Problem 6

Ch 7  Question 5

(a) Recall the kinetic energy \( K = \frac{1}{2} mv^2 \)
the velocity \( v = \frac{dx}{dt} \) (the slope of the \( x-t \) curve)
Thus, at time \( t \),
\( V_A > V_B > V_C \) (speed)
and \( K_A > K_B > K_C \) (kinetic energy)

(b) At time \( t_1 \), the same criterion with (a), one gets
\( V_C > V_B > V_A \)
and \( K_C > K_B > K_A \)

(c) Recall the Work-Kinetic Energy Theorem.
\( W = \Delta K \)
Look at each \( x-t \) curve, one may find that

<table>
<thead>
<tr>
<th>Velocity at time ( t_1 )</th>
<th>Velocity at time ( t_2 )</th>
<th>Change of kinetic energy ( \Delta K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( V_{A_1} )</td>
<td>( V_{A_2} )</td>
</tr>
<tr>
<td>B</td>
<td>( V_{B_1} )</td>
<td>( V_{B_2} )</td>
</tr>
<tr>
<td>C</td>
<td>( V_{C_1} )</td>
<td>( V_{C_2} )</td>
</tr>
</tbody>
</table>
Therefore, the net work done on the three boxes should be

\[ W_c > W_a > W_h \]

(d) The sign of net work tells us how the energy is transferred.

A: \[ W_a < 0 \] ——— (2)

B: \[ W_a = 0 \] ——— (3)

C: \[ W_a > 0 \] ——— (1)

Ch. 7 Question 8

If one use \( W_f \) to denote the work done by "your force", 
\( W_g \) to denote the work done by "the gravitational force of the armadillo".

According to the Work – Kinetic Energy theorem,

\[ W_f + W_g = 0 \] (here we assume the armadillo is in rest at the beginning and the end of the process)

Where \( W_g = mg \cdot h \cos(180^\circ) = -mgh \) \( (h \) is the height) 

Thus \( W_f = mgh \). From here, one can conclude that the work done by "your force" \( W_f \)

depends on \( (i) \) \( (2) \) \( (3) \)

does not depend on \( (d) \) \( (e) \)
(a) Can think about the force diagram of the free massless pulley.

When speed is constant, the forces on the pulley should be in balance, according to Newton’s 2nd law,

\[2T - mg = 0\]

- the tension on the cord \(T = \frac{1}{2} mg\)
- the force on the free end of the cord \(F = T = \frac{1}{2} mg = \frac{1}{2} \times 20 \times 9.8 = 98 \text{ N}\)

(b) If the anister is lifted by 2.0 cm, two segments of the cord at the two sides of the free pulley should shorten by the same amount. Thus, the distance pulled at the free end of the cord should be \(2.0 \text{ cm} \times 2 = 4.0 \text{ cm} = 0.04 \text{ m}\)

(c) The work done by “your force”

\[W_F = F \cdot \vec{d} = F \cdot d \cos(0) = Fd = 98 \times 0.04 = 3.92 \text{ N}\]

(d) The work done by the gravitational force on the anister

\[W_g = mg \cdot d' \cos(180°) = -mg \cdot d' = -20 \times 9.8 \times 0.02 = -3.92 \text{ N}\]

where \(d\) — the distance moved at the free end of the cord
\(d'\) — the distance moved by the anister
(a) Here, we can make a unit conversion first.

\[ k = 2.5 \text{ N/cm} = \frac{2.5 \text{ N}}{0.01 \text{ m}} = 250 \text{ N/m} \]

\[ d = 12 \text{ cm} = 0.12 \text{ m} \]

\[ M = 250 \text{ g} = 0.25 \text{ kg} \]

The work done by the gravitational force is

\[ W_g = mgd \cos(0) = mgd = 0.25 \times 9.8 \times 0.12 = 0.29 \text{ J} \]

(b) The work done by the spring force is

\[ W_F = \int_0^{0.12} kx \cos(180^\circ) \, dx = -\int_0^{0.12} kx \, dx = -\frac{1}{2} kx^2 \bigg|_0^{0.12} \]

\[ = -\frac{1}{2} \times 250 \times 0.12^2 = -1.8 \text{ J} \]

(c) Denote \( v \) as the speed of the block just before it hits the spring. One can apply the Work – Kinetic Energy theorem to the block during the process from its just hitting the spring to "momentarily stopping".

\[ W_{\text{net}} = \Delta K \]

\[ W_F + W_g = 0 - \frac{1}{2} m v^2 \]

\[ -1.8 + 0.29 = -\frac{1}{2} \times 0.25 \times v^2 \]

\[ v = 3.48 \text{ m/s} \]
(d) If the speed is doubled \( V' = 2 \times 3.48 = 6.96 \text{ m/s} \)

Now, denote \( d' \) as the maximum distance of the compression of spring. The analogues of \( W_F, W_A \) are

\[
W_F' = -\frac{1}{2}kd'^2 \\
W_A = mgd'
\]

Still use Work - Kinetic Energy theorem

\[
W_F' + W_A = 0 - \frac{1}{2}mV'^2
\]

\[
mgd' - \frac{1}{2}kd'^2 = -\frac{1}{2}mV'^2
\]

\[
0.25 \times 9.8 \times d' - \frac{1}{2} \times 250 \times d'^2 = -\frac{1}{2} \times 0.25 \times 6.96^2
\]

\[
125d'^2 - 2.45d' - 6.05 = 0
\]

One only needs the positive solution, since \( d' > 0 \)

\[
d' = \frac{2.45 + \sqrt{2.45^2 + 4 \times 125 \times 6.05}}{2 	imes 125} = 0.23 \text{ m}
\]
Denote $W_g$ as the work done by the gravitational force of the total mass.

$W_m$ as the work done by the motor

$W_c$ as the work done by the counterweight

Apply the Work-Kinetic Energy theorem,

$$W_g + W_m + W_c = \Delta K = 0$$

Then

$$W_m = -W_g - W_c$$

And

$$W_g = -mgd = -1200 \times 9.8 \times 54 = -635040 \text{ J}$$

$$W_c = mgd = 950 \times 9.8 \times 54 = 502740 \text{ J}$$

$$W_m = 635040 - 502740 = 132300 \text{ J}$$

The average power of the motor is

$$P_m = \frac{W_m}{t} = \frac{132300}{3 \times 60} = 735 \text{ W}$$
For region "AB" and "BC", one can consider the conservation of mechanical energy. For region "CD", the frictional force will do work and cost the total mechanical energy.

<table>
<thead>
<tr>
<th>External Force (friction)</th>
<th>Total Mechanical Energy</th>
<th>Gravitational Potential Energy</th>
<th>Kinetic Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>NO</td>
<td>Constant</td>
<td>decreases</td>
</tr>
<tr>
<td>BC</td>
<td>NO</td>
<td>Constant</td>
<td>increases</td>
</tr>
<tr>
<td>CD</td>
<td>non-zero</td>
<td>decreases</td>
<td>constant</td>
</tr>
</tbody>
</table>
(a) From P to Q
\[ W_1 = mg \cdot (5R - R) = mg \cdot 4R = 4mgR \]

(b) From P to the top of the loop
\[ W_2 = mg \cdot (5R - 2R) = mg \cdot 3R = 3mgR \]

(c) At P
\[ U = mg \cdot (5R - 0) = 5mgR \]

(d) At Q
\[ U = mg \cdot (R - 0) = mgR \]

(e) At the top of the loop
\[ U = mg \cdot (2R - 0) = 2mgR \]

(f) If one looks at the formula to calculate the work done by the gravitational force and the potential energy, they don’t depend on the speed. Thus, the result of (a) through (e) remain the same.
(a) If denote "h" as the height achieved by the truck when it momentarily stops on the ramp, the mechanical energy is conserved since the ramp is frictionless.

\[ \frac{1}{2} m v^2 = m g h \]

\[ h = \frac{v^2}{2g} \]

The minimum length "L" has the relation with "h".

\[ \frac{h}{L} = \sin 15^\circ \]

\[ L = \frac{h}{\sin 15^\circ} = \frac{v^2}{2g \sin 15^\circ} = \frac{36.11^2}{2 \times 9.8 \times \sin 15^\circ} = 237 \text{ m} \]

(Notice \( v = 130 \text{ km/h} = 36.11 \text{ m/s} \))

(b) Since \( L = \frac{v^2}{2g \sin 15^\circ} \) does not depend on the truck's mass, "L" remains the same with the decrease of mass.

(C) The "L" will decrease as the speed "v" decreases.
(a) The elastic potential energy
\[ U = \frac{1}{2} k x^2 = \frac{1}{2} \times 1960 \times 0.2^2 = 39.2 \text{ J} \]

(Note \( k = 19.6 \text{ N/m} = \frac{19.6 \text{ N}}{0.01 \text{ m}} = 1960 \text{ N/m} \))

(b) Consider the conservation of mechanical energy.
\[ \Delta K + \Delta U_E + \Delta U_G = 0 \]
\[ \Delta K \quad \text{change of kinetic energy} \]
\[ \Delta U_E \quad \text{change of elastic potential energy} \]
\[ \Delta U_G \quad \text{change of gravitational potential energy} \]

\[ \Delta K = 0 \]
\[ \Delta U_E = 0 - 39.2 = -39.2 \text{ J} \]
\[ \Delta U_G = -\Delta K - \Delta U_E = 39.2 \text{ J} \]

(c) If "h" is the height achieved by the block
\[ \Delta U_G = m g h = 39.2 \text{ J} \]

\[ h = \frac{39.2}{m g} = \frac{39.2}{2 \times 9.8} = 2 \text{ m} \]

\[ L \sin 30^\circ = h \]
\[ L = \frac{h}{\sin 30^\circ} = \frac{2}{0.5} = 4 \text{ m} \]
(a) Suppose Tarzan could move to the bottom, check the force on the vine.

At the bottom, the speed is \( v \). Consider the conservation of mechanical energy.

\[ mgh = \frac{1}{2} m v^2 \]

\[ \therefore m v^2 = 2 mgh \]

Look at the force diagram at this moment.

\[ T - mg = \frac{m v^2}{r} \]

\[ \therefore T = mg + \frac{mv^2}{r} = mg + \frac{2mh^2}{r} \]

\[ = mg \left( 1 + 2 \frac{h}{g} \right) = 688 \times \left( 1 + 2 \times \frac{3.2}{9.8} \right) \]

\[ = 932.63 \text{ N} \]

\[ T < 950 \text{ N}, \text{ the vine will not break} \]

(b) Tarzan will have largest speed at bottom, since he has the maximum kinetic energy at this moment. And the force achieves the greatest value at this time, i.e. 932.63 N
If the gravitational potential of the skier at the ground is taken to be zero.

The mechanical energy at the beginning is
\[ mgh + \frac{1}{2}mv^2 = 60 \times 14 \times 9.8 + \frac{1}{2} \times 60 \times 2.4^2 = 2551.2 \, J \]

The mechanical energy at the end is
\[ \frac{1}{2}mv^2 = \frac{1}{2} \times 60 \times 22^2 = 14520 \, J \]

\[ \therefore \text{the reduced mechanical energy is} \]
\[ 2551.2 - 14520 = -11968 \, J \]
The frictional force should be \( f = mg \mu \).

The work done by the frictional force is equal to the change of mechanical energy:

\[
\Delta E_{me} = \frac{1}{2} m v_f^2 - mgh
\]

\[
f \cdot d = mg \mu \cdot d = \frac{1}{2} m v_f^2 - mgh
\]

\[
d = \frac{\frac{1}{2} v_f^2 - gh}{g \mu} = \frac{v_f^2}{2g \mu} - \frac{h}{\mu} = \frac{6^2}{2 \times 9.8 \times 0.6} - \frac{1.1}{0.6}
\]

\[
= 1.2 \text{ m}
\]