

# Physics 7501: Homework Set No. 4

**Due date: Tuesday, September 22, 2015, 5:00pm  
in PRB 3018 (Fuyan Lu's office)**

**Total point value of set: 100 points**

**Problem 1 (30 pts.):** For a function  $f(x) = \langle x|f\rangle$ ,  $x$  real, with  $\langle f|f\rangle = 1$ , compute the following matrix elements (5 pts. for each one of them) of the position operator  $\hat{X}$  and momentum operator  $\hat{P} = \hbar\hat{K}$  in the eigenbases of  $\hat{X}$  and  $\hat{K}$ , respectively:

$$\langle k|\hat{X}^2|f\rangle; \quad \langle x|\hat{X}^2|f\rangle; \quad \langle k|\hat{P}^2|f\rangle; \quad \langle x|\hat{P}^2|f\rangle; \quad \langle k|\frac{1}{2}(\hat{X}\hat{P} + \hat{P}\hat{X})|f\rangle; \quad \text{and} \quad \langle x|\frac{1}{2}(\hat{X}\hat{P} + \hat{P}\hat{X})|f\rangle.$$

**Problem 2 (20 pts.):** Exercise 4.2.1 in Shankar (p.129). The answers are given in Shankar, but you will be graded on the correctness of each step in your work leading to the answers.

**Problem 3 (10 pts.):** Exercise 4.2.2 in Shankar (p.139).

**Problem 4 (10 pts.):** Exercise 4.2.3 in Shankar (p.139).

**Problem 5 (30 pts. total, 5 pts. for each subquestion):**

- (a) Write down the  $x$ -components of the eigenvectors  $|p\rangle$  of the momentum operator  $\hat{P} = \hbar\hat{K}$ .
- (b) Compute the  $x$ -components of the Hilbert space vector obtained by adding all such vectors corresponding to eigenvalues between  $p$  and  $p + \Delta p$  and dividing the result by  $\sqrt{\Delta p}$ .
- (c) Divide the result (b) once more by  $\sqrt{\Delta p}$  and take the limit  $\Delta p \rightarrow 0$  — what do you get?
- (d) What is the norm of the vector obtained in (b)?
- (e) Taking two such vectors, one obtained as in (b), the other by performing an analogous “average” of  $\hat{P}$ -eigenvectors over the interval  $[p', p' + \Delta p]$  where  $|p - p'| = \Delta p$ , compute their inner product.
- (f) Give a physical interpretation of your result.