Physics 7501: Homework Set No. 2

Due date: Tuesday, September 8, 2015, 5:00pm in PRB 3018 (Fuyan Lu's office)

Total point value of set: 90 points

Problem 1 (5 pts.): Exercise 1.9.2 (Shankar p.55)

Problem 2 (5 pts.): Exercise 1.9.3 (Shankar p.55)

Problem 3 (5 pts.): Exercise 1.10.2 (Shankar p.63)

Problem 4 (5 pts.): Exercise 1.10.3 (Shankar p.63)

Problem 5 (30 pts.): For $\hat{H}$ Hermitean, $\hat{U}, \hat{V}$ unitary, show (each item is worth 5 points):

(i) $\hat{U}\hat{H}\hat{U}^{-1}$ is Hermitean

(ii) $(\hat{U})^n$ is unitary

(iii) $\hat{U}\hat{V}$ is unitary

(iv) If $f(x)$ is real and has no poles, $f(\hat{H})$ is Hermitean

(v) $\mathrm{i}\dfrac{\hat{U}-\hat{I}}{\hat{U}+\hat{I}}$ is Hermitean

(vi) $\dfrac{\hat{I}-\mathrm{i}\hat{H}}{\hat{I}+\mathrm{i}\hat{H}}$ is unitary

Problem 6 (20 pts.): Show that $e^{\hat{L}}\hat{M}e^{-\hat{L}} = \sum_{n=0}^{\infty} \frac{1}{n!}[\hat{L},\hat{M}]_{(n)}$ where $[\hat{L},\hat{M}]_{(0)} = \hat{M}$, $[\hat{L},\hat{M}]_{(n)} = [\hat{L},[\hat{L},\hat{M}]_{(n-1)}]$.
(We will need this later for $\hat{L} = i\hat{H}t$ where $\hat{H}$ is the Hamiltonian.)
Hint: Consider $\hat{F}(\alpha) = e^{\alpha\hat{L}}\hat{M}e^{-\alpha\hat{L}}$ and expand it into a Taylor series around the origin.

Problem 7 (20 pts.): Show that $e^{\hat{L}+\hat{M}} = e^{\hat{L}}e^{\hat{M}}e^{-\frac{1}{2}[\hat{L},\hat{M}]}$ if $[\hat{L},[\hat{L},\hat{M}]] = 0 = [\hat{M},[\hat{L},\hat{M}]]$.
Hint: Consider $\hat{F}(\alpha) = e^{\alpha(\hat{L}+\hat{M})}$. Show that $\hat{F}\hat{L} = \hat{L}\hat{F} - \alpha[\hat{L},\hat{M}]\hat{F}$ (see problem 6). Use this to bring $\frac{d\hat{F}}{d\alpha}$ into a form that you can integrate to give, for $\alpha = 1$, the r.h.s. of the relation you want to prove. Fill in the rest.