Question 1
Let $x \sim N(0, \Sigma)$ where $x = (x_1, \ldots, x_p)'$. Write down a Gibbs sampler for $x$ that samples each component of $x$ individually (i.e., each full conditional distribution is univariate). After you have the general form:

(a) Let $p = 3$ and

$$
\Sigma_1 = \begin{pmatrix}
3 & -1 & 3 \\
-1 & 1 & -3 \\
3 & -3 & 11
\end{pmatrix}.
$$

Obtain 1,000 exact samples of $x$ from the joint distribution (using R or another computing package) and make a scatterplot matrix. Run the Gibbs sampler for 1,000 iterations and provide trace plots for each of the components of $x$. Comment on how well the Gibbs sampler has explored the state space through 1,000 iterations.

(b) Now let

$$
\Sigma_2 = \begin{pmatrix}
3 & -0.2 & 3 \\
-0.2 & 1 & -3 \\
3 & -3 & 11
\end{pmatrix},
$$

where the (marginal) covariance between $x_1$ and $x_2$ has been reduced to $-0.2$, meaning that $x_1$ and $x_2$ are closer to being (marginally) independent. As in (a), obtain 1,000 exact samples of $x$ from the joint distribution and make a scatterplot matrix. Comment on any similarities or differences with the joint distribution in (a). Run the Gibbs sampler for 1,000 iterations and provide trace plots for each of the components on $x$. Comment on how well the Gibbs sampler has explored the state space through 1,000 iterations.

(c) Why does the Gibbs chain for (a) move more freely than the chain for (b)? Be somewhat specific in your answer.