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Measurement of $\mathcal{B}(B^-\to D^0\pi^-)$ and $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ and Isospin Analysis of $B\to D\pi$ Decays

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Abstract

We present new measurements of branching fractions for the color-favored decays $B^-\to D^0\pi^-$ and $\overline{B}^0\to D^+\pi^-$. Using 9.67×10^6 $B\overline{B}$ pairs collected with the CLEO detector, we obtain the branching fractions $\mathcal{B}(B^-\to D^0\pi^-)=(49.7\pm1.2\pm2.9\pm2.2)\times10^{-4}$ and $\mathcal{B}(\overline{B}^0\to D^+\pi^-)=(26.8\pm1.2\pm2.4\pm1.2)\times10^{-4}$. The first error is statistical, the second is systematic, and the third is due to the experimental uncertainty on the production ratio of charged and neutral B mesons in $\Upsilon(4S)$ decays. These results, together with the current world average for the color-suppressed branching fraction $\mathcal{B}(\overline{B}^0\to D^0\pi^0)$, are used to determine the cosine of the strong phase difference δ_I between the $I=1/2$ and $I=3/2$ isospin amplitudes. We find $\cos\delta_I=0.863^{+0.024+0.036+0.038}_{-0.023-0.035-0.030}$, and obtain a 90% confidence interval of $16.5^\circ<\delta_I<38.1^\circ$. This non-zero value of δ_I suggests the presence of final state interactions in the $D\pi$ system.

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This paper presents the results of measurements of the branching fractions for $B^-\to D^0\pi^-$ and $\overline{B}^0\to D^+\pi^-$ and the extraction of the strong phase difference δ_I between the $I=1/2$ and $I=3/2$ isospin amplitudes in the $D\pi$ system. These decays are an excellent testing ground for the theoretical description of hadronic B-meson decays. Our understanding of these decays has improved considerably during the past few years with the development and application of Heavy Quark Effective Theory (HQET) [1, 2] and Soft Collinear Effective Theory (SCET) [3]. Originating from the simple, but very effective, idea of color-transparency [4], the factorization hypothesis has been put on a more solid basis, and in the case of $\overline{B}\to D\pi$, has been proven within the framework of SCET.

The recent observation [5, 6] of the color-suppressed $\overline{B}^0\to D^0\pi^0$ decay[1] completed the measurement of the $D\pi$ final states and was used to determine the cosine of the strong phase difference $\cos\delta_I = 0.89\pm0.08$, a value which is consistent with one. A value of $\cos\delta_I$ inconsistent with one would signal the presence of final-state interactions in the $\overline{B}\to D\pi$ process [7, 8]. In this paper, we present improved measurements of branching fractions for the color-favored decays $B^-\to D^0\pi^-$ and $\overline{B}^0\to D^+\pi^-$ based on a larger data set than that from which the previous results were obtained, as well as a new evaluation of $\cos\delta_I$ which takes into account the correlations among the various contributions to the overall systematic error.

This analysis uses e^+e^- annihilation data recorded with the CLEO detector at the Cornell Electron Storage Ring. The integrated luminosity of the data sample is 9.15 fb^{-1} collected on the $\Upsilon(4S)$ (on-resonance), corresponding to 9.67×10^6 $B\overline{B}$ pairs, and 4.35 fb^{-1} collected 60 MeV below the $B\overline{B}$ threshold (off-resonance), which is used for background studies. The results we present in this paper for $\mathcal{B}(B^-\to D^0\pi^-)$ and $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ supersede those in the CLEO publication, Ref. [9], which were based on a 1.3 fb^{-1} subset of the data used in the present analysis. Data were recorded with two detector configurations, CLEO II [10] and CLEO II.V [11]. Cylindrical drift chambers in a 1.5 T solenoidal magnetic field measure momentum and specific ionization (dE/dx) of charged particles. Photons are detected using a CsI(Tl) crystal electromagnetic calorimeter, consisting of a barrel-shaped central part of 6144 crystals and 1656 crystals in the forward regions of the detector (endcaps). In the

[1] Throughout this paper, charge conjugation is implied.

CLEO II.V configuration, the innermost tracking chamber was replaced by a three-layer, double-sided silicon microvertex detector, and the main drift chamber gas was changed from argon-ethane to a helium-propane mixture.

In our analysis, we impose quality requirements on charged particle tracks and improve the purity of pion and kaon used to reconstruct D mesons by using dE/dx information if the particle momentum is less than 800 MeV/c. The neutral D mesons are reconstructed using three decay modes: $K^-\pi^+$, $K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$. Charged D mesons are similarly reconstructed via the mode $K^-\pi^+\pi^+$. In each case, D meson candidates are required to have a mass within 3σ (standard deviations) of the PDG D mass [12] before kinematic fitting. Resolutions for the various D modes range from 6 to 12 MeV.

Each B meson candidate is reconstructed using the four-momentum of the mass-constrained D meson and an additional charged track in the event (assumed to be a pion). Candidates are then identified using the beam-constrained mass $M_B = \sqrt{E^2_{\rm beam} - p^2_B}$, where $E_{\rm beam}$ denotes the beam energy and p_B the candidate momentum, and the energy difference ΔE defined by $\Delta E \equiv E_D + E_\pi - E_{\rm beam}$, where E_D and E_π are the D meson and π energies, respectively. Preselection of B candidates requires $M_B > 5.24$ GeV/c^2 and ΔE to be between -50 and 50 MeV. Additionally, we calculate the sphericity vectors [13] of the B daughter particles and of the rest of the event. We require the absolute value of the cosine of the angle between these two vectors to be less than 0.8. The distribution of this angle is strongly peaked at ± 1 for continuum background and is nearly flat for $B\overline{B}$ events. We also require events to satisfy $R_2 < 0.45$, where R_2 is the ratio of the second to zeroth Fox-Wolfram moments of the event [14]. Finally, for events with more than one B meson candidate, the candidate with the smallest $|\Delta E|$ is chosen.

To obtain event yields for $\overline{B}\to D\pi^-$ for each D meson decay mode, the M_B distribution of candidates surviving the above slection cuts are fit using a binned maximum likelihood fit. The function used is a Gaussian for the signal plus an empirical background function, $f(M_B) = AM_B\sqrt{1-(M_B/E_{\rm beam})^2}\exp a(1-(M_B/E_{\rm beam})^2)$, having a fixed $E_{\rm beam} = 5.29$ GeV. All other parameters in both background and signal functions are allowed to float in the fit. The fitted M_B distributions for each of the D meson decay modes are presented in Fig. 1.

A small, non-negligible background from the decay $\overline{B}\to DK^-$ contributes to the yields

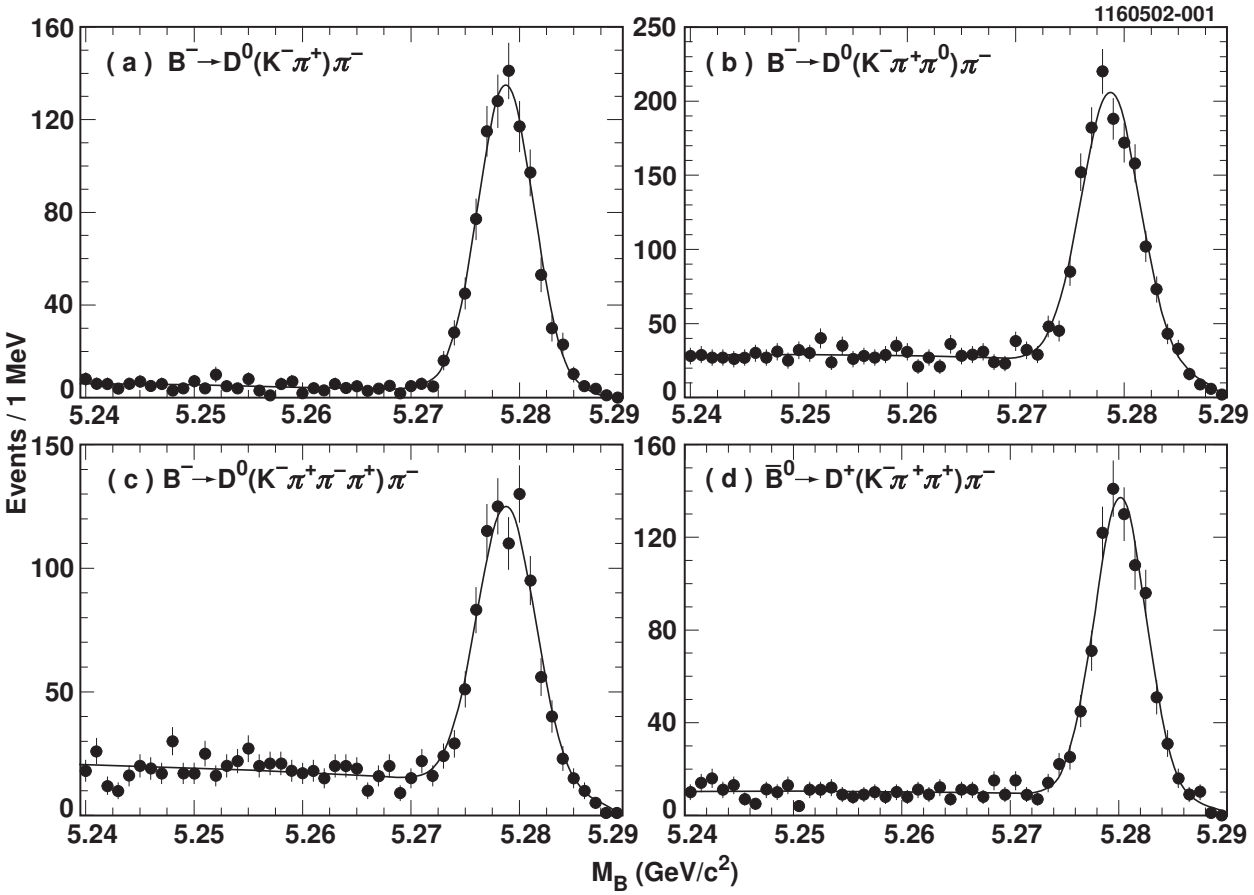

FIG. 1: Fitted M_B distributions in for a) $B^-\to D^0(K^-\pi^+)\pi^-$ b) $B^-\to D^0(K^-\pi^+\pi^0)\pi^-$ c) $B^-\to D^0(K^-\pi^+\pi^-\pi^+)\pi^-$ and d)$\overline{B}^0\to D^+(K^-\pi^+\pi^+)\pi^-$.

obtained by the fit procedure described above. We have, therefore, simulated this background via Monte Carlo to determine the fraction of feed-through to the $D\pi^-$ sample, and performed a subtraction using the average of the two measurements [15, 16] of $\mathcal{B}(B^-\to D^0K^-)/\mathcal{B}(B^-\to D^0\pi^-) = 0.071 \pm 0.009$ and the recent measurement [16] of $\mathcal{B}(\overline{B}^0\to D^+K^-)/\mathcal{B}(\overline{B}^0\to D^+\pi^-) = 0.068 \pm 0.017$. The amount of DK feed-through is found to be approximately $(4 \pm 1)\%$ of the $D\pi$ yield. We then reduce the event yields obtained in the fit to the data by this fraction.

Using efficiencies determined by applying the above method of analysis to samples of signal Monte Carlo events, we obtain the branching fractions for the processes under investigation

TABLE I: Results for the branching fractions $\mathcal{B}(B^-\to D^0\pi^-)$ and $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$. Fit yields with errors and efficiencies are obtained as described in the text. The errors given for the efficiencies correspond to the Monte Carlo statistical errors for each mode. The D mode branching fractions and the branching fraction errors have been taken from the PDG [12]. The errors reported for the measured B branching fraction are the statistical errors only. The current PDG average values for the two branching fractions have been included for comparison.

$B^-\to D^0\pi^-$				
D^0 Decay Mode	Yield	Efficiency (%)	D^0 mode $\mathcal{B}$(%)	$\mathcal{B}(B^-\to D^0\pi^-)(\times 10^{-3})$
$K^-\pi^+$	820 ± 31	45.4 ± 0.3	3.83 ± 0.09	4.90 ± 0.18
$K^-\pi^+\pi^0$	1200 ± 45	17.1 ± 0.2	13.9 ± 0.9	5.20 ± 0.19
$K^-\pi^+\pi^-\pi^+$	740 ± 33	20.9 ± 0.3	7.49 ± 0.31	4.91 ± 0.22
PDG				5.3 ± 0.5
$\overline{B}^0\to D^+\pi^-$				
D^+ Decay Mode	Yield	Efficiency (%)	D^+ mode $\mathcal{B}$(%)	$\mathcal{B}(\overline{B}^0\to D^+\pi^-)(\times 10^{-3})$
$K^-\pi^+\pi^+$	764 ± 33	32.8 ± 0.4	9.0 ± 0.6	2.68 ± 0.12
PDG				3.0 ± 0.4

from the event yields corrected for the DK feed-through:

$$\mathcal{B}(\overline{B}\to D\pi) = \frac{\text{Corrected Yield}}{\epsilon \times \mathcal{B}(D\to f.s.) \times N(\Upsilon(4S)) \times 2f}, \tag{1}$$

where f represents f_{+-} or f_{00}, the charged or neutral B meson production ratios at the $\Upsilon(4S)$, as appropriate. The corrected yields, efficiencies and final branching fraction obtained for each D decay mode are shown in Table I. We have assumed $f_{+-} = f_{00} = 0.5$.

The three $\overline{B}\to D\pi$ decay branching fractions (the two color-favored modes, which we report measurements for in the present paper, as well as the color-suppressed mode $\overline{B}^0\to D^0\pi^0$) form a complete set of branching fractions with which we may calculate $\cos\delta_I$, the cosine of the strong phase angle difference between the two isospin amplitudes $I = 1/2$ and $I = 3/2$ that contribute to the decay process. The expression for $\cos\delta_I$, following Ref. [7], is:

$$\cos\delta_I = \frac{3\Gamma(D^-\pi^+) + \Gamma(\overline{D}^0\pi^+) - 6\Gamma(\overline{D}^0\pi^0)}{4|\mathcal{A}_{1/2}\mathcal{A}_{3/2}|}, \tag{2}$$

where the isospin amplitudes $\mathcal{A}_{3/2}$ and $\mathcal{A}_{1/2}$ are given by

$$|\mathcal{A}_{3/2}|^2 = \Gamma(\overline{D}^0\pi^+), \qquad \text{and} \tag{3}$$

$$|\mathcal{A}_{1/2}|^2 = \frac{3}{2}\left(\Gamma(D^-\pi^+) + \Gamma(\overline{D}^0\pi^0)\right) \; -\tfrac{1}{2}\Gamma(\overline{D}^0\pi^+). \tag{4}$$

The calculation of $\cos\delta_I$ in the $D\pi$ system takes into account correlations of systematic errors between the two color-favored decay modes $B^-\to D^0\pi^-$ and $\overline{B}^0\to D^+\pi^-$. It also considers the fact that some of the systematic errors in the measurement of $\mathcal{B}(B^-\to D^0\pi^-)$ using the three D^0 decay modes are correlated. Further, apart from the errors on f_{00} and f_{+-} (which are anticorrelated), we treat the errors between the two color-favored $\overline{B}\to D\pi$ modes and the color-suppressed $\overline{B}^0\to D^0\pi^0$ mode as uncorrelated. This treatment is justified since the systematic error on the color-suppressed mode is dominated by the background parameterization and fit uncertainties, whereas such contributions are not dominant for the color-favored modes [5, 6].

We estimate the following systematic error contributions to our results for these measurements: 1% per track for track finding and fitting, 2% for the total number of $B\overline{B}$ pairs, 2% per track for which dE/dx is used, 2.5% for the cuts used in the analysis and 1% for the DK feed-through subtraction. Other systematic errors include 2% for π^0 finding in the case of the $D\to K^-\pi^+\pi^0$ submode, $2.3\% - 7\%$ for D-meson branching fractions, $2 - 3\%$ for background parameterization and fitting, and $0.7\% - 2\%$ for Monte Carlo statistics. The experimental errors of 4.5% on the individual quantities f_{+-} and f_{00} [17] are reported as a separate systematic error in our final result.

The overall systematic error for our measurement of $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ is obtained by standard error propagation of the individual contributions. However, in order to extract the correct overall systematic errors for $\mathcal{B}(B^-\to D^0\pi^-)$ and for $\cos\delta_I$, we must take into account the correlation among the systematic error contributions for each of the D submodes. To do this, we perform Monte Carlo experiments in which we vary the measured branching fractions by their various systematic errors. In each experiment and for each systematic error contribution, we generate multiplicative correction factors according to a Gaussian distribution. The combined $B^-\to D^0\pi^-$ branching fraction and $\cos\delta_I$ are then calculated from the values which have been varied as described above for each Monte Carlo experiment. From the complete ensemble of 2×10^6 Monte Carlo experiments, we obtain the probability

distribution functions and errors for $\mathcal{B}(B^-\to D^0\pi^-)$ and $\cos\delta_I$, which are shown in Fig. 2.

We thus obtain the following final results for the color-favored branching fractions.

$$\begin{aligned}\mathcal{B}(B^-\to D^0\pi^-) &= (49.7\pm 1.2\pm 2.9\pm 2.2)\times 10^{-4},\\ \mathcal{B}(\overline{B}^0\to D^+\pi^-) &= (26.8\pm 1.2\pm 2.4\pm 1.2)\times 10^{-4}\end{aligned}$$

In each measured quantity, the first error is statistical and the second is systematic. The third error is a separate systematic error which corresponds to the experimental uncertainty of the production fraction of charged (or neutral, as appropriate) B mesons in $\Upsilon(4S)$ decays.

Our results for $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ and for $\mathcal{B}(B^-\to D^0\pi^-)$ each reflect improvement with respect to the present PDG average values [12]. Our result for $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ may be directly compared with the prediction of Ref. [1] for this decay. Their prediction of 32.7×10^{-4} is marginally consistent with our result.

The largest contribution to the overall systematic error in our result for $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$ is the 6.6% relative systematic error due to the D branching fraction. We therefore report the following result which is independent of the $D^+\to K^-\pi^+\pi^+$ branching fraction: $\mathcal{B}(\overline{B}^0\to D^+\pi^-)\times\mathcal{B}(D^+\to K^-\pi^+\pi^+) = (2.41\pm 0.11\pm 0.15\pm 0.11)\times 10^{-4}$.

Using the CLEO measurement [5] of the color-suppressed branching fraction, $\mathcal{B}(\overline{B}^0\to D^0\pi^0) = 2.74^{+0.36}_{-0.32}\pm 0.55\times 10^{-4}$, and the PDG(2002) ratio of B lifetimes, $\tau(B^+)/\tau(B^0) = 1.083\pm 0.017$, we obtain

$$\cos\delta_I = 0.877\pm 0.030^{+0.046+0.039}_{-0.044-0.031}.$$

The error distributions derived from the ensemble of Monte Carlo experiments for $\cos\delta_I$ and δ_I are shown in Fig. 2. Integrating the δ_I distribution over the physical region $|\cos\delta_I|\le 1$, we obtain a 90% confidence interval:

$$13.6^\circ < \delta_I < 38.3^\circ.$$

Our final results for $\cos\delta_I$ and δ_I are based on the average of both measurements of $\mathcal{B}(\overline{B}^0\to D^0\pi^0) = 2.92\pm 0.45\times 10^{-4}$ [5, 6]. Using this average, we obtain

$$\cos\delta_I = 0.863^{+0.024+0.036+0.038}_{-0.023-0.035-0.030}.$$

Similarly, we obtain our final result for δ_I, a 90% confidence interval of

$$16.5^\circ < \delta_I < 38.1^\circ.$$

Using our results for $\mathcal{B}(B^-\to D^0\pi^-)$ and $\mathcal{B}(\overline{B}^0\to D^+\pi^-)$, we also calculate the ratio of the $I=1/2$ and $I=3/2$ isospin amplitudes, $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = 0.69\pm0.03\pm0.06\pm0.06$. In the heavy quark limit, $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = 1$.[2] Corrections to this are $\mathcal{O}(\Lambda_{\rm QCD}/m_c)$, which is consistent with our result.

In summary, we have measured the branching ratios for the color-favored $B\to D\pi$ decays, and used these measurements, together with the current average of measurements of $\mathcal{B}(\overline{B}^0\to D^0\pi^0)$, to determine the value of the cosine of the strong phase difference δ_I in the $D\pi$ system, and the ratio of $I=3/2$ and $I=1/2$ isospin amplitudes. Our result for $\cos\delta_I$ differs from one by approximately 2.3σ and thus suggests the presence of final-state interactions in $\overline{B}\to D\pi$ decays.

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[2] The ratio given here is based on the formalism of Ref.[7]. It is equivalent to $\mathcal{A}_{1/2}/(\sqrt{2}\mathcal{A}_{3/2})$ according to that of Ref. [8]

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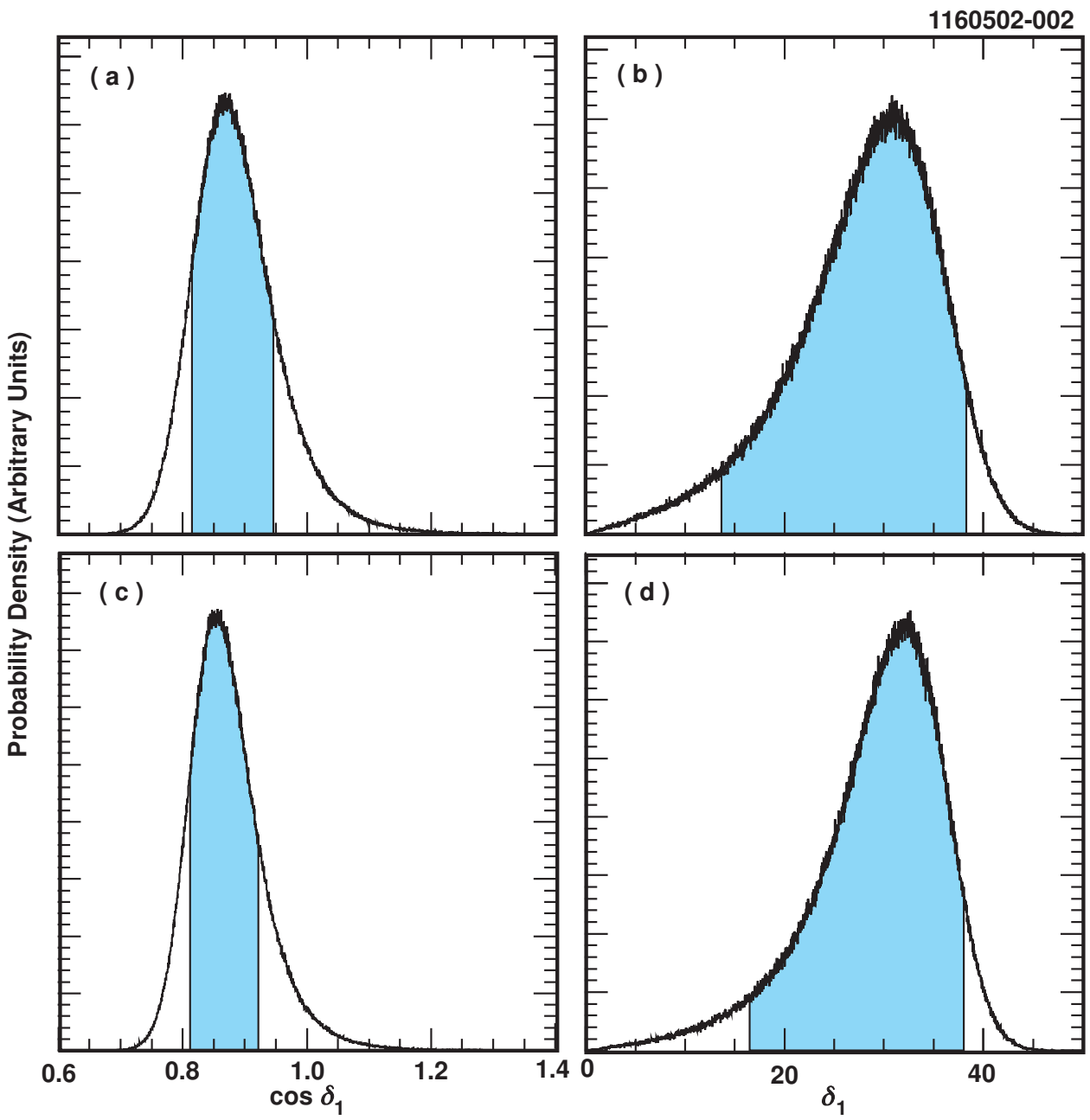

FIG. 2: The error distributions for $\cos\delta_I$ and δ_I obtained from the ensemble of 2×10^6 Monte Carlo experiments described in the text. The shaded area in the $\cos\delta_I$ plots is the $\pm1\sigma$ window (the 90% C.L. region in the δ_I plots). The upper two plots show the distributions for a) $\cos\delta_I$ and b) δ_I obtained using only the CLEO measurement of $\mathcal{B}(\overline{B}^0\to D^0\pi^0)$. The lower two plots are distributions for c) $\cos\delta_I$ and d) δ_I obtained using both CLEO and Belle measurements of $\mathcal{B}(\overline{B}^0\to D^0\pi^0)$.