Statistics 632: Homework #4
Due February 6, 2012

To be turned in for credit:

IV.1:
E8: “Suppose that the social classes of successive generations...” (4 pts)
P1: “Five balls are distributed between two urns, labeled A and B...” (4 pts)
P5: “The four towns, A, B, C, and D...” (4 pts)
P13: “A Markov chain has the transition probability matrix...After a long period of time...” (5 pts)

IV.2:
E6: “A component of a computer has an active life...” (4 pts)
P1: “Consider a discrete-time periodic review inventory model...” (5 pts)

Additional Problem 1 (to be turned in for credit) (4 pts):
Recall the “umbrella” problem from class: a person has $r$ umbrellas, allocated between home and the office. Every morning, if it is raining and there is at least one umbrella at home, the person takes an umbrella to the office. In the evening, if it is raining and there is at least one umbrella at the office, the person takes one home. Assume that it rains in the morning with probability $p$, and that it rains in the afternoon, independently of the morning, with probability $p$. We found the one-step transition probability matrix to be

\[
\begin{pmatrix}
0 & 1 & 2 & \cdots & r-1 & r \\
0 & 1 & 0 & 0 & \cdots & 0 & 1 \\
1 & 0 & 0 & 0 & \cdots & q & p \\
2 & 0 & 0 & 0 & \cdots & p & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
 q & p & 0 & 0 & \cdots & 0 & 0 \\
 p & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
q & p & 0 & 0 & \cdots & 0 & 0 \\
\end{pmatrix} = P
\]

where $q = 1 - p$. We also showed that $P$ is a regular matrix, implying the existence of a nonzero limiting distribution. Find this limiting distribution.

30 total points

Suggested exercises*:

IV.2:
P6: “Consider a computer system that fails on a given day with probability...”

*These questions will not be graded but solutions will be provided.