Conservation of energy: \( \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (m_1 + m_2) v_e^2 + \frac{1}{2} k x^2 \) \( \text{(i)} \)

and \( v_2 = 0 \) because at the beginning the spring is uncompressed.

\[ x = \text{maximum compression of the spring} \]

From (i)

\[ x = \frac{1}{k} \sqrt{m_1 v_1^2 + m_2 v_2^2 - (m_1 + m_2) v_e^2} \] \( \text{(ii)} \)

Find \( v_e \) using conservation of total momentum

\[ p_0 = p_f \]

\[ m_1 u_1 + m_2 u_2 = (m_1 + m_2) v_e \]

\[ v_e = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \] \( \text{(iii)} \)

Plug (iii) into (ii)

\[ x = \frac{1}{k} \sqrt{m_1 u_1^2 + m_2 u_2^2 - (m_1 + m_2) \left( \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} \right)^2} \]

Plug in values given by the problem

\[ x = 0.25 \text{ m} \]
\[ v_{1s} = \text{velocity of ball right before collision} \]
\[ v_{1f} = \text{velocity of ball after} \]
\[ v_{2b} = \text{velocity of block before} \]
\[ v_{2f} = \text{velocity of block after} \]

- Find \( v_{1s} \) by using conservation of energy
  \[ \frac{1}{2} m_1 v_{1s}^2 = m_1 g h \]
  where \( h \) = height of the ball before it was released = 70.0 cm
  \[ v_{1s} = \sqrt{2gh} = \sqrt{2(9.8 \text{ m/s}^2)(0.7 \text{ m})} = 5.27 \text{ m/s} \]

- Use equation \((10-50)\) in book, valid for elastic collisions in one dimension, when one of the objects is initially at rest
  \[ v_{1f} = \frac{m_2 - m_1}{m_2} v_{1s} \quad (10-50) \]
  \[ v_{1f} = \frac{0.5 \text{ kg} - 2.5 \text{ kg}}{0.5 \text{ kg} + 2.5 \text{ kg}} = -2.47 \text{ m/s} \]

- Use equation \((10-51)\) to find \( v_{2f} \)
  \[ v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1s} \]
  \[ v_{2f} = \frac{2 \text{ kg}}{2 \text{ kg} + 0.5 \text{ kg}} = 1.23 \text{ m/s} \]
(a) Use conservation of momentum.

\[ v_f = \frac{m_1 - m_2}{m_1 + m_2} \]

(b) Use the problem statement: 

\[ v_f = \frac{v_{ic}}{2} \]

\[ m_1 \cdot v_{ic} + m_2 \cdot 5m \cdot v_{ic} = (m_1 + m_2) \cdot v_f \]

\[ m_0 = 1.2 \text{ kg} \]

For a closed, isolated system, the velocity of the center of mass is not affected by collisions; so we can use equation (1) with the velocities before the collision:

\[ v_{cm} = \frac{(2 \text{ kg})(4 \text{ m/s})}{(2 \text{ kg}) + (1.2 \text{ kg})} \]

\[ v_{cm} = 2.5 \text{ m/s} \]
before collision

\[ u_1 \]

\[ \rightarrow \]

\[ u_2 \]

after collision

\[ v_{1x} \]

\[ v_{2x} \]

\[ v_{1y} \]

\[ v_{2y} \]

Conservation of momentum in x-direction

\[ \mu_1 u_{1x} = \mu_2 u_{2x} \cos \theta_1 + \mu_2 u_{2x} \cos \theta_2 \]

Conservation of momentum in y-direction

\[ 0 = u_{1y} \sin \theta_1 - u_{2y} \sin \theta_2 \]

(a)

(b) \[ u_{2y} = \frac{u_{1x} \sin \theta_1}{\sin \theta_2} \]

(c) into (1)

\[ u_{1x} = u_{1x} \cos \theta_1 + \frac{u_{1y} \sin \theta_1}{\sin \theta_2} \cos \theta_2 \]

Solve for \( \theta_2 \)

\[ \theta_2 = \arctan \left( \frac{u_{1x} \sin \theta_1}{u_{1x} - u_{1x} \cos \theta_1} \right) \]

\[ \theta_2 \approx 30^\circ \]

Using (3)

\[ v_{1x} = 1.9 \text{ m/s} \]

[See figure above]

continued...
E_i = E_f
\frac{1}{2} m v_{i1}^2 = \frac{1}{2} m v_{f1}^2 + \frac{1}{2} m v_{f2}^2
v_{f1}^2 = v_{i1}^2 + v_{i2}^2
(2.2)^2 = (1.1)^2 + (1.905255...)^2

we verify in this way that energy was conserved.
\Rightarrow elastic collision
Chapter 11

Question 2

(1) The initial direction of rotation is clockwise. (-)
(2) The final direction of rotation is counterclockwise (+).
(3) The disk momentarily stops since there is a line when \( \omega = 0 \).
(4) The angular acceleration is the slope of the \( \omega \) vs. \( t \) line, which is positive.
(5) The slope of the line is constant \( \Rightarrow \) the angular acceleration is constant.

Chapter 11

Question 2

\[ \Sigma \tau = I \alpha \]
\[ m \cdot F \cdot \sin \phi = \frac{1}{2} Im \cdot \sin \phi \]

The greatest angular acceleration is going to be max when torque is max.

Torque is max when \( \sin \phi \) is max.

\( \sin \phi \) is greatest when \( \phi = 90^\circ \) and the same for \( \phi = 75^\circ \) and \( 115^\circ \).
5) We need an expression for \( \omega_0 \) already in part a.

\[
\omega_0 = \frac{\theta c}{(k t)^2} - \frac{1}{2} \alpha (k t) 
\]

\[
\omega_0 = \frac{\theta c}{(k t)^2} - \frac{1}{2} \frac{90}{(2 \times 3)^2} = 27 \text{ rad/s}
\]

4) How long was the wheel spinning before the 3.5 s interval?

Find expression for \( \omega_0 \)

\[
\theta c = \theta_0 + \omega_0 (k t) + \frac{1}{2} \alpha (k t)^2 
\]

\[
\omega_0 = \frac{\theta c}{(k t)^2} - \frac{1}{2} \alpha (k t)
\]

In terms of the starting time \( t_s \)

\[
\omega_s = \omega_0 + \alpha t_s 
\]

\[
\omega_s = \frac{\theta c}{(k t)^2} - \frac{1}{2} \alpha (k t) + \alpha t_s 
\]

\[
t_s = \frac{\theta c}{\alpha k} - \frac{1}{2} k t 
\]

\[
t_s = \frac{90}{(2 \times 3)} - \frac{3}{2} = 13.5 \text{ s}
\]
\( \omega_0 = 15 \text{ rad/s} + 0.239 \text{ rad/s} \)

\[
\omega = \omega_0 + \alpha t
\]

\[
\alpha = \frac{\omega^2 - \omega_0^2}{2(\theta)} = -7.12 \times 10^{-4} \text{ rad/s}^2 = -4.5 \times 10^{-5} \text{ rad/s}^2
\]

\[
t = \frac{\omega - \omega_0}{\alpha} = 335 \text{ sec to come to rest}
\]

\( a = \omega_0^2 + 2 \alpha \theta \)

\[
\alpha = \frac{-\omega_0^2}{2(\theta)} = -7.12 \times 10^{-4} \text{ rad/s} = -4.5 \times 10^{-5} \text{ rad/s}^2
\]

\( \beta_f = \beta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \)

\[
t = -\frac{\beta_1 - \beta_0 - 2\beta_0 x}{\alpha} = -0.239 \pm \sqrt{0.239^2 + 2(10)(-0.12 \times 10^{-2})}
\]
The eqn has two positive roots 
\[ t = 97.8 \text{ s} \] and \[ t = 5.72 \text{ s} \]

It should take longer for 40 revolutions than for 20, we throw out \[ t = 5.72 \text{ s} \]
since it is greater than 38.5.

\[ t = 97.8 \text{ s} \]

\[ r = 2.83 \text{ cm} \]
\[ \alpha = 14.2 \text{ rad/s}^2 \]
\[ \omega_0 = 0 \]
\[ \omega_f = 2760 \text{ rev/min} \]
\[ \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) \left( \frac{60 \text{ sec}}{\text{min}} \right) = 289 \text{ rad/sec} \]

\( a_t = \alpha r = (14.2 \text{ rad/s}^2)(2.83 \text{ cm}) = 40.2 \text{ cm/s}^2 \)

\( a_r = \omega_0^2 r = (2.89)^2 (2.83 \text{ cm}) = 23.6 \times 10^{-3} \text{ m/s}^2 \)

\( \theta = \omega t = \frac{2\pi}{289 \text{ rad/sec}} = 204 \times 10^{-3} \text{ rad} \)
The linear displacement is
\[ s = \theta = (0.0283)(2.94 \times 10^3) = 83.24 \text{ m} \]

The angular velocity of the pulsar
\[ \omega = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T} \]

So, the angular acceleration is
\[ \alpha = \frac{\Delta \omega}{\Delta t} = \frac{2\pi}{T^2} \]

From the problem,
\[ \frac{\Delta t}{t} = \frac{1.26 \times 10^5 \text{ s}}{3.16 \times 10^5 \text{ s}} = 0.40 \times 10^{-3} \]

Therefore,
\[ \alpha = \left( \frac{2\pi}{(0.0283)^2} \right) \left( 4.00 \times 10^{-3} \right) = 2.3 \times 10^9 \text{ rad/s}^2 \]

To find the time it will take to stop
\[ \omega_f = \omega_0 + \alpha \Delta t \]
where \( \omega_f = 0 \), \( \omega_0 = \frac{2\pi}{T} \)
\[ -\frac{2\pi}{\alpha} = -\frac{2\pi}{(2.3 \times 10^9)(0.3)} = 8.3 \times 10^{10} \text{ s} \]
\[ \frac{8.3 \times 10^{10}}{3.16 \times 10^8} \approx 2600 \text{ years} \]
The initial speed of the pulsar was
\[ v(0) = \omega_0 + \alpha \Delta t \]

The time is
\[ (2005 - 1954) \text{ years} = \frac{3.16 \times 10^7 \text{ s}}{1 \text{ year}} \]
\[ = 2.9 \times 10^7 \text{ s} \]

So
\[ \omega(t) = \alpha t = 2.9 \times 10^7 \text{ rad/s} \]

It's period was
\[ T = \frac{2\pi}{\omega} = \frac{2\pi}{2.9 \times 10^7} = \frac{6.28}{2.9 \times 10^7} \text{ s} \]