Physics 7502: Homework Set No. 8

Due date: Tuesday, March 22, 2016, 5:00pm in PRB M2039 (Bowen Shi's office)

Total point value of set: 80 points

Problem 1 (10 pts.): Exercise 18.2.3 (Shankar, p. 478)

Problem 2 (10 pts.): Exercise 18.2.5 (Shankar, p. 478)

Problem 3 (10 pts.): Exercise 18.2.6 (Shankar, p. 478)

Problem 4 (20 pts.): The forced harmonic oscillator is defined by the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 - Q(t)\hat{x} - P(t)\hat{p} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + f(t)\hat{a} + f^*(t)\hat{a}^\dagger,$$

where

$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi}\, f(\omega')\, e^{i\omega' t} \equiv -\sqrt{\frac{\hbar}{2m\omega}}Q(t) + i\sqrt{\frac{\hbar m\omega}{2}}P(t).$$

For this Hamiltonian, the exact expression for the transition probability is given by the Poisson distribution (see Merzbacher, Sect. 6.6)

$$P_{0\to n}(\infty) = \left|\langle n^0|\hat{U}_I(\infty, -\infty)|0^0\rangle\right|^2 = \frac{e^{-\langle n\rangle}\langle n\rangle^n}{n!}$$

with mean excitation number $\langle n\rangle = |f(\omega)/\hbar|^2$.

Compute this transition probability in first-order time-dependent perturbation theory and compare with the exact result. Compute also the total energy transferred to the oscillator by the interaction with the external force, both exactly (using the exact probability distribution quoted above) and in first-order perturbation theory. Explain the agreement.

Problem 5 (10 pts.): Using the result from first order perturbation theory for the time-dependent transition probability $P^{(1)}_{i\to f}(t)$ from state i to $f \neq i$ and conservation of probability, compute the probability $P_{i\to i}(t)$ that the system remains in the initial state. Compare this to the probability $P_{i\to i}(t)$ directly computed from time-dependent perturbation theory and show that the two results agree only if in the direct calculation you keep the second-order terms.

Problem 6 (20 pts.): Derive the unitary operator that transforms states between the Heisenberg and interaction pictures. What's its equation of motion? What is the correct transformation between operators in the Heisenberg and interaction pictures? Starting from the Heisenberg equation of motion for the operators in the Heisenberg picture, use this transformation to derive the equation of motion of operators in the interaction picture, without going through the Schrödinger picture.