurement time. If $\omega_1/Q \ll \omega_b \ll \omega_1$ (where Q is the quality factor of the cantilever) one cannot observe oscillations of the cantilever tip near its equilibrium position. In this case, one can observe only the relaxation of the cantilever to its equilibrium position.

The experimental setup proposed here could detect single-spin flips caused by the relaxation processes. We assume that the coercivity of the ferromagnetic particle is larger than 0.28 T, the value of the magnetic field, B_F, at the paramagnetic atom. We also assume that a single-spin relaxation time is much larger than the relaxation time of the cantilever, T_r. (For the cantilever reported in [12], the frequency is $\omega_1/2\pi = 1.7$ kHz, and the quality factor is 6700, so the relaxation time is $T_r \approx Q/\omega_1 \approx 0.63$ s. The electron spin relaxation time for paramagnetic impurities in a diamagnetic host can be of the order of 1 hour [15].)

Suppose that an experimenter reverses the direction of the external magnetic field, B_0. If $B_0 < B_F = 0.28$ T, then the directions of m_F and B_F do not change due to the coercivity of the ferromagnetic particle. Next, suppose that the total magnetic field at the paramagnetic atom, $B_F - B_0$, is reduced to the value, $2\mu_B(B_F-B_0) < k_B T$. (For $T=1$ mK the difference (B_F-B_0) must be less than or of the order of 200 μT.) In this case, the paramagnetic spin will randomly change its direction. The average time between jumps will determine the spin relaxation time. After each jump, the equilibrium position of the cantilever tip changes. Thus, each spin flip generates damped oscillations of the cantilever near the new equilibrium position. In this case, an experimenter might observe a sequence of short-time cantilever oscillations such as

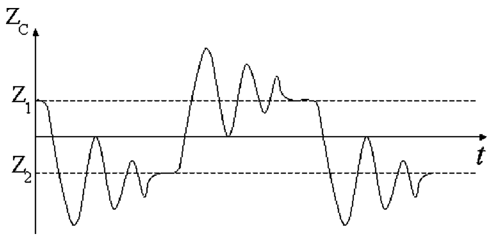

FIG. 3. Damped oscillations of a cantilever caused by random jumps of the paramagnetic spin. z_1 and z_2 are the equilibrium positions of the cantilever for the two directions of the spin.

that shown schematically in Fig. 3. If the bandwidth, ω_b, of the measuring device is less than $\omega_1(\omega_1/Q \ll \omega_b \ll \omega_1)$, one can observe a smooth change of the equilibrium position of the cantilever tip with characteristic time, $T_r \approx Q/\omega_1$ rather than the damped oscillations shown in Fig. 3. We should note that the experimental observation of the single-spin flips is possible only if the spectral density of this ''spin noise'' is greater than the spectral density of the $1/f$ noise.

III. ESTIMATION OF THE DECOHERENCE TIME

In this section we argue that at currently available temperatures, the cantilever acts on a single spin as a classical measuring device because the decoherence time of the cantilever is too small. Consider what will happen if an electron spin, e.g., accidentally, appears in the superposition of two stationary states. In this case, the ''average spin'' rotates in the x-y plane due to the different phase advance for the two stationary states. At the same time, the two stationary states of the electron spin in the inhomogeneous magnetic field correspond to two different equilibrium positions of the cantilever. If the effect of cantilever decoherence is absent, then after a time interval of the order of the relaxation time, T_r, the cantilever will appear in a static Schrödinger-cat state: the cantilever will be in two equilibrium positions simultaneously.

Our rough estimate of the decoherence time, T_d, for the cantilever Schrödinger-cat state is based on the uncertainty relation. Instead of the cantilever we consider a particle with an effective mass, $m = m_c/4$, attached to a spring which is initially in two equilibrium positions separated by the distance, Δz. We assume that a Schrödinger-cat wave function collapses when the diffusion in the momentum space $\overline{\delta p^2}(t)$, becomes close to the ''Schrödinger-cat momentum uncertainty'': $\Delta p^2 \sim (\hbar/\Delta z)^2$.

For a particle interacting with a thermostat, the characteristic fluctuation of energy, δE, during the characteristic time of the fluctuation can be estimated as $k_B T$. If the particle is in equilibrium with the thermostat, the average value of momentum is zero, so $\delta E = \delta p^2/2m$. Thus, $\delta p^2 \sim m k_B T$. The characteristic duration of the particle's fluctuations in the equilibrium position can be estimated as the relaxation time, T_r. The diffusion coefficient in the momentum space is: $D = \delta p^2/T_r \sim m k_B T/T_r$. This is the same expression that appears in the equation for the Wigner function, $W(z,p,t)$, in the model based on the interaction between a particle and thermal excitations of a quantum scalar field [16]. Assume that after the creation of a Schrödinger-cat state (at $t=0$), the diffusion, $\overline{\delta p^2}(t)$, can be estimated as

$$\overline{\delta p^2}(t) \sim Dt \sim \frac{m k_B T}{T_r} t. \tag{8}$$

Decoherence occurs when

$$\overline{\delta p^2}(t) \approx \Delta p^2 \sim (\hbar/\Delta z)^2. \tag{9}$$

It happens at time

$$t = T_d \sim T_r \frac{\hbar^2}{m k_B T \Delta z^2}. \tag{10}$$

This expression (within a factor of 2) also coincides with formula for decoherence time presented in [16]. Setting $\Delta z = 2z_c$, $m = m_c/4$, and using expressions (5) and (6), we present Eq. (10) in the form

$$T_d \sim \frac{T_r}{m_c k_B T} \left(\frac{\hbar k_c}{\mu_B \partial B_z/\partial z} \right)^2. \tag{11}$$

Setting $z_c = 6 \times 10^{-11}$ m, $m_c = 4k_c/\omega_1 = 6 \times 10^{-14}$ kg, $T = 1$ mK, we have the estimate: $T_d/T_r \sim 10^{-10}$. This value is very small due to the relatively large effective mass, m. According to this estimate, the static Schrödinger-cat state cannot appear in our system. An important fact is that the superpositional spin state (which could generate the Schrödinger-cat state of the cantilever) must also rapidly collapse.

Note that our example demonstrates the connection between the measurement process and decoherence. Namely, due to the decoherence the cantilever becomes a classical measuring device that destroys (collapses) the wave function of the spin, forcing the spin to occupy one of the two stationary spin states.

We should mention that this estimate (11) is derived under the assumption that the Schrödinger-cat state was initially created with a given value, Δz. If fact, that is not the case. The creation of the Schrödinger-cat state due to the interaction between the spin and the cantilever occurs simultaneously with the process of decoherence. For correct description of this phenomenon one must solve the master equation for the density matrix of the cantilever-spin system. These calculations are now in progress.

IV. APPLICATION TO QUANTUM COMPUTATION

Finally, we consider the possibility of using static single-spin measurements for quantum computation. Recently, we proposed a solid-state nuclear spin quantum computer based on (MRFM) [17,18]. In this proposal, a qubit is represented by the nuclear spin of a paramagnetic ion. A chain of paramagnetic ions is placed on the surface of a diamagnetic host. All electron spins are assumed to be in their ground states $(2\mu_B B_0 \gg k_B T)$. The system of nuclear spins can be in any superpositional state.

The important part of the proposal [17,18] is the measurement of a nuclear-spin state using MRFM and the hyperfine interaction between the electron spin and nuclear spin in a paramagnetic ion. To perform this measurement, a small ferromagnetic particle is placed on the tip of the cantilever. This particle targets a paramagnetic ion, changing the electron spin resonance (ESR) frequency of the ion. The ESR frequency depends on the state of the nuclear spin in the ion. Applying a periodic sequence of resonant electromagnetic pulses whose frequencies correspond to the ground state of the nuclear spin, one can generate electron spin transitions and the driven vibrations of the cantilever only if the nuclear spin of the targeted ion is in its ground state.

To measure a nuclear-spin state using an electron single-spin static measurement considered in this paper, one should execute the following steps:

(a) target a paramagnetic ion with the ferromagnetic particle and measure the equilibrium position of the cantilever.

(b) apply a resonant π-pulse which drives the electron spin of the selected ion into its excited state if its nuclear spin is in the ground state.

(c) measure the new equilibrium position of the cantilever. The change of the equilibrium position means that the electron spin changed its state. Consequently, the nuclear spin of the targeted ion is determined to be in its ground state.

(d) apply a second π-pulse that returns the electron spin to its ground state, if it was transferred to its excited state by the first π-pulse.

To test the feasibility of this proposal we should estimate the deviation of the ESR frequency due to the thermal vibrations of the cantilever. The deviation of the magnetic field at $T = 1.7$ mK can be estimated as

$$\Delta B = \left| \frac{\partial B_z}{\partial z} \right| z_{\rm rms} \approx 2.5 \times 10^{-3} \text{ T}.$$

The corresponding deviation of the ERS frequency is

$$\Delta f \approx (\gamma_e / 2\pi) \Delta B \approx 70 \text{ MHz}.$$

To provide the inversion of the electron spin, the electron Rabi frequency (the nutation frequency) of a π-pulse must be greater than 70 MHz. From the other hand, the electron Rabi frequency must be less than the hyperfine ESR splitting. For a single-ionized tellurium-125 ion, which has been proposed in [18] for implementation of quantum computation, the hyperfine ESR splitting is 3.5 GHz. Thus, both conditions can be easily satisfied.

At first sight, the static method of a single-spin measurement appears to be more cumbersome than the resonant approach based on MRFM [14]. It does not take advantage of the quality factor of the cantilever and can only be applied in the millikelvin region. However, if the temperature range is not a limiting factor, then the static measurement can be considered as an alternative opportunity. It requires only two electromagnetic pulses per measurement instead of the periodic sequence of pulses required by the MRFM method.

V. SUMMARY

In conclusion, we discussed the opportunity of the static Stern-Gerlach effect implementation using a magnetic force microscopy. We have shown that the detection of a single electron spin in a diamagnetic host could be achieved at millikelvin temperatures. We presented a simple estimate for the decoherence time of a cantilever. Experimental implementation of static detection of a single electron spin would allow direct observation of the single-spin jumps caused by relaxation processes, if the spectral density of the "spin noise" is greater than the spectral density of the $1/f$ noise. Also, it could provide single-qubit measurements required for quantum computation.

ACKNOWLEDGMENTS

This work was supported by the Department of Energy (DOE) under Contract No. W-7405-ENG-36, by the National Security Agency (NSA), and Advanced Research and Development Activity (ARDA).

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