

Physics 7502: Homework Set No. 5

Due date: Tuesday, Feb. 16, 2016, 5:00pm
in PRB M2039 (Bowen Shi's office)

Total point value of set: 100 points

Problem 1 (20 pts.): Obtain a variational estimate for the energy of a nonrelativistic particle moving in 1 dimension in the potential $V(x) = \lambda x^4$, using the trial wave function $\psi(x; \alpha) = e^{-\alpha x^4}$, and compare it with the estimate based on a Gaussian trial function (see (16.1.5) in Shankar). (*Hint:* Use a suitable variable transformation to convert the expectation value integrals into Gaussian integrals that you can do analytically. You should find that the two upper limits for the ground state energy agree with each other roughly within 10%. Which of the two is the better estimate you have to find out for yourself.)

Problem 2 (10 pts.): Use the variational method to estimate the energy of the first excited state for a nonrelativistic particle moving in 1 dimension in the potential $V(x) = \lambda x^4$. What are the expected general features of the wave function for this state? Make a suitable 1-parameter ansatz for the trial wave function, inspired by the ansatz (16.1.3) in Shankar for the ground state, but ensuring that the first excited state is always orthogonal on the ground state. Compute and minimize the energy expectation value in your trial wave function. Compare with Eqs. (16.1.4/5) and interpret your result.

Problem 3 (10 pts.): Show the work needed to arrive at the right hand side of Eq. (16.1.15) in Shankar.

Problem 4 (10 pts.): Exercise 16.1.5 (Shankar, p. 435)

Problem 5 (10 pts.): Exercise 16.2.4 (Shankar, p. 445)

Problem 6 (10 pts.): Exercise 16.2.5 (Shankar, p. 449)

Problem 7 (10 pts.): Exercise 16.2.7 (Shankar, p. 449)

Problem 8 (20 pts.): Exercise 16.2.8 (Shankar, p. 449)