

GPU group 2/3/2012

Average Reciprocal Distance (ARD)

Design matrix $X = (n \times p)$

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1p} \\ x_{21} & x_{22} & \dots & x_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{np} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$X = (x_i)_{i=1,2,\dots,n} \quad x_i \in \mathbb{R}^p \text{ (or } [0,1]^p \text{)}$$

$$\text{ARD}(X) = \left[\frac{1}{\binom{n}{2} \cdot (2^p - 1)} \sum_{j=1}^p \sum_{k=1}^{\binom{p}{j}} \sum_{x_k^* \in X_{k,j}} \left(\frac{j^{j/2}}{\sum_{x_i^* \in X_{k,j}} 1} \right)^p \right]^{1/p}$$

* That is, for each $j=1 \dots p$, there are $\binom{p}{j}$ choices of projecting X onto j dim

* This gives a total of $2^p - 1$ projections

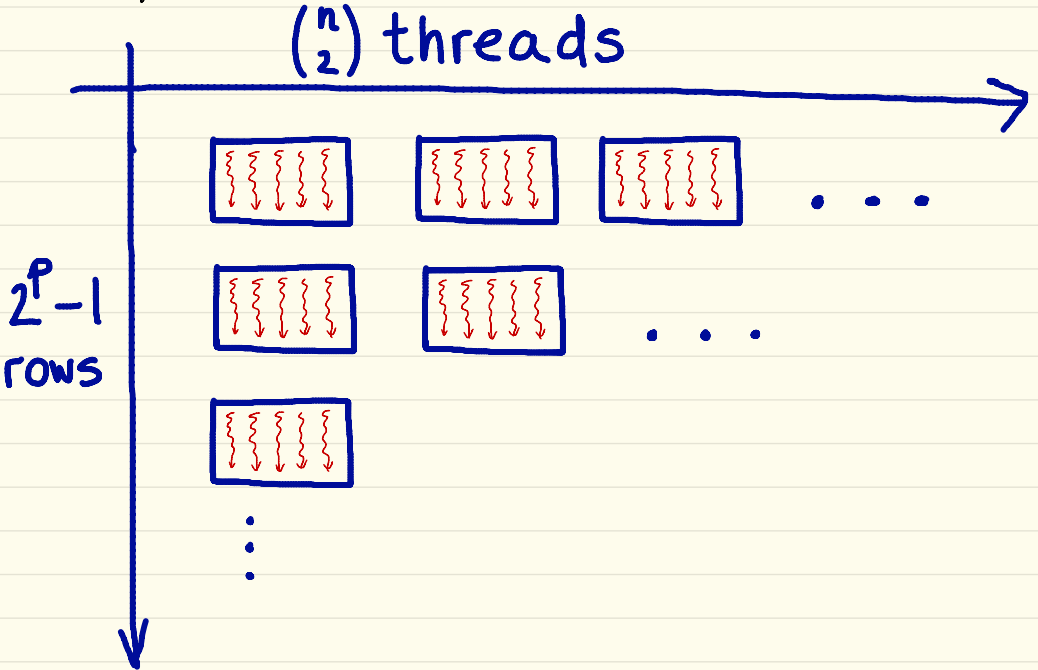
* For each projection there are $\binom{n}{2}$ pairwise distances to be computed between the rows of the projected design.

* take the average

Organizing work

✖ each thread will compute one distance $\rho_{\pm}(x_k^*, x_i^*)$ for a given $j \in \{1, \dots, p\}$ and $k=1, \dots, \binom{p}{j}$

Block grid



Each row corresponds to a (j, k) combo

Given $A = (\text{blockIdx.y} + 1)$ look at the binary representation of A (on p digits)

$$A = \underbrace{(0 \ 1 \ 0 \ 0 \ 1 \ \dots \ 0 \ 0 \ 1 \ 0)}$$

of 1s gives the dimension $j \in \{1, \dots, p\}$
location of 1s gives which of $\binom{p}{j}$ projection is selected