Gravitation

We have already talked about the force of gravity:

\[ mg \]

Where:
- \( m \) is the mass of the person
- \( g \) is a constant

The main assumption needed to use the above expression for the force of gravity is that the person needed to be very close to the surface of the earth.

However, the force of gravity acts beyond just the surface of the earth. The force of gravity is responsible for:
- Keeping the moon in orbit around the earth
- Keeping the earth in orbit around the sun
- Keeping the solar system as part of the galaxy
- etc

Gravity acts between all things with mass.

Newton’s Law of Universal Gravitation

The gravitational force between any two bodies of mass \( m_1 \) and \( m_2 \), separated by a distance \( r \), is given by:

\[ F_G = \frac{Gm_1 m_2}{r^2} \]

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 \]

Note:
1) Object \( m_1 \) attracts object \( m_2 \) with force \( F_{12} \)
2) This force is attractive, and is directed towards \( m_1 \) \[ |F_{12}| = |F_{21}| = F_G \]
3) Object \( m_2 \) attracts object \( m_1 \) with force \( F_{21} \)
4) This force is also attractive and directed towards \( m_2 \)
Gravity Near the Surface of the Earth

The force of gravity (due to the earth) on an object with mass \( m \) very near the earth’s surface:

\[
F = \frac{Gm_\text{E}m}{r_\text{E}^2} = \left( \frac{Gm_\text{E}}{r_\text{E}^2} \right) m = mg
\]

\[
\left( \frac{Gm_\text{E}}{r_\text{E}^2} \right) \equiv a_g = 9.83 \text{m/s}^2
\]

If the person always stays very close to the surface of the earth, then \( r_\text{E} \) can be approximated as a constant.

Interesting things about Gravity

\[
F_G = \frac{Gm_1m_2}{r^2}
\]

- \( G \) is a measure of the “strength” of gravity. It is very very small.
- So the force of gravity will be large only when at least one of the two masses in the above expression are large.
- But the force of gravity has unlimited range. \( F \) becomes zero only in the limit that \( r \) goes to infinity.
- This last statement implies that you are currently exerting a (small) force on galaxies far, far away.
- The above expression is only strictly true for “pointlike” particles. How is it that we can use it for “extended” objects like the earth, a car, ourselves…?
**Principle of Superposition**

Principle of superposition: The net effect is the sum of individual effects.

The total gravitational force on particle \( m_1 \) due to particles \( m_2, m_3, \) and \( m_4, \) is the vector sum of the gravitational forces:

\[
\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}
\]

Or more generally for “n” particles:

\[
\vec{F}_1 = \sum_{i=2}^{n} \vec{F}_{i1}
\]

If we want to know the force that a real object (made up of a continuous distribution of particles) exerts on another particle of mass \( m, \) we need to replace the above discrete sum by a continuous integral:

\[
F_1 = \int dF = \int \frac{Gm_1}{r^2} dm
\]

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**Example: Force on a Particle due to a Ring:**

A particle of mass \( m_1 \) is located a distance \( z \) away from a thin ring of mass \( m_2, \) along its axis. The radius of the ring is \( R. \)

Determine the magnitude and direction of the gravitational force of the ring, on the particle \( m_1. \)
Example: Force on a Particle due to a Ring:

First consider a small element \( dm_2 \) of the ring.

\[
\text{d}F_{Gz} = \left( \frac{G m_1 dm_2}{r^2} \right) \cos \theta
\]

This is the \( z \)-component of the force of gravity due to mass element \( dm_2 \).

Notice that the component of the force of gravity \textit{perpendicular} to this will be canceled by a mass element on the opposite side of the ring.

The \( z \)-component of the force of gravity exerted on mass \( m_1 \) by mass element \( dm_2 \) is:

\[
dF_{Gz} = \left( \frac{G m_1 dm_2}{r^2} \right) \cos \theta
\]

Since:

\[
r^2 = z^2 + R^2
\]

and

\[
\cos \theta = \frac{z}{r}
\]

then

\[
dF_{Gz} = \frac{G m_1 dm_2}{r^2} \cos \theta = \frac{z G m_1 dm_2}{\left( z^2 + R^2 \right)^{3/2}}
\]

The force of gravity exerted on mass \( m_1 \) by the whole ring is then:

\[
F_{Gz} = \int dF_{Gz} = \frac{z G m_1 m_2}{\left( z^2 + R^2 \right)^{3/2}}
\]
Ex: Force on mass due to a thin Ring

NOTE:
1) If the mass $m_1$ is located at $z=0$, the gravitational force goes to zero.
2) Newton showed using similar methods that a uniform spherical shell of mass $m$ attracts particles outside of the shell as if all of the shell’s mass were located at its center.
   The same can be demonstrated for a uniform sphere.
3) If a particle is located anywhere inside a uniform shell, the net gravitational attraction on it is zero.

Example: Gravitational Attraction between Two Spheres

Both spheres have mass $m=100$kg, and are separated by a distance of 1.00m.

$$F_G = \frac{Gm_1m_2}{r^2} = \frac{(6.67 \times 10^{-11})(100)(100)}{(1.00)^2}$$

$$F_G = 6.67 \times 10^{-7} \text{N}$$

This is small. About the weight of a baby flea. Will not pull a lineman offside.
Example: Gravitational Attraction between The Earth and the Moon

\[ F_G = \frac{G m_e m_m}{r^2} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})(5.98 \times 10^{24})}{(384 \times 10^3)^2} \]

\[ F_G = 1.99 \times 10^{20} \text{N} \]

This is big. Equivalent to the thrust of 6 million Saturn 5 moon rockets.